$$\frac{\partial w[n]}{\partial \eta[n-1]} = \frac{\partial}{\partial \eta[n-1]} \left\{ w[n-1] + \eta[n-1]\, \underline{u}[n-1]\, e[n-1] \right\}$$

$$= \frac{\partial}{\partial \eta[n-1]} \left\{ \eta[n-1]\, \underline{u}[n-1]\, e[n-1] \right\}$$

$$= \underline{u}[n-1]\, e[n-1]$$

$$\frac{\partial \eta[n-1]}{\partial \beta[n-1]} = \frac{\partial}{\partial \beta[n-1]} \left\{ \frac{\rho}{\|u[n-1]\|^2 + \beta[n-1]} \right\}$$

$$= \frac{-\rho}{\left(\|u[n-1]\|^2 + \beta[n-1] \right)^2}$$

Combining all terms :

$$\underline{w}[n+1] = w[n] + \eta[n]\, \underline{u}[n]\, e[n],$$

$$\eta[n] = \frac{\rho}{\|u[n]\|^2 + \beta[n]}.$$

$$\beta[n+1] = \beta[n] - \frac{\delta \rho\, e[n] e[n-1]\, u^T[n]\, \underline{u}[n-1]}{\left(\|u[n-1]\|^2 + \beta[n-1] \right)^2}$$

The parameter δ controls the learning rate for updating the offset parameter recursion.