ECE - 541, Fall 2010

Example: Independent Increments Process

$$F_{X(t_n)\,|\,X(t_{n-1})=x_{n-1},\,X(t_{n-2})=x_{n-2},\,\ldots\ldots}$$

$$= P_r\{X(t_n) \le x_n \,|\, X(t_{n-1}) = x_{n-1},\, X(t_{n-2}) = x_{n-2},\, \ldots\ldots,\, X(t_1) = x_1,\, X(t_0) = 0\}$$

$$= P_r\{X(t_n) - X_{n-1} \le x_n - x_{n-1} \,|\, X(t_{n-1}) = x_{n-1},\, X(t_{n-2}) = x_{n-2},\, \vdots,\, X(t_1) = x_1,\, X(t_0) = 0\}$$

$$= P_r\{X(t_n) - X(t_{n-1}) \le x_n - x_{n-1} \,|\, X(t_{n-1}) = x_{n-1},\, X(t_{n-2}) = x_{n-2},\, \vdots,\, X(t_1) = x_1,\, X(t_0) = 0\}$$

$$= P_r\{X(t_n) - X(t_{n-1}) \le x_n - x_{n-1} \,|\, X(t_{n-1}) - X(t_{n-2}) \le x_{n-1} - x_{n-2},\, \vdots,\, X(t_1) - X(t_0) = x_1\}$$

$$= P_r\{X(t_n) - X(t_{n-1}) \le x_n - x_{n-1}\}$$

$$= P_r\{X(t_n) - x_{n-1} \le x_n - x_{n-1} \,/_{X(t_{n-1}) = x_{n-1}}\}$$

$$= P_r\{X(t_n) \le x_n \,/_{X(t_{n-1}) = x_{n-1}}\}$$

$$= F_{X(t_n)\,/\,X(t_{n-1}) = x_{n-1}}$$

$\Rightarrow$ $X(t)$ is Markov

$\Rightarrow$ Independent increments implies Markov property.