Newton's Method Vs. SDA

- Higher-order Taylor series expansion of cost-function:

$$J(\mathbf{w}) \approx J(\mathbf{w}[n]) + (\mathbf{w}-\mathbf{w}[n])^H \nabla_w(\mathbf{w}[n]) + \frac{1}{2}(\mathbf{w}-\mathbf{w}[n])^H \mathbf{H}[n](\mathbf{w}-\mathbf{w}[n]).$$

- Gradient and Hessian of cost-function:

$$\mathbf{g}[n] = \nabla_w(J(\mathbf{w}[n]), \quad \mathbf{H}(\mathbf{w}[n]) = \nabla_w(\nabla_w(J(\mathbf{w})))|_{\mathbf{w}=\mathbf{w}[n]}.$$

- Optimal Solution:

$$2(\mathbf{H}[n]\mathbf{w} - \mathbf{H}[n]\mathbf{w}[n] + \mathbf{g}[n]) = 0.$$

- Weight update:

$$\mathbf{w}[n+1] = \mathbf{w}[n] - \mathbf{H}^{-1}[n]\mathbf{g}[n].$$

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- For optimal filtering problem:

$$\nabla_w(J(\mathbf{w}[n])) = -2(\mathbf{r}_{du} - \mathbf{R}_{uu}\mathbf{w}[n]), \quad \mathbf{H}[n] = 2\mathbf{R}_{uu}$$

- Weight update for optimal filtering problem:

$$\mathbf{w}[n+1] = \mathbf{w}[n] - \frac{1}{2}\mathbf{R}^{-1}(2\mathbf{R}_{uu}\mathbf{w}[n] - 2\mathbf{r}_{du})$$

- Tap-weights converge in a single iteration:

$$\mathbf{w}[n+1] = \mathbf{w}_{\mathrm{opt}}.$$

- Computational complexity price for faster convergence.