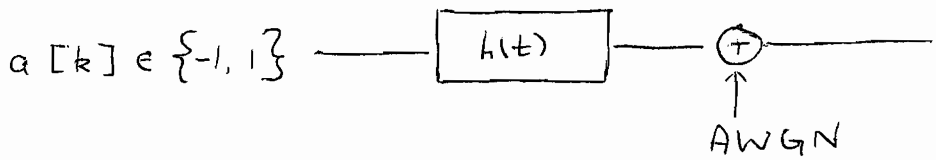
ECE-542
Spring 05

Equalization



Assuming baseband BPSK

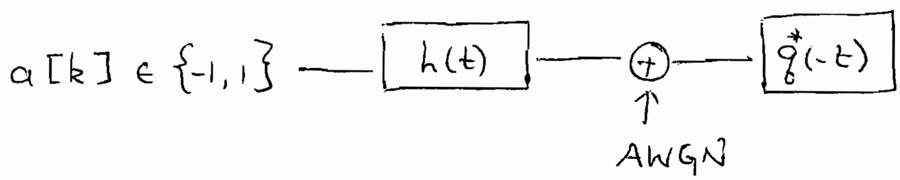
$$x_{PAM}(t) = \sum_{k=-\infty}^{\infty} a[k]\, p(t-kT)$$

Received baseband wave form

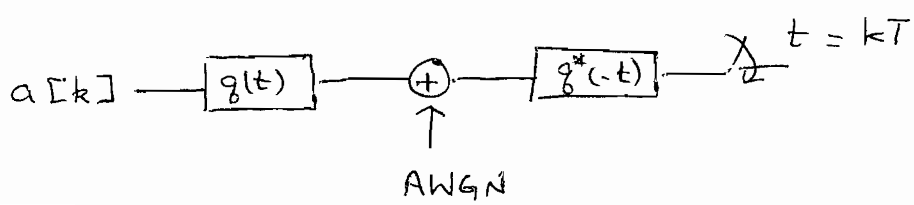
$$r(t) = \left\{ \sum_{k=-\infty}^{\infty} a[k]\, p(t-kT) \right\} * h(t)$$

$$r(t) = \sum_{k=-\infty}^{\infty} a[k]\, g(t-kT), \quad \text{where} \quad g(t) = p(t) * h(t)$$

With Matched Filtering



With Sampled Matched Filtering



For Signal component

$$a[k] \longrightarrow \boxed{q(t) * q^*(-t)} \longrightarrow \;\; t = kT$$

For noise component

$$N_o \longrightarrow \boxed{q^*(-t)} \longrightarrow \;\; t = kT$$

$$\mathcal{F}\left\{ q(t) * q^*(-t) \right\} = |Q(j\Omega)|^2$$

$$\mathcal{F}\left\{ (q(t) * q^*(-t)) \Big|_{t=kT} \right\}$$

$$= \frac{1}{T} \sum_{p=-\infty}^{\infty} \left| Q\left(\frac{\omega - 2k\pi}{T} \right) \right|^2 = S_h(e^{j\omega})$$

$$|\omega| \leq \pi$$

PSD of noise at SMF output

$$N_o \frac{1}{T} \sum_{p=-\infty}^{\infty} \left| Q\left(\frac{\omega - 2k\pi}{T} \right) \right|^2$$

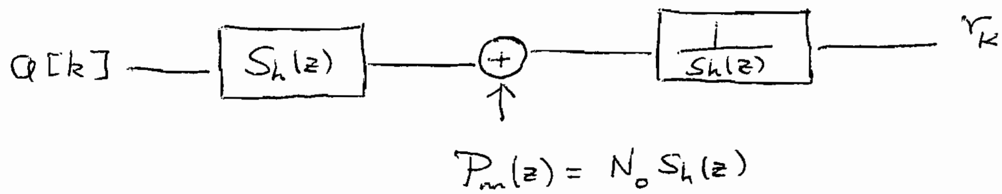
Equivalent discrete model

$$a_k \longrightarrow \boxed{S_h(z)} \longrightarrow \oplus \longrightarrow r_k$$

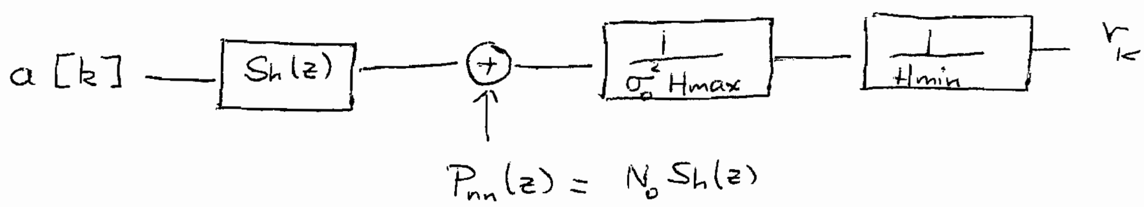
$$\uparrow$$

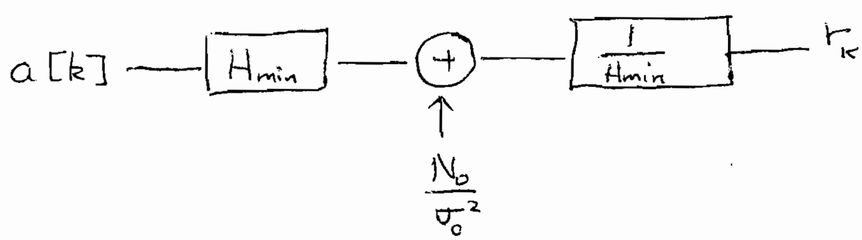
$$P_{ww}(z) = N_o S_h(z)$$

Zero forcing Equalizer



WSMF model





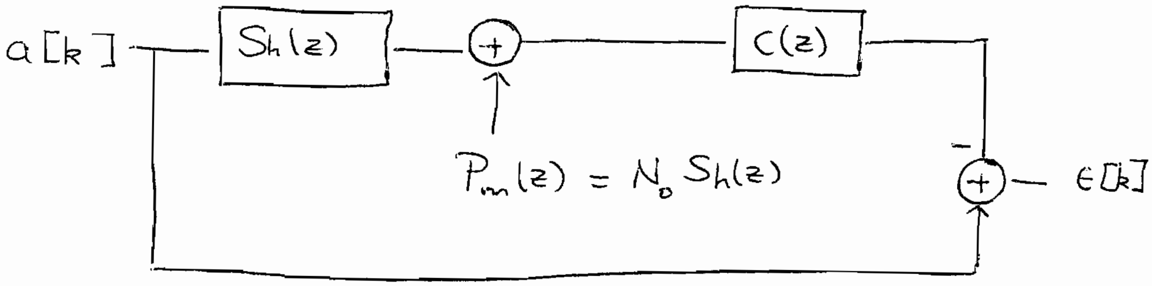
In this case ISI is cancelled but we are inverse filtering noise

$$P_{ww}(z) = \frac{N_0}{\sigma_0^2} \frac{1}{H_{min}(z)} \frac{1}{H^*_{min}(1/z^*)} = \frac{N_0}{S_h(z)}$$

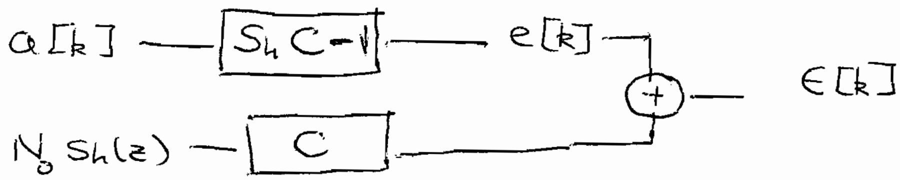
Error incured :

$$\sigma_\epsilon^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{N_0}{S_h(e^{j\omega})} d\omega = N_0 A \left\{ \frac{1}{S_h(z)} \right\}$$

MMSE Equalizer



Error Analysis:



$$P_{\epsilon\epsilon}(e^{j\omega}) = \sigma_a^2 \, |S_h C - 1|^2 + N_0 S_h |C|^2$$

Choose $C \ni P_{\epsilon\epsilon}(e^{j\omega})$ is smallest

$$P_{\epsilon\epsilon}(e^{j\omega}) = \sigma_a^2 \left(|S_h C|^2 + 1 - 2\,\mathrm{re}\{S_h C\}\right) + N_0 S_h |C|^2$$

$$P_{\epsilon\epsilon}(e^{j\omega}) = S_h |C|^2 (N_0 + \sigma_a^2 S_h) + \sigma_a^2 - \sigma_a^2 \, 2\,\mathrm{Re}\{S_h C\}$$

$$P_{\epsilon\epsilon}(e^{j\omega}) = S_h (N_0 + \sigma_a^2 S_h)\left\{|C|^2 - \frac{2\sigma_a^2\,\mathrm{Re}\{\cancel{S_h} C\}}{(N_0 + \sigma_a^2 S_h)\cancel{S_h}}\right\} + \sigma_a^2$$

$$P_{\epsilon\epsilon}(e^{j\omega}) = S_h(N_0 + \sigma_a^2 S_h)\left(\left|C - \frac{\sigma_a^2}{\sigma_a^2 S_h + N_0}\right|^2\right)$$

$$- S_h(N_0 \cancel{+ \sigma_a^2 S_h}) \frac{\sigma_a^4}{(\sigma_a^2 S_h + N_0)^{\cancel{2}}} + \sigma_a^2$$

$$P_{\epsilon\epsilon}(e^{j\omega}) = S_h(N_0 + \sigma_a^2 S_h)\left|C - \frac{\sigma_a^2}{\sigma_a^2 S_h + N_0}\right|^2$$

$$+ \sigma_a^2 - \frac{\sigma_a^4 S_h}{\sigma_a^2 S_h + N_0}$$

$$P_{\epsilon\epsilon}(e^{j\omega}) = S_h(N_0 + \sigma_a^2 S_h)\left|C - \frac{\sigma_a^2}{\sigma_a^2 S_h + N_0}\right|^2$$

$$+ \frac{\sigma_a^2 N_0}{\sigma_a^2 S_h + N_0}$$

First term forced to zero if $C = \dfrac{\sigma_a^2}{\sigma_a^2 S_h + N_0}$

Residual noise $P_{ww}(e^{j\omega}) = \dfrac{\sigma_a^2 N_0}{\sigma_a^2 S_h + N_0}$

$$C(z) = \frac{1}{S_h + \frac{N_0}{\sigma_a^2}} = \frac{1}{S_h(z) + \eta}$$

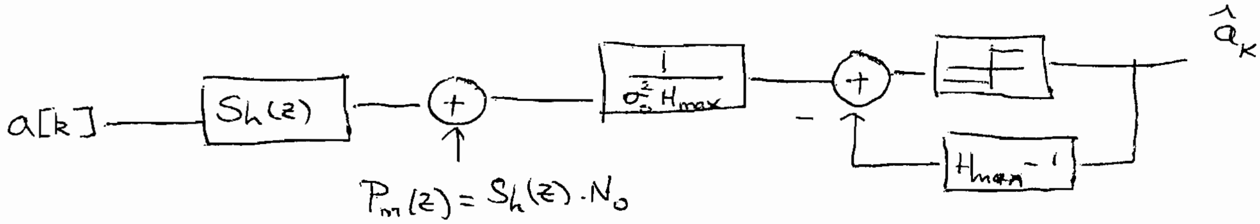
$$\sigma_\epsilon^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} P_{ww}(e^{j\omega})\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{N_0}{S_h(e^{j\omega}) + \eta}\,d\omega$$

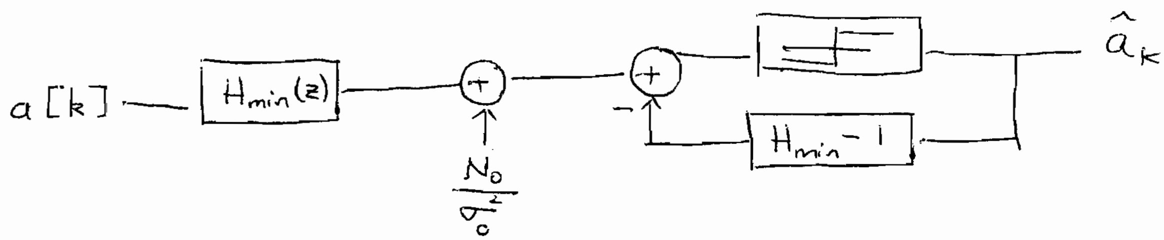
$$\sigma_\epsilon^2 = \frac{N_0}{2\pi}\int_{-\pi}^{\pi} C(e^{j\omega})\,d\omega = N_0\, c[0]$$

NOTE: Here the desired signal $a[k]$ is treated as a random signal.

Decision Feedback Equalization

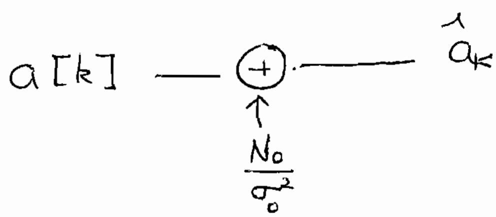
Here we move the slicer into a feedback loop that feeds decisions made back to the output of equalizer





If correct decisions are fed back then GLTF of the feedback loop is $\dfrac{1}{1 + H_{min} - 1} = \dfrac{1}{H_{min}(z)}$

The noise at the input to slicer is still white



In this case $\sigma_\epsilon^2 = \dfrac{N_0}{\sigma_0^2} = \dfrac{N_0}{GM\{S_h(z)\}}$

Contrasting this with the performance of the ZF-LE we have

$$\sigma_\epsilon^{2\,(ZF-LE)} > \sigma_\epsilon^{2\;ZF-DFE}$$