Overview of FIR Wiener Filtering

For the optimal FIR Wiener filter, the estimate of the *signal of interest* (SOI) takes the form:

$$\hat{d}[n] = \sum_{k=0}^{L-1} w[k]x[n-k].$$

The corresponding estimation error is given via:

$$e[n] = d[n] - \sum_{k=0}^{L-1} w[k]x[n-k].$$

The orthogonality principle associated with this MMSE optimization problem is given by:

$$E\{e[n]x^*[n-l]\} = 0, \quad 0 \leq l \leq L-1.$$

The *Wiener–Hopf* equations for the optimal FIR Wiener filter derived from the orthogonality principle are given by:

$$\sum_{k=0}^{L-1} w_{\text{opt}}[k]R_{xx}[l-k] = R_{dx}[l], \quad 0 \leq l \leq L-1$$

Reformulated into a matrix system the normal equations can be written as:

$$\mathbf{R}_x \mathbf{w}_{\text{opt}} = \mathbf{r}_{dx},$$

where $\mathbf{R}_x$ is the autocorrelation matrix of the observations and $\mathbf{r}_{dx}$ is the cross-correlation vector between the SOI and the observations.

If a unique solution for the optimal FIR filtering problem is to exist, $\mathbf{r}_{dx} \in \text{Range}(\mathbf{R}_x)$, i.e., the matrix $\mathbf{R}_x$ must be invertible and from the properties of covariance and correlation matrices, also has to be positive–definite:

$$\mathbf{a}^T \mathbf{R}_x \mathbf{a} > 0, \quad \mathbf{a} \in \mathbf{R}^L.$$

The corresponding *minimum mean-squared error* (MMSE) for the optimal FIR filter is given via:

$$\epsilon^2_{\min} = R_{dd}[0] - \mathbf{w}^T_{\text{opt}}\mathbf{r}_{dx} = R_{dd}[0] - \mathbf{w}^T_{\text{opt}}\mathbf{R}_x\mathbf{w}_{\text{opt}} = R_{dd}[0] - \mathbf{r}^T_{dx}\mathbf{R}_x^{-1}\mathbf{r}_{dx}.$$

In the context of linear transversal filtering seen in digital communication applications, the linear FIR–MMSE filter is sometime referred to as the *finite tap equalizer*. Note that eventhough the FIR Wiener filter is suboptimal compared to the non-causal and causal IIR filters, it is more practical for real-time applications such as equalization, echo-cancellation, and interference mitigation. Furthermore there are no stability problems associated with the FIR–MMSE filtering that one encounters with IIR filtering.