PS #1 , Fall 2018
Probability Theory & Stochastic Processes, ECE-541
Instructor: Balu Santhanam
Date Assigned: 08/28/2018
Date Due: 09/04/2018

Problem # 1.0

A continuous-time stochastic process $X(t)$ with $t \in [-1, 1]$ is defined via:

$$X(t, \omega) = A(\omega) \cos\left(\Upsilon(\omega)t + \Theta(\omega)\right),$$

where the random variables $\Theta \sim U([-\pi, \pi)]$, $\Upsilon \sim N(\Omega_c, \sigma^2)$, and $A \sim U([-A, A])$ are independent of each other. For this process:

1.a) plot two sample realizations $x_1(t)$ and $x_2(t)$.

1.b) Determine the mean $\mu_x(t)$ and the variance $\sigma_x^2(t)$ associated with this process. Is this process first-order stationary? Justify your answer properly. HINT: You may use the result that the characteristic function associated with a Gaussian random variable is given by the expression:

$$\Psi_N(j\Omega) = \exp\left(j\mu_N t\right) \exp\left(-\frac{1}{2}\sigma_N^2 t^2\right).$$

1.c) Determine the auto-correlation $R_{xx}(t_1, t_2)$ and the auto-covariance $C_{xx}(t_1, t_2)$ associated with this process. Is this process second-order stationary? Justify your answer properly.

Problem # 2.0

A discrete-time stochastic process is defined by the expression:

$$X[n, \omega] = \omega n, \quad n \geq 0,$$

where ω is selected at random, i.e., uniformly from the interval $(0, 1)$. For this environment:

2.c) Draw to sample functions $x_1[n]$ and $x_2[n]$ of the process.

2.b) Find the first-order CDF of $X_x(\omega)$.

2.c) Find the joint CDF for $X_n(\omega)$ and $X_{n+1}(\omega)$.

2.d) Find the mean $\mu_x[n]$, variance $\sigma_x^2[n]$ and autovariance function $C_{xx}[p, q]$.

Problem # 3.0

Suppose a stochastic process $S[n, \omega)$ is defined by the expression:

$$S_n(\omega) = \sum_{k=1}^{n} X_k(\omega),$$

where the $X_n(\omega)$ are i.i.d. Poisson random variables with mean α. For this description:

3.a) Find the PMF of the sum process $S_n(\omega)$.

3.b) Find the mean $\mu_s[n]$ and variance $\sigma_s^2[n]$.

3.c) Find the joint PMF of $S_n(\omega)$ and $S_{n+k}(\omega)$.

3.d) Find $r_{ss}[n, k]$.