Example : MS Derivative

Consider the Wiener process discussed in class:

$$R_{xx}(t_1, t_2) = \alpha \min(t_1, t_2)$$

$$= \begin{cases} \alpha t_1 & t_1 < t_2 \\ \alpha t_2 & t_2 < t_1 \\ \alpha t & t_1 = t_2 \end{cases}$$

$$\frac{\partial R_{xx}(t_1, t_2)}{\partial t_1} = \begin{cases} \alpha & t_1 < t_2 \\ 0 & t_2 < t_1 \end{cases}$$

$$= \alpha \, u(t_2 - t_1)$$

Similarly

$$\frac{\partial R_{xx}(t_1, t_2)}{\partial t_2} = \begin{cases} \alpha, & t_2 < t_1 \\ 0, & t_1 < t_2 \end{cases}$$

$$= \alpha \, u(t_1 - t_2)$$

$$\frac{\partial^2 R_{xx}(t_1, t_2)}{\partial t_1 \partial t_2} = \alpha \, \delta(t_2 - t_1)$$

This does not exist in the usual sense but exists in the generalized function sense

The Wiener process is therefore not MS differentable in the usual sense