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Random Signal Processing, EECE-541

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Abstract

This is intended to be a guide for review of material relating to random variables and probability from EECE-340 which is a prerequisite for this course. I will just briefly overview material in class in these areas but you are expected to know this material. Further material in this course will build on these concepts:

- Conditional Probability and Bayes Theorem.
- Independence and uncorrelatedness.
- Random Variables: CDF & PDF.
- Mean and Variance of random variables.
- Transformation of random variables.
- Characteristic functions.

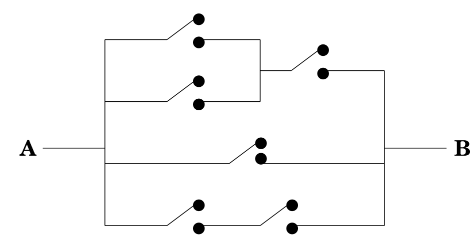
Problem 0.1: Conditional Probability

The input to a communication channel is the random variable X and the output is another random variable Y. The joint probability mass function of X and Y is given by:

Y/X	-1	0	1
-1	0.25	0.125	0
0	0	0.25	0
1	0	0.125	0.25

1. Determine the marginal PMF's of X and Y.
2. Determine $\Pr(Y = 1|X = 1)$.
3. Determine $\Pr(X = 1|Y = 1)$.
4. Determine the correlation coefficient ρ_{XY} between the input and the output of the channel.

Problem 0.2: Switching Circuits



In the circuit diagram shown above each switch is in a closed state with probability p and open state with probability 1 - p. Assuming that the state of one switch is independent of the other, find the probability that a closed path can be maintained between A and B.

Problem 0.3: Discrete random variables

Binary data is transmitted over a noisy communication channel in blocks of 16 bits. The probability that a received bit is in error due to channel noise is 0.1. Assume that the occurrence of a bit error does not influence the error in another bit within the same block.

1. Determine the average number of bit errors per block.
2. Find the variance of the number of bit errors per block.
3. Find the probability that the number of bit errors per block is greater than or equal to 3.

Problem 0.4: Transformation of Random Variables

The continuous random variable X is uniformly distributed on the interval: $[-\pi, \pi]$. The random variable Y is obtained via transformation of the variable X as $Y = \sin(X)$. With regard to this new variable:

1. Determine the domain of definition of the the new random variable Y.
2. Determine the *cumulative distribution function* (CDF) of the transformed variable Y, $F_Y(y)$ by mapping the probabilities. Determine the *probability density function* (PDF) of the transformed variable Y from the CDF.
3. Determine the *probability density function* (PDF) of the transformed variable Y, $f_Y(y)$ using the gradient method. Compare this with the result from the previous part.
4. Determine the probability that the transformed variable lies in the interval $[y_1, y_2]$.

Problem 0.5: Characteristic functions

The *probability density function* (PDF) of a Laplacian random variable X is given by

$$f_X(x) = \frac{a}{2} \exp(-a|x|), \; a, x \in \mathbf{R}, a > 0.$$

1. Determine the mean and variance of the random variable X.
2. Compute the characteristic function $\Psi_X(\omega)$ of X.
3. Determine the mean and variance of X from $\Psi_X(\omega)$ and compare this to the result obtained in part (1).
4. Write X as the sum of two independent random variables Z and W. Determine their PDFs: $f_W(w)$ and $f_Z(z)$. Is this decomposition unique?