Problem # 6.0
ECE-541, Fall 2015
Probability Theory and Stochastic Processes
Date Assigned: 11/03/2015
Date Due: 11/10/2015

Background

In this project, we will implement three types of quantizers that were discussed in the class: (a) uniform quantization, (b) CDF-based quantization, and (c) optimal Lloyd-Max quantization. Our goal is to study and compare their performance.

Uniform Quantization

Using the MATLAB function `fxquant.m` posted on the course webpage, we will study the effect of the various parameters on the performance of this quantizer. Towards this purpose generate an input signal $x[n]$ that is uniformly distributed in the interval $[-1, 1]$. We seek to establish some of the results seen in the class:

1. Plot the cross-correlation between the quantization error $e_q[n]$ and the input signal $x[n]$, i.e., $E\{x[n]e_q[n]\}$ for three word lengths $B = 4, 8, 16$. Use the MATLAB funtion `xcorr.m` for this purpose. Comment on the emerging pattern.

2. Plot the power spectral density of the quantization error for the three different word-lengths using the MATLAB function `psd.m`. Calculate the sample variance of the quantization error using the MATLAB function `var.m` and compare it to the theoretical value.

3. Plot the histogram of the quantization error using the MATLAB function `hist.m`. Comment on the emerging pattern.

4. Plot the output SNR of the uniform quantizer for the word lengths $B = 2, 4, 8, 10, 12, 16$. Comment on the emerging pattern.

CDF based Non-uniform Quantization

Here we implement and study the performance of the CDF-based non-uniform quantizer:

1. Pick your choice of a heavy-tailed distribution, i.e., Erlang, Pareto, Beta, Gamma, Weibull and use the built-in MATLAB functions from the statistics toolbox to generate samples of this non-uniform source $x[n]$. Use the MATLAB function `hist.m` to confirm this.

2. Using the built-in functions for the CDF of a distribution, transform the non-uniform source to a uniform source $z[n]$. Confirm this fact by plotting the histogram of the transformed source.

3. Use the MATLAB function `fxquant.m` to quantize the source with various word-lengths. Remember to subtract of the mean of the source before quantization and to add it back after quantization.

4. Use the inverse CDF functions in MATLAB to transform the quantized uniform signal $z_q[n]$ to the quantized non-uniform signal $x_q[n]$.

5. Plot the output SNR of the CDF quantizer for various word-lengths. Comment on the emerging pattern.

Optimal Non-uniform Quantization

Using the non-uniform source $x[n]$ generated in the previous part we wish to quantize the source using Lloyd-Max quantization implemented via the MATLAB commands `lloyds.m` and `quantiz.m`. Plot the output SNR of this quantizer for the word-lengths $B = 2, 4, 6, 8, 12, 14$. Compare the performance of the three quantizers using the non-uniform source. For the uniform quantizer, use a $X_{\max} = 2\sigma_x$ rule for the quantization. Use a sufficient number of input samples to be able to detect errors.