EXAMPLE: CYCLOSTATIONARY PROCESS

ECE-541, FALL 2004

$$X(t) = A\cos(\omega_c t) + B\sin(\omega_c t)$$

$$\left.\begin{matrix} A \sim N(\mu_1, \sigma_1^2) \\ B \sim N(\mu_2, \sigma_2^2) \end{matrix}\right\} \text{A \& B are uncorrelated}$$

First-order Statistics:

$$t_0 \in T \; (\mathbb{R}^1)$$

$$X(t_0) = A\cos(\omega_c t_0) + B\sin(\omega_c t_0)$$

$$\mu_x(t_0) = E\{A\}\cos(\omega_c t_0) + E\{B\}\sin(\omega_c t_0)$$

$$\mu_x(t_0) = \mu_1\cos(\omega_c t_0) + \mu_2\sin(\omega_c t_0)$$

$$\mu_x(t), \; t \in \mathbb{R}^1 = \mu_1\cos(\omega_c t) + \mu_2\sin(\omega_c t)$$

$$\boxed{\begin{aligned} \mu_x(t) &= \sqrt{\mu_1^2+\mu_2^2}\,\cos\left(\omega_c t - \tan^{-1}\left(\frac{\mu_2}{\mu_1}\right)\right) \\ \mu_x(t) &= \mu_x(t + 2\pi/\omega_c) \end{aligned}}$$

$$E\{X^2(t)\} = \mu_1^2\cos^2(\omega_c t) + \mu_2^2\sin^2(\omega_c t) + \sigma_1^2\cos^2(\omega_c t) + 2\mu_1\mu_2\cos(\omega_c t)\sin(\omega_c t) + \sigma_2^2\sin^2(\omega_c t)$$

$$\boxed{\begin{aligned} \sigma_x^2(t) &= E\{X^2(t)\} - \mu_x^2(t) \\ &= \sigma_1^2\cos^2(\omega_c t) + \sigma_2^2\sin^2(\omega_c t) \end{aligned}}$$

$$\sigma_x^2(t) = \frac{\sigma_1^2 + \sigma_2^2}{2} + \frac{\sigma_1^2}{2}\cos(2\omega_c t) - \frac{\sigma_2^2}{2}\sin(2\omega_c t)$$

$$\sigma_x^2(t) = \sigma_x^2\left(t + \frac{2\pi}{2\omega_c}\right) \quad \left(\begin{array}{c}\text{First-order} \\ \text{Cyclostationary}\end{array}\right)$$

<u>Second--order Statistics</u>

$$R_{xx}(t,\tau) = E\{X(t)X(t-\tau)\}$$

$$= E\{[A\cos(\omega_c t) + B\sin(\omega_c t)][\cos(\omega_c t - \omega_c\tau)A + B\sin(\omega_c t - \omega_c \tau)]\}$$

$$R_{xx}(t,\tau) = \frac{(\sigma_1^2 + \mu_1^2)}{2}\{\cos(\omega_c\tau) + \cos(2\omega_c t - \omega_c\tau)\} + \frac{(\sigma_2^2 + \mu_2^2)}{2}\{\cos(\omega_c\tau) - \cos(2\omega_c t - \omega_c\tau)\} + \mu_1\mu_2\{\sin(2\omega_c t - \omega_c\tau)\}$$

$$R_{xx}(t,\tau) = R_{xx}\left(t + \frac{2\pi}{2\omega_c}, \tau\right) \quad (2^{\text{nd}} \text{ order cyclostationary})$$

$$\boxed{\tilde{R}_{xx}(\tau) = \frac{\sigma_1^2 + \mu_1^2}{2}\cos(\omega_c\tau) + \frac{\sigma_2^2 + \mu_2^2}{2}\sin(\omega_c\tau)}$$

Time - averaged ACF :

$$\boxed{T_0^{(1)} = \frac{2\pi}{2\omega_c}, \quad T_0^{(2)} = \frac{2\pi}{2\omega_c}}$$

Covariance :

$$C_{xx}(t,\tau) = R_{xx}(t,\tau) - \mu_x(t)\mu_x(t-\tau)$$

$$= \left(\frac{\sigma_1^2 + \mu_1^2}{2}\right)\left\{\cos(\omega_c\tau) + \cos(2\omega_c t - \omega_c\tau)\right\}$$

$$+ \left(\frac{\sigma_2^2 + \mu_2^2}{2}\right)\left\{\cos(\omega_c\tau) - \cos(2\omega_c t - \omega_c\tau)\right\}$$

$$+ \mu_1\mu_2 \sin(2\omega_c t - \omega_c\tau)$$

$$- \left(\mu_1\cos(\omega_c t) + \mu_2\sin(\omega_c t)\right)\left(\mu_1\cos(\omega_c t - \omega_c\tau) + \mu_2\sin(\omega_c t - \omega_c\tau)\right)$$

$$= \frac{\sigma_1^2}{2}\left[\cos(\omega_c\tau) + \cos(2\omega_c t - \omega_c\tau)\right]$$

$$+ \frac{\sigma_2^2}{2}\left[\cos(\omega_c\tau) - \cos(2\omega_c t - \omega_c\tau)\right]$$

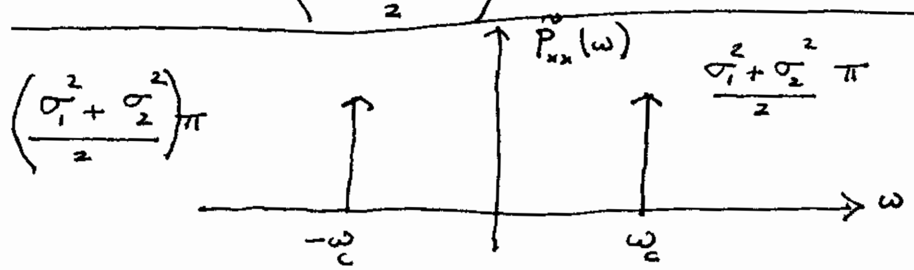
Time - averaged Covariance:

$$\tilde{C}_{xx}(\tau) = \frac{\sigma_1^2}{2}\cos(\omega_c\tau) + \frac{\sigma_2^2}{2}\cos(\omega_c\tau)$$

$$\tilde{C}_{xx}(\tau) = \frac{\sigma_1^2 + \sigma_2^2}{2}\cos(\omega_c\tau)$$

Time - averaged PSD:

$$\boxed{\tilde{P}_{xx}(\omega) = \left(\frac{\sigma_1^2 + \sigma_2^2}{2}\right)\cdot\pi\left\{\delta(\omega - \omega_c) + \delta(\omega + \omega_c)\right\}}$$



If $\mu_1 = \mu_2 = 0$ & $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$$\Rightarrow \quad \mu_x(t) = 0 \qquad \Big| \qquad R_{xx}(t,\tau) = \sigma^2 \cos(\omega_c \tau)$$

$$\sigma_x^2(t) = \sigma^2$$

$\Rightarrow$ $X(t)$ becomes WSS