Overview of Causal Wiener Filtering

For the optimal causal IIR filter, the estimate of the *signal of interest*(SOI) takes the form:

$$\hat{d}[n] = \sum_{k=0}^{\infty} w[k]x[n-k].$$

Note that the summation limits are from 0 to ∞. The corresponding estimation error is given via:

$$e[n] = d[n] - \sum_{k=0}^{\infty} w[k]x[n-k].$$

Also note that the space of optimal causal IIR filters is a subset of the more general space of optimal IIR filters and consequently is a suboptimal filter as compared to the general optimal IIR filter.

Assuming that the observation process $x[n]$ corresponds to a zero-mean, WSS, finite variance random sequence that has a rational PSD, it has a power-spectral factorization of the form:

$$P_{xx}(z) = \sigma_o^2 H(z) H^* \left(\frac{1}{z^*}\right), \quad \rho < |z| < \frac{1}{\rho},$$

where $H(z)$ is the monic-minimum-phase part of $P_{xx}(z)$. The optimal realizable IIR filter is terms of this factorization is:

$$H_{\text{opt}}^{(c)}(z) = \frac{1}{\sigma_o^2 H(z)} \left(\frac{P_{dx}(z)}{H^*\left(\frac{1}{z^*}\right)} \right)_+ ,$$

where the + suffix denotes the time–causal part. The corresponding *minimum mean-squared error* (MMSE) for this causal IIR filter is given via:

$$\epsilon_{\min}^2 = R_{dd}[0] - \sum_{k=0}^{\infty} w_{\text{opt}}[k] R_{dx}[k]$$

Note that only positive-sided correlations between the SOI and the observations is being subtracted off in the expression for the MMSE. Consequently if there does exist a correlation between the SOI and the observations for negative indices, these are not taken into account. Comparing the expression of the causal IIR filter to the form of the optimal non-causal IIR filter:

$$H_{\text{opt}}^{(nc)}(z) = \frac{P_{dx}(z)}{\sigma_o^2 H(z) H^*\left(\frac{1}{z^*}\right)}$$

we can see the the difference is essentially in the time–causal part of the causal IIR filter.