Least Squares Lattice (LSL)

- **Forward prediction error:**

$$f_m[i] = u[i] - \mathbf{w}_{f,m}^T[i]\mathbf{u}_m[i-1]$$

- **Cost function:**

$$J_f^{(m)}[n] = \sum_{i=1}^{n} \lambda^{n-i} f_m^2[i]$$

- **Orthogonality principle:**

$$\sum_{i=1}^{n} \lambda^{n-i} f_m[i]\mathbf{u}_m[i-1] = 0$$

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LSL: Backward Prediction

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- **Backward prediction error:**

$$g_m[i] = u[i-m] - \mathbf{w}_{b,m}^T[n]\mathbf{u}_m[i]$$

- **Cost function:**

$$J_m^{(b)}[n] = \sum_{i=1}^{n} g_m^2[i]$$

- **Orthogonality principle:**

$$\sum_{i=1}^{n} \lambda^{n-i} g_m[i]\mathbf{u}_m[i] = 0$$

LSL: Adaptive Backward Predictor

- **RLS backward gain vector:** $\mathbf{g}_m^{(b)}[n] = \mathbf{g}_m[n]$
- **RLS backward innovations process:**

$$\alpha_m^{(b)}[n] = u[n-m] - \mathbf{w}_{b,m}^T[n-1]\mathbf{u}_m[n].$$

- **RLS backward tap-weight update:**

$$\mathbf{w}_{b,m}[n] = \mathbf{w}_{b,m}[n-1] + \mathbf{g}_m^{(b)}[n]\alpha_m^{(b)}[n]$$

- **MSE update:**

$$E_m^{(b)}[n] = E_m^{(b)}[n-1] + \alpha^{(b)}[n]\mathbf{g}_m^{(b)}[n]$$

LSL: Forward Adaptive Predictor

- **RLS forward gain vector:** $\mathbf{g}_m^{(f)}[n] = \mathbf{g}_m[n-1]$
- **RLS forward innovations process:**

$$\alpha_m^{(f)}[n] = u[n] - \mathbf{w}_{f,m}^T[n-1]\mathbf{u}_m[n-1]$$

- **RLS forward tap-weight update:**

$$\mathbf{w}_{f,m}[n+1] = \mathbf{w}_{f,m}[n] + \mathbf{g}_m^{(f)}[n]\alpha_m^{(f)}[n]$$

- **MSE update:**

$$E_m^{(f)}[n] = E_m^{(f)}[n-1] + \alpha_m^{(f)}[n]f_m[n]$$

LSL: Forward Predictor

- **Optimal forward reflection coefficient:**

$$\kappa_m^{(f)}[n] = \sum_{i=1}^{n} \lambda^{n-i} f_{m-1}[i] g_{m-1}[i-1] / \sum_{i=1}^{n} \lambda^{n-i} g_{m-1}^2[i-1]$$

- **Weighted cross correlation:**

$$\Delta_{m-1}[n] = \sum_{i=1}^{n} \lambda^{n-i} g_{m-1}[i-1] f_{m-1}[i]$$

- **Updating of reflection coefficient:**

$$\kappa_m^{(f)}[n] = -\frac{\Delta_{m-1}[n]}{E_{m-1}^{(b)}[n-1]}$$

LSL: Special Features

- $\kappa^{(f)}_m$**[n]: projection coefficient of forward prediction errors** $f_{m-1}[n]$**, onto reverse errors** $g_{m-1}[n-1]$**.**

- **Orthogonality of backward prediction errors:**

$$< b_p[n], b_q[n] > \; = \; \sum_{i=1}^{n} \lambda^{n-i} b_p[i] b_q[i] = 0, \quad q > p$$

- **Conversion factor: cosine of angle**

$$\gamma_m[n] = \cos(\phi_m[n]) = (1 - \mathbf{g}_m^T[n]\mathbf{u}_m[n]) < 1$$

LSL : Reverse Predictor

- **Optimal reverse reflection coefficient:**

$$\kappa_m^{(b)}[n] = \sum_{i=1}^{n} \lambda^{n-i} f_{m-1}[i] g_{m-1}[i-1] / \sum_{i=1}^{n} \lambda^{n-i} f_{m-1}^2[i]$$

- **Updating reflection coefficient:**

$$\kappa_m^{(b)}[n] = -\frac{\Delta_{m-1}[n]}{E_{m-1}^{(f)}[n]}$$

- **Time-update:**

$$\Delta_{m-1}[n] = \lambda \Delta_{m-1}[n-1] + \alpha_{m-1}^{(f)}[n] g_{m-1}[n-1]$$