

3-0235 — 50 SHEETS — 5 SQUARES
 3-0236 — 100 SHEETS — 5 SQUARES
 3-0237 — 200 SHEETS — 5 SQUARES
 3-0137 — 200 SHEETS — FILLER

COMET

ECE-541, FALL 2010
 Prob. Theory & Stochastic Processes

EXAMPLE : M.S. CONVERGENCE

Consider a sequence of R.V's defined on $(\Omega, \mathcal{F}, \mathbb{P})$ that converge in probability to the R.V $X(\omega)$:

$$\lim_{n \rightarrow \infty} \mathbb{P}_r \{ |X_n(\omega) - X(\omega)| > \epsilon \} = 0, \text{ for any } \epsilon > 0$$

Suppose the PDF of $X_n(\omega)$ is \ni

$$f_{X_n}(x) = 0, |x| > x_0, n > N$$

Since $X_n(\omega) \xrightarrow{D} X(\omega)$, we have :

$$f_X(x) = 0, |x| > x_0 \text{ (compact PDF)}$$

Define $|X_n(\omega) - X(\omega)| \triangleq Z_n(\omega)$

$$\begin{aligned} & \mathbb{P}_r \{ |X_n(\omega) - X(\omega)| > 2x_0 \} \\ & \leq \mathbb{P}_r \{ |X_n(\omega)| > x_0 \} + \mathbb{P}_r \{ |X(\omega)| > x_0 \} \\ 0 \leq & \lim_{n \rightarrow \infty} \mathbb{P}_r \{ Z_n(\omega) > 2x_0 \} \leq \lim_{n \rightarrow \infty} \mathbb{P}_r \{ |X_n(\omega)| > x_0 \} \\ & + \mathbb{P}_r \{ |X(\omega)| > x_0 \} \\ \Rightarrow & \lim_{n \rightarrow \infty} \mathbb{P}_r \{ Z_n(\omega) > 2x_0 \} = 0 \end{aligned}$$

Let us now evaluate:

$$0 \leq E\{[X_n(\omega) - x(\omega)]^2\} = E\{Z_n^2(\omega)\}$$

$$E\{Z_n^2(\omega)\} \stackrel{\Delta}{=} \int_{-\infty}^{\infty} z^2 f_{Z_n}(z) dz = \int_0^{\infty} z^2 f_{Z_n}(z) dz$$

$$E\{Z_n^2(\omega)\} = \int_0^{\epsilon} z^2 f_{Z_n}(z) dz + \int_{\epsilon}^{2x_0} z^2 f_{Z_n}(z) dz$$

$$\begin{aligned} 0 \leq E\{Z_n^2(\omega)\} &\leq \epsilon^2 \int_0^{\epsilon} f_{Z_n}(z) dz + 4x_0^2 \int_{\epsilon}^{2x_0} f_{Z_n}(z) dz \\ &\leq \epsilon^2 + 4x_0^2 P\{Z_n > \epsilon\} \end{aligned}$$

First applying limit as $n \rightarrow \infty$

$$\begin{aligned} \lim_{n \rightarrow \infty} E\{Z_n^2(\omega)\} &\leq \epsilon^2 + 4x_0^2 \lim_{n \rightarrow \infty} P\{Z_n > \epsilon\} \\ &= \epsilon^2 \end{aligned}$$

Next apply limits as $\epsilon \rightarrow 0_+$

$$0 \leq \lim_{\substack{n \rightarrow \infty \\ \epsilon \rightarrow 0_+}} E\{Z_n^2(\omega)\} \leq \lim_{\substack{n \rightarrow \infty \\ \epsilon \rightarrow 0_+}} \epsilon^2 = 0$$

$$\Rightarrow X_n(\omega) \xrightarrow[\text{m.s.}]{n \rightarrow \infty} X(\omega)$$

→ If $X_n(\omega) \xrightarrow[\text{m.s.}]{\text{i.p.}} X(\omega)$ and $X_n(\omega)$ and $X(\omega)$ have a compact PDF then $X_n(\omega) \xrightarrow{\text{m.s.}} X(\omega)$