GNLMS Derivation

NLMS update:

$$\underline{w}[n+1] = \underline{w}[n] + \frac{\mu}{\|\underline{u}[n]\|^2 + \beta} e[n]\,\underline{u}[n]$$

or

$$\underline{w}[n+1] = \underline{w}[n] + \eta[n]\, e[n]\,\underline{u}[n]$$

Instead of just varying $\eta[n]$ by dividing by $\|\underline{u}[n]\|^2$, vary β also

$$\beta[n+1] = \beta[n] - s \nabla_{\beta[n-1]} J[n],$$

$$J[n] = \frac{1}{2} e^2[n]$$

$$= \frac{1}{2} (d[n] - \underline{w}^T[n]\,\underline{u}[n])^2$$

$$\frac{\partial J}{\partial e[n]} = e[n]$$

$$\frac{\partial e[n]}{\partial \underline{w}[n]} = -\underline{u}^T[n]$$

$$\frac{\partial e[n]}{\partial y[n]} = -1$$

$$\frac{\partial y[n]}{\partial w[n]} = -u^T[n]$$