Sequences with Rational Power Spectra

Consider the class of zero-mean, finite variance, WSS, random sequence with rational PSD. These random signals are in general obtained by the processing of a zero-mean, WSS, white, random signal through a LTI system and consequently the general form for the PSD of these signals is:

$$P_{xx}(z) = \sigma_o^2 \frac{B(z)B^*\left(\frac{1}{z^*}\right)}{A(z)A^*\left(\frac{1}{z^*}\right)}$$

If the PSD of the signal contains just P poles, i.e, there are no zeroes or in other words $B(z) = K$ then the PSD takes the form:

$$P_{xx}(z) = \frac{K\sigma_o^2}{A(z)A^*\left(\frac{1}{z^*}\right)}$$

These processes are termed as *autoregressive* or AR(p) random signals. It is not difficult to show that the ACF of the random sequence $x[n]$ in this case takes the form:

$$R_{xx}[k] = \sum_{i}^{p} c_i a_i^{|k|}, \quad |a_i| < 1$$

Note that Gaussian AR random signals are ergodic in the general sense because:

$$\sum_{k=-\infty}^{\infty} |R_{xx}[k]| < \infty.$$

If the PSD of the random signal has just q zeroes in it, i.e, $A(z) = 1$ then the PSD takes the form:

$$P_{xx}(z) = \sigma_o^2 B(z)B^*\left(\frac{1}{z^*}\right).$$

These processes are termed as *moving average* or MA(q) random signals and the ACF of the random sequence can be shown to satisfy:

$$R_{xx}[k] = 0, \quad |k| > q$$

Signals with both poles and zeroes in their PSD are referred to as *autoregressive moving average* or ARMA(p, q) random signals.

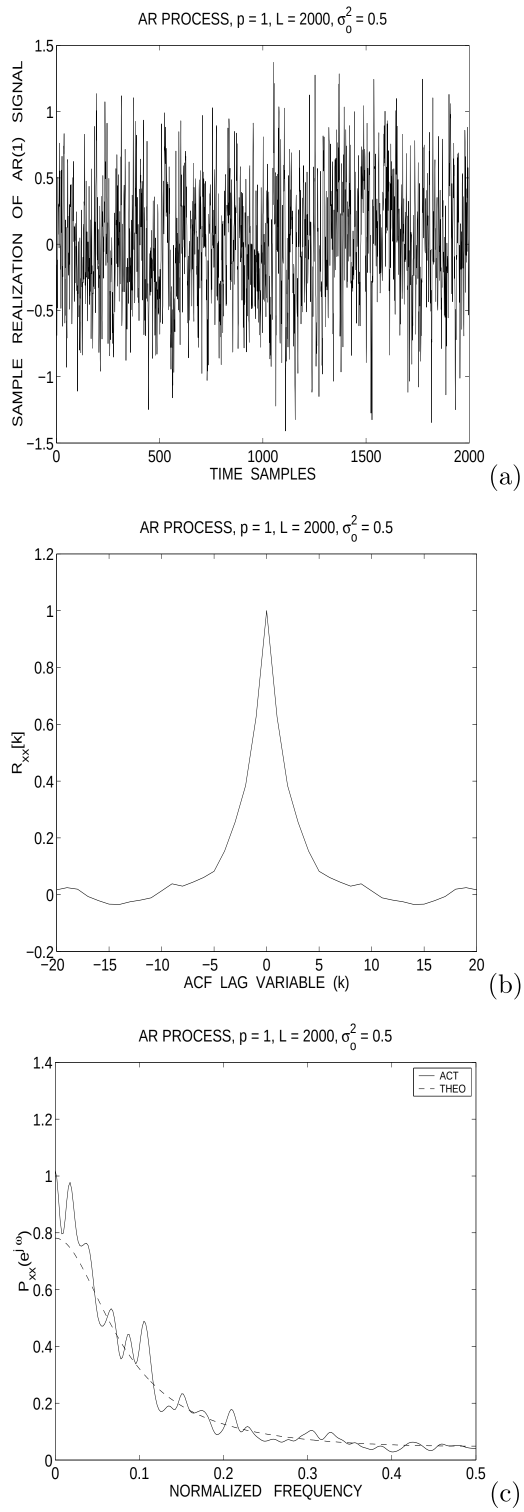

Figure 1: AR(1) example: (a) sample realization of a AR(1) process, (b) sample ACF of the process, and (c) estimated PSD of the process.