NLMS: SVD solution

- **NLMS cost function:**

$$J(\mathbf{w}[n]) = ||\mathbf{w}[n+1]-\mathbf{w}[n]||^2+\lambda(d[n]-\mathbf{w}^T[n+1]\mathbf{u}[n]).$$

- **Constraint system:**

$$\in [n] = d[n]-\mathbf{w}^T[n+1]\mathbf{u}[n] = e[n]-\delta(\mathbf{w}[n])^T\mathbf{u}[n].$$

- **Constraint satisfied when :**

$$\mathbf{u}^T[n]\delta(\mathbf{w}[n]) = e[n]$$

NLMS: SVD Solution

- **Outer-product singular with unit rank:**

$$\mathbf{u}\mathbf{u}^T[n]\mathbf{u}[n] = ||\mathbf{u}[n]||^2\mathbf{u}[n]$$

- **Minimum norm/SVD solution to constraints:**

$$\delta(\mathbf{w}[n]) = \frac{\mathbf{u}[n]}{||\mathbf{u}[n]||^2}e[n]$$

- **Tap-weight update for minimum norm solution:**

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \frac{\mu}{||\mathbf{u}[n]||^2}e[n]\mathbf{u}[n]$$