3-0235 — 50 SHEETS — 5 SQUARES
3-0236 — 100 SHEETS — 5 SQUARES
3-0237 — 200 SHEETS — 5 SQUARE
3-0137 — 200 SHEETS — FILLER

COMET

ECE-541, Fall 2010
Probability & Stochastic Processes

Example: Modes of Convergence

Consider a sequence of R.V's defined on $(\Omega, \mathcal{F}, \mathbb{P})$ that converges in distribution to a constant c, i.e

$$X_n(\omega) \xrightarrow[n\to\infty]{\mathcal{D}} X(\omega) = c$$

Specifically this implies that

$$\lim_{n\to\infty} F_{X_n}(x) = F_X(x) = \begin{cases} 1, & x \geq c \\ 0, & \text{otherwise} \end{cases}$$

For any $\epsilon > 0$, let us look at

$$\Pr\{|X_n(\omega) - c| > \epsilon\}$$

$$= \Pr\{X_n > c+\epsilon\} + \Pr\{X_n < c-\epsilon\},$$

since $\{\omega : |X_n(\omega) - c| > \epsilon\}$

$$= \{\omega : X_n > c+\epsilon\} \cup \{\omega : X_n < c-\epsilon\} \quad \text{(Disjoint)}$$

$$\Pr\{|X_n(\omega) - c| > \epsilon\} \leq 1 - F_{X_n}(c+\epsilon) + F_{X_n}(c-\epsilon)$$

Taking the limit as $n \to \infty$

$$0 \le \Pr\{ |X_n(\omega) - c| > \epsilon \} \le 1 - F_{X_n}(\epsilon + c) + F_{X_n}(c - \epsilon)$$

$$0 \le \lim_{n\to\infty} \Pr\{ |X_n(\omega) - c| > \epsilon \} \le \lim_{n\to\infty} 1 - F_{X_n}(c+\epsilon) + F_{X_n}(c-\epsilon)$$

Using Convergence in Distribution

$$\left.\begin{aligned} \lim_{n\to\infty} F_{X_n}(c+\epsilon) &= 1 \\ \lim_{n\to\infty} F_{X}(c-\epsilon) &= 0 \end{aligned}\right\} \text{ since } \epsilon > 0$$

$$\Rightarrow \quad \lim_{n\to\infty} \Pr\{ |X_n(\omega) - c| > \epsilon \} = 0$$

$$\Rightarrow \quad X_n(\omega) \xrightarrow[\text{i.p.}]{n\to\infty} X(\omega) = c$$

$\Rightarrow$ If $X_n(\omega)$ converges in distribution to a constant then it converges in probability to the same constant.