NLMS Algorithm: Motivation

- Want tap-weight update to reflect changes in statistics of u[n], specifically average power.
- Want convergence of the algorithm to be relatively independent of $\chi(\mathbf{R})$.
- Want to retain the steepest descent flavor of the LMS algorithm.

NLMS Algorithm

- Cost function:

$$J(\mathbf{w}[n]) = ||\mathbf{w}[n+1]-\mathbf{w}[n]||^2+\lambda(d[n]-\mathbf{w}^T[n+1]\mathbf{u}[n]).$$

- Gradient w.r.s.t. $\mathbf{w}[n+1]$:

$$\nabla_{\mathbf{w}[n+1]}(J) = 2(\mathbf{w}[n+1]-\mathbf{w}[n]) - \lambda\mathbf{u}[n] = 0.$$

- Gradient w.r.s.t. λ

$$\frac{\partial J}{\partial \lambda} = d[n] - \mathbf{w}^T[n+1]\mathbf{u}[n] = 0.$$

NLMS Algorithm

- Tap-weight update:

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \frac{\beta}{||\mathbf{u}[n]||^2+\delta}e[n]\mathbf{u}^*[n]$$

- Variable step-size:

$$\tilde{\mu}[n] = \frac{\beta}{||\mathbf{u}[n]||^2+\delta}$$

- Norm update:

$$||\mathbf{u}[n+1]||^2 = ||\mathbf{u}[n]||^2+u^2[n-L+1]-u^2[n+1].$$

- Normalized step-size:
 $0 < \beta < 2$

NLMS Algorithm

- Off-set parameter δ used to avoid divide by zero problems.
- Direction of tap-weight update still in the direction of steepest descent.
- Normalization of step-size removes sensitivity to eigenvalue spread $\chi(\mathbf{R})$.
- Convergence characteristics superior to the LMS.

NLMS : Step-size Analysis

- **Mean-square stability:**

$$0 < \beta < 2\frac{E\{< \epsilon[n], \mathbf{u}[n] > e[n]/||\mathbf{u}[n]||^2\}}{E\{e^2[n]/||\mathbf{u}[n]||^2\}}$$

- **Optimal step-size:**

$$\mu_{\text{opt}} = \frac{D[n]R_{uu}[0]}{J[n]}$$