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ECE - 541 : Fall 2007
Probability Theory & Stochastic Processes

On Indicator Functions

Let S be a subset of $\mathcal{B}$ on the real line and define the function:

$$I_S(x) = \begin{cases} 1, & x \in S \\ 0, & x \notin S \end{cases}$$

This function will be refered to as the indicator function of S. Note that the range of $I(\cdot)$ contains only 2 points. This function is measurable since:

$$I_S^{-1}(1) = S \in \mathcal{B}$$
$$I_S^{-1}(0) = S^c \in \mathcal{B}$$

Now consider a function f and a linear combination of indicator functions of disjoint measurable sets :

$$f = \sum_{j=1}^{n} c_j I_{S_j}$$

This is also a measurable function

Now consider a set of disjoint intervals in the range of $X(\omega)$:

$$A_i \triangleq \left\{ \frac{i}{2^n} \leq X(\omega) \leq \frac{i+1}{2^n} \right\}, \text{ i.e,}$$

the set of values $X(\omega)$ takes in the interval $\left(\frac{i}{2^n}, \frac{i+1}{2^n} \right]$

The indicator function of each A_i defined via:

$$I_{A_i}(\omega) = \begin{cases} 1, & X(\omega) \in A_i \\ 0, & X(\omega) \notin A_i \end{cases}$$

is a measurable function

The random variable $X(\omega)$, a measurable function can then be expanded as

$$X(\omega) = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{2^n} \right) I_{A_i}(\omega)$$

The mean of the R.V. $X(\omega)$

$$E\{X(\omega)\} = \mu_L(X) = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{2^n} \right) E\{I_{A_i}(\omega)\}$$

$$\text{or} \quad \mu_L(X) = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{2^n} \right) \Pr\{A_i\}$$