Extended Kalman Filter (EKF)

- State model for EKF:

$$\begin{aligned} \mathbf{x}[n+1] &= \mathbf{A}[n]\mathbf{x}[n] + \mathbf{f}[n] \\ y[n] &= h(\mathbf{x}[n], n) + v[n]. \end{aligned}$$

- Similar assumptions as the DKF regarding noise sources and observability of the states.
- Measurement equation nonlinear in state parameters.
- Assume nonlinearity is differentiable and continuous.

Extended Kalman Filter (EKF)

- Taylor series expansion of h(.) about prior estimate:

$$h(\mathbf{x}[n], n) \approx h(\hat{\mathbf{x}}_-[n], n) - \frac{\partial h(\mathbf{x}[n], n)}{\partial \mathbf{x}[n]}|_{\mathbf{x}=\hat{\mathbf{x}}_-}\mathbf{e}_-[n]$$

- Linearized model:

$$\begin{aligned} h(\mathbf{x}[n], n) &- h(\hat{\mathbf{x}}_-[n], n) = \mathbf{C}(\hat{\mathbf{x}}_-[n], n)\mathbf{e}_-[n] \\ \mathbf{C}(\hat{\mathbf{x}}_-[n], n) &= -\frac{\partial h(\mathbf{x}[n], n)}{\partial \mathbf{x}[n]}|_{\mathbf{x}=\hat{\mathbf{x}}_-} \end{aligned}$$

- Linearization valid only if :

$$||\mathbf{P}_+[n]|| < ||\mathbf{P}_-[n]|| \longleftrightarrow ||(\mathbf{I}-\mathbf{K}_2^{\text{opt}}[n]\mathbf{C}(\hat{\mathbf{x}}_-[n], n))|| < 1$$

Extended Kalman Filter (EKF)

- Computing Kalman Gain:

$$\mathbf{K}_2^{\text{opt}}[n] = \mathbf{P}_-[n]\mathbf{C}(\hat{\mathbf{x}}_-[n], n)(\mathbf{R}_{vv}+\mathbf{C}(\hat{\mathbf{x}}_-[n], n)\mathbf{P}_-[n]\mathbf{C}^T(\hat{\mathbf{x}}_-[n], n))^{-1}$$

- EKF state estimate:

$$\hat{\mathbf{x}}_+[n] = \hat{\mathbf{x}}_-[n] + \mathbf{K}_2^{\text{opt}}[n](y[n] - h(\hat{\mathbf{x}}_-[n], n))$$

- Error covariance update:

$$\mathbf{P}_+[n] = (\mathbf{I} - \mathbf{K}_2^{\text{opt}}[n]\mathbf{C}(\hat{\mathbf{x}}_-[n], n))\mathbf{P}_-[n]$$

- State extrapolation:

$$\hat{\mathbf{x}}_-[n+1] = \mathbf{A}[n]\hat{\mathbf{x}}_+[n],\ \mathbf{P}_-[n+1] = \mathbf{A}[n]\mathbf{P}_+[n]\mathbf{A}^T[n]+\mathbf{R}_{ff}$$

Extended Kalman Filter (EKF)

- DKF equations valid with linearized state model.
- Error surface multi-modal due to nonlinearity.
- Choice of initial conditions : unbiased estimate

$$\hat{\mathbf{x}}_-[0] = E\{\mathbf{x}[0]\} \longleftrightarrow\ E\{\mathbf{e}_-[n]\} = 0.$$

- EKF diverges if initial conditions are not chosen ∋

$$||\mathbf{I} - \mathbf{K}_2^{\text{opt}}[n]\mathbf{C}(\hat{\mathbf{x}}_-[n], n))|| < 1$$