ECE - 595, FALL 2011

ADAPTIVE FILTER THEORY

Application of the Inversion Lemma to the RLS, exponential sliding window formulation:

$$\tilde{\mathbb{R}}_{uu}[n] = \sum_{i=0}^{n} \lambda^{n-i} \underline{u}[i]\, \underline{u}^T[i]$$

$$= \lambda\, \tilde{\mathbb{R}}_{uu}[n-1] + \underline{u}[n]\, u^T[n]$$

$$\tilde{r}_{du}[n] = \sum_{i=0}^{n} \lambda^{n-i} \underline{u}[i]\, d[i]$$

$$= \lambda\, \tilde{r}_{du}[n] + \underline{u}[n]\, d[n]$$

$$P[n] \triangleq \left(\tilde{\mathbb{R}}_{uu}[n]\right)^{-1} = \left(\underbrace{\lambda\, \mathbb{R}_{uu}[n-1]}_{A} + u[n]\,u^T[n]\right)^{-1}$$

Choosing $C = 1$, $B = \underline{u}[n]$, $D = \underline{u}^T[n]$

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B\,(C^{-1} + DA^{-1}B)^{-1}\,DA^{-1}$$

$$P[n] = \frac{1}{\lambda} P[n-1] - \frac{\frac{1}{\lambda} P[n-1]\,\underline{u}[n]\,u^T[n]\,\frac{1}{\lambda} P[n-1]}{1 + u^T[n]\,\frac{1}{\lambda} P[n-1]\,\underline{u}[n]}$$

$$P[n] = \frac{1}{\lambda} P[n-1] - \frac{\frac{1}{\lambda} P[n-1]\, u[n]\,u^T[n]\, P[n-1]}{\lambda + u^T[n]\, P[n-1]\, u[n]}$$