ECE-541, Fall 2007
Prob. Theory & Stochastic Proc.

Example: PSD factorization

Consider a PSD of the form:

$$P_{xx}(j\Omega) = \frac{\Omega^2}{(\Omega^2+a^2)(\Omega^2+b^2)}, \quad b > a, \quad \Omega \in \mathbb{R}^1$$

This expression is real and positive on the $j\Omega$ axis and the $j\Omega$ axis lies in the ROC, so the criteria for PSD factorization hold and consequently by analytic continuation

$$P_{xx}(s) = \frac{\left(\frac{s}{j}\right)^2}{\left(\left(\frac{s}{j}\right)^2+a^2\right)\left(\left(\frac{s}{j}\right)^2+b^2\right)}$$

$$P_{xx}(s) = \frac{-s^2}{(a^2-s^2)(b^2-s^2)},$$

$$-\min(a,b) < \mathrm{Re}\{s\} < \min(a,b)$$

$$P_{xx}(s) = \frac{s^2}{(s^2-a^2)(b^2-s^2)}, \quad |\mathrm{Re}\{s\}| < \min(a,b)$$

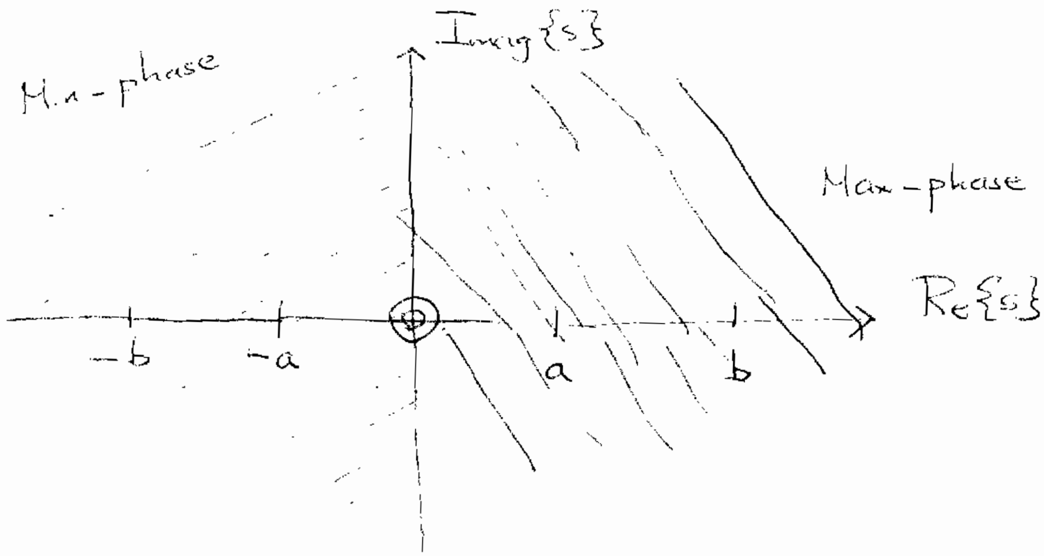
$$P_{xx}(s) = \frac{s^2}{(s+a)(s-a)(b+s)(b-s)}, \quad |\mathrm{Re}\{s\}| < \min(a,b)$$

$$P_{xx}(s) = \left(\frac{s}{(s+a)(s+b)}\right)\left(\frac{s}{(s-a)(b-s)}\right)$$

$$H_{min}(s) = \frac{s}{(s+a)(s+b)}, \quad Re\{s\} > -\min(a,b)$$

$$H_{max}(s) = \frac{s}{(s-a)(b-s)}, \quad Re\{s\} < \min(a,b)$$

$$\sigma_0^2 = 1$$



$$P_{xx}(s) = \frac{k_1}{(a^2-s^2)} + \frac{k_2}{(b^2-s^2)}$$

$$= \frac{\left(\frac{-a^2}{b^2-a^2}\right)}{a^2-s^2} + \frac{\left(\frac{-b^2}{a^2-b^2}\right)}{(b^2-s^2)}$$

$$P_{xx}(s) = \frac{-a}{2(b^2-a^2)}\left(\frac{2a}{a^2-s^2}\right) + \frac{-b}{2(a^2-b^2)}\left(\frac{2b}{b^2-s^2}\right)$$

$$R_{xx}(\tau) = \frac{-a}{2(b^2-a^2)} e^{-a|\tau|} + \frac{-b}{2(a^2-b^2)} e^{-b|\tau|}$$

$$P_{ave}^{x} = \frac{1}{2(a+b)}$$