

DKF: Assumptions



- Known state model for desired process:
 - $\begin{aligned} \mathbf{x}[n+1] &= \mathbf{A}[n]\mathbf{x}[n] + \mathbf{f}[n] \\ y[n] &= \mathbf{C}[n]\mathbf{x}[n] + v[n]. \end{aligned}$
- Underlying state-model observable:

 $\text{det}(O(C,A)) \neq 0.$

- f[n] & v[n] are zero mean, uncorrelated Gaussian noise sources.
- Associated covariance matrices: R_{vv}, R_{ff}





Linear state estimate:

$$\hat{\mathbf{x}}_{+}[n] = \mathbf{K}_{1}[n]\hat{\mathbf{x}}_{-}[n] + \mathbf{K}_{2}[n]y[n]$$

Prior/posteriori state estimation errors:

$$\mathbf{e}_{+}[k] = \hat{\mathbf{x}}_{+}[k] - \mathbf{x}[k]$$
$$\mathbf{e}_{-}[k] = \hat{\mathbf{x}}_{-}[k] - \mathbf{x}[k]$$

Unbiased prior/posteriori state estimates provided:

$$\mathbf{K}_1[n] = \mathbf{I} - \mathbf{K}_2[n]\mathbf{C}[n]$$



DKF: State Estimation



- DKF state estimate:
 - $\hat{\mathbf{x}}_{+}[n] = (\mathbf{I} \mathbf{K}_{2}[n]\mathbf{C}[n])\hat{\mathbf{x}}_{-}[n] + \mathbf{K}_{2}[n]y[n]$
- State estimate: alternative form

 $\hat{\mathbf{x}}_{+}[n] = \hat{\mathbf{x}}_{-}[n] + \mathbf{K}_{2}[n](y[n] - \mathbf{C}[n]\hat{\mathbf{x}}_{-}[n])$

- Conversion factor: $e_{+}[n] = (I - K_{2}[n]C[n])e_{-}[n] + K_{2}[n]v[n]$
- Error covariance update:

 $\mathbf{P}_{+}[n] = (\mathbf{I} - \mathbf{K}_{2}[n]\mathbf{C}[n])^{T}\mathbf{P}_{-}[n](\mathbf{I} - \mathbf{K}_{2}[n]\mathbf{C}[n]) + \mathbf{K}_{2}^{T}[n]\mathbf{R}_{vv}\mathbf{K}_{2}[n]$



DKF: State Estimation



Cost function: Trace of error covariance

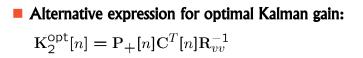
 $J(\mathbf{K}_{2}[n], n) = E\{\mathbf{e}_{+}^{T}[n]\mathbf{e}_{+}[n]\} = \mathsf{Trace}(E\{\mathbf{e}_{+}[n]\mathbf{e}_{+}^{T}[n]\})$

Optimal solution:

 $\mathbf{K}_{2}^{\mathsf{opt}}[n] = \mathbf{P}_{-}[n]\mathbf{C}^{T}[n](\mathbf{R}_{vv} + \mathbf{C}[n]\mathbf{P}_{-}[n]\mathbf{C}^{T}[n])^{-1}$

Optimized error covariance:
P₊[n] = (I - K₂[n]C[n])P₋[n]





Optimal state extrapolation/prediction:

 $\hat{\mathbf{x}}_{-}[n] = \mathbf{A}[n-1]\mathbf{x}_{+}[n-1]$

Error covariance after prediction:

$$\mathbf{P}_{-}[n] = \mathbf{A}[n-1]\mathbf{P}_{+}[n-1]\mathbf{A}^{T}[n-1] + \mathbf{R}_{ff}$$



DKF: Innovations



- Alternative form of state estimate: $\hat{\mathbf{x}}_{+}[n] = \hat{\mathbf{x}}_{-}[n] + \mathbf{K}_{2}[n]\alpha[n].$
- Innovations process or measurement residual: $\alpha[n] = y[n] C[n] \hat{x}_{-}[n]$
- For smoothing problem: convex combination

 $\hat{\mathbf{x}}_{+}[n] = (\mathbf{I} - \mathbf{K}_{2}[n])\hat{\mathbf{x}}_{+}[n-1] + \mathbf{K}_{2}[n]y[n]$

Innovations : an orthonormal basis for regular or unpredictable part of y[n].



]

DKF: State Estimation



Prior/Posteriori output errors:

 $e[n] = y[n] - \mathbf{C}[n]\hat{\mathbf{x}}_{-}[n] - \mathbf{C}[n]\mathbf{K}_{2}[n]\alpha[n]$

Conversion factor:

 $e[n] = (1 - \mathbf{C}[n]\mathbf{K}_2[n])\alpha[n].$

Conversion factor:

 $\gamma[n] = 1 - \frac{\mathbf{C}[n]\mathbf{P}_{-}[n]\mathbf{C}^{T}[n]}{\sigma_{v}^{2} + \mathbf{C}[n]\mathbf{P}_{-}[n]\mathbf{C}^{T}[n]}$



DKF: State Estimation



- Orthogonality principle:
 - $E\{\hat{\mathbf{x}}_{+}[n]\mathbf{e}_{+}^{T}[n]\}=\mathbf{0}$
- Choice of weight matrices guarantees no bias.
- Convergence in MS sense: I K₂[n]C[n] is a stable matrix.
- Optimal Kalman gain matrix K₂[n] weights prior observations and current data automatically.