ECE-541, FALL 2018

PROBABILITY THEORY & STOC. PROCESSES

EXAMPLE: MS Integrability

Consider a continuous, zero-mean white-noise process with ACF

$$R_{xx}(t_1, t_2) = \sigma^2 \delta(t_1 - t_2)$$

The necessary and sufficient condition for MS integrability is

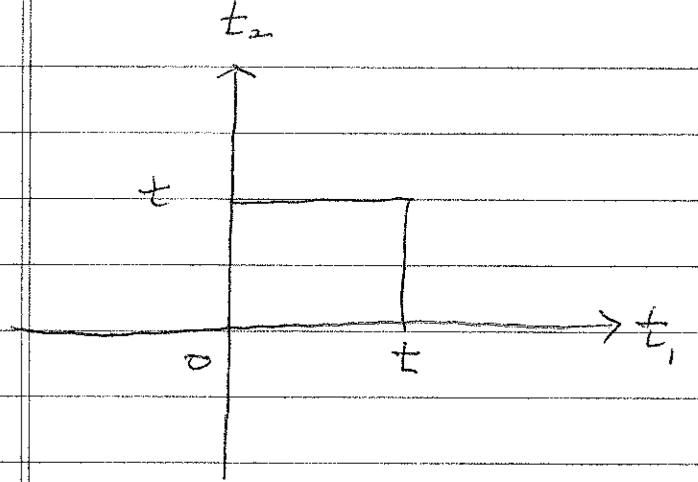
$$\int_{t_i}^{t_f} \int_{t_i}^{t_f} R_{xx}(t_1, t_2)\, dt_1\, dt_2 < \infty$$

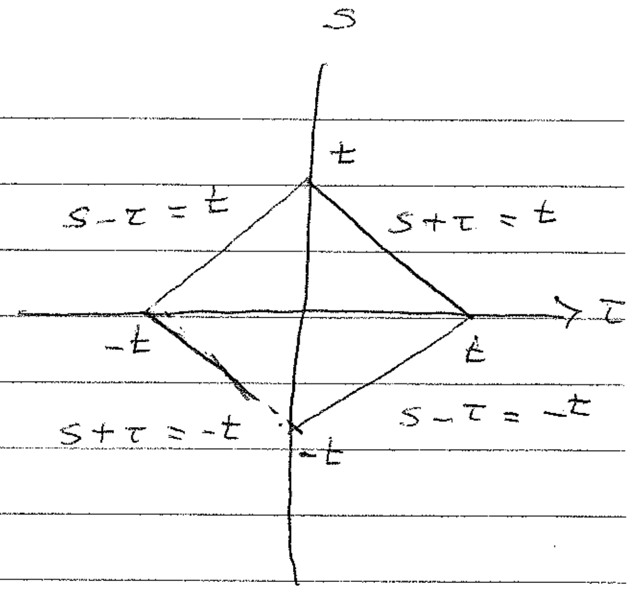
In our case this becomes

$$\int_0^t \int_0^t \sigma^2 \delta(t_1 - t_2)\, dt_1\, dt_2 = I$$

Substitution of variables:

$$\tau = t_1 - t_2 \qquad d\tau\, ds = 2\, dt_1\, dt_2$$
$$s = t_1 + t_2$$





$$I = \int_{-t}^{0}\int_{-t-\tau}^{t+\tau} \sigma^2 \delta(s) \frac{1}{2} d\tau\, ds$$

$$+ \int_{0}^{t}\int_{-t+\tau}^{t-\tau} \sigma^2 \delta(s) \frac{1}{2} d\tau\, ds$$

$$I = \frac{1}{2}\sigma^2 t + \frac{1}{2}\sigma^2 t = \sigma^2 t$$

Assuming that $\sigma^2 < \infty$

$I = \sigma^2 t < \infty$ for finite t

$\Rightarrow$ White-noise is MS integrable

$\Rightarrow$ MS Integral of white noise is the Wiener process