ECE-541, Fall 2007
Prob. Theory & Stoch. Proc.

Example: Schur-Cohn Stability

In class when we looked at the Levinson-Durbin recursion we had:

$$\underline{a}^{(p+1)} = \begin{pmatrix} \underline{a}^{(p)} \\ 0 \end{pmatrix} + \Gamma_{p+1} \begin{pmatrix} 0 \\ a_R^{(p)} \end{pmatrix}$$

Reversing the order:

$$a_R^{(p+1)} = \begin{pmatrix} 0 \\ a_R^{(p)} \end{pmatrix} + \Gamma_{p+1} \begin{pmatrix} \underline{a}^{(p)} \\ 0 \end{pmatrix}$$

Suppose we look at the system function

$$H(z) = \frac{1}{1 + \frac{1}{2} z^{-1} - 0.1 z^{-2} - \frac{1}{2} z^{-3}}$$

Denoting $A_3(z) = 1 + \frac{1}{2} z^{-1} - 0.1 z^{-2} - \frac{1}{2} z^{-3}$

$B_3(z) = A_3^R(z) = -\frac{1}{2} - 0.1 z^{-1} + \frac{1}{2} z^{-2} + z^{-3}$

and $\Gamma_3 = -\frac{1}{2}$

$$\begin{pmatrix} 1 \\ \frac{1}{2} \\ -0.1 \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ a_2[1] \\ a_2[2] \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 \\ a_2[2] \\ a_2[1] \\ a_2[0] \end{pmatrix}$$

This yields the system:

$$a_2[1] - \frac{1}{2} a_2[2] = \frac{1}{2}$$
$$a_2[2] - \frac{1}{2} a_2[1] = -0.1$$

Rearranging:

$$\begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} a_2[1] \\ a_2[2] \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -0.1 \end{pmatrix}$$

This yields:

$$\begin{pmatrix} a_2[1] \\ a_2[2] \end{pmatrix} = \frac{\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ -0.1 \end{pmatrix}}{1 - \frac{1}{4}}$$

0.25
0.10
0.15
0.05
$\cancel{0.15} \cdot \frac{4}{3}$
$= 0.2$

or
$$a_2[1] = \frac{1}{\cancel{2}}(0.9) \cdot \frac{\overset{2}{\cancel{4}}}{3} = 0.6$$
$$a_2[2] = \left(\frac{1}{4} - .1\right) \cdot \frac{4}{3} = 0.2$$

$$A_2(z) = 1 + 0.6 z^{-1} + 0.2 z^{-2}$$
$$\Gamma_2 = 0.2$$

$$\begin{pmatrix} 1 \\ 0.6 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 1 \\ a_1[1] \\ 0 \end{pmatrix} + 0.2 \begin{pmatrix} 0 \\ a_1[1] \\ 1 \end{pmatrix}$$

which yields $1.2\, a_1[1] = 0.6$

or $a_1[1] = \frac{1}{2}$

$A_1(z) = 1 + \frac{1}{2} z^{-1}$ and $\Gamma_1 = \frac{1}{2}$

Since $\Gamma_1 = \frac{1}{2}$, $\Gamma_2 = 0.2$, $\Gamma_3 = -\frac{1}{2}$ are all less than 1 in magnitude, the system is stable