Overview of FIR Wiener Filtering

For the optimal FIR Wiener filter, the estimate of the signal of interest (SOI) takes the form:

\[ \hat{d}[n] = \sum_{k=0}^{L-1} w[k]x[n - k]. \]

The corresponding estimation error is given via:

\[ e[n] = d[n] - \sum_{k=0}^{L-1} w[k]x[n - k]. \]

The orthogonality principle associated with this MMSE optimization problem is given by:

\[ E\{e[n]x^*[n - l]\} = 0, \quad 0 \leq l \leq L - 1. \]

The Wiener–Hopf equations for the optimal FIR Wiener filter derived from the orthogonality principle are given by:

\[ \sum_{k=0}^{L-1} w_{\text{opt}}[k]R_{xx}[l - k] = R_{dx}[l], \quad 0 \leq l \leq L - 1 \]

Reformulated into a matrix system the normal equations can be written as:

\[ R_x w_{\text{opt}} = r_{dx}, \]

where \( R_x \) is the autocorrelation matrix of the observations and \( r_{dx} \) is the cross-correlation vector between the SOI and the observations.

If a unique solution for the optimal FIR filtering problem is to exist, \( r_{dx} \in \text{Range}(R_x) \), i.e., the matrix \( R_x \) must be invertible and from the properties of covariance and correlation matrices, also has to be positive-definite:

\[ a^T R_x a > 0, \quad a \in \mathbb{R}^L. \]

The corresponding minimum mean-squared error (MMSE) for the optimal FIR filter is given via:

\[ \epsilon_{\text{min}}^2 = R_{dd}[0] - w_{\text{opt}}^T r_{dx} = R_{dd}[0] - w_{\text{opt}}^T R_x w_{\text{opt}} = R_{dd}[0] - r_{dx}^T R_x^{-1} r_{dx}. \]

In the context of linear transversal filtering seen in digital communication applications, the linear FIR–MMSE filter is sometime referred to as the finite tap equalizer. Note that even though the FIR Wiener filter is suboptimal compared to the non-causal and causal IIR filters, it is more practical for real-time applications such as equalization, echo-cancellation, and interference mitigation. Furthermore there are no stability problems associated with the FIR–MMSE filtering that one encounters with IIR filtering.