DKF and RLS Filters

- RLS state model:

$$\begin{aligned} d[n] &= \mathbf{w}_{\mathrm{mod}}^T \mathbf{u}[n] + \epsilon_o[n] \\ \mathbf{x}[n+1] &= \mathbf{x}[n], \mathbf{x}[n] = \mathbf{w}_{\mathrm{mod}}, \mathbf{C}[n] = \mathbf{u}^T[n] \end{aligned}$$

- Unforced dynamical system:

$$\mathbf{P}_-[n] = \mathbf{A}[n-1]\mathbf{P}_+[n-1]\mathbf{A}^T[n-1] = \mathbf{P}_+[n-1]$$

- Kalman gain for unit variance v[n]:

$$\mathbf{K}_2^{\mathrm{opt}}[n] = \frac{\mathbf{P}_-[n]\mathbf{C}^T[n]}{1 + \mathbf{C}[n]\mathbf{P}_-[n]\mathbf{C}^T[n]}$$

DKF and RLS filters

- RLS gain vector:

$$\mathbf{g}[n] = \frac{\lambda^{-1}\mathbf{P}[n-1]\mathbf{u}[n]}{1 + \lambda^{-1}\mathbf{u}^T[n]\mathbf{P}[n-1]\mathbf{u}[n]}$$

- One-one correspondence if :

$$\mathbf{P}^-[n] = \mathbf{P}_+[n-1] = \lambda^{-1}\mathbf{P}[n-1]$$

- DKF state estimate:

$$\hat{\mathbf{x}}_+[n] = \mathbf{A}[n-1]\hat{\mathbf{x}}_+[n-1] + \mathbf{K}_2[n]\alpha_{\mathrm{kal}}[n]$$

DKF and RLS Filters

- RLS Tap-weight update:

$$\mathbf{w}[n] = \mathbf{w}[n-1] + \mathbf{g}[n]\alpha_{rls}[n]$$

- Relate DKF state estimate and RLS tap-weights:

$$\hat{\mathbf{x}}_+[n] = \phi[n]\mathbf{w}[n]$$

- Solution for one-one correspondence:

$$\begin{aligned} \phi[n] &= \lambda^{-(n+1)/2}, \quad \mathbf{A}[n-1] = \lambda^{-1/2}\mathbf{I} \\ \mathbf{K}_2[n] &= \lambda^{-1/2}\mathbf{g}[n], \quad \alpha_{\mathrm{kal}}[n] = \lambda^{-n/2}\alpha_{\mathrm{rls}}[n] \end{aligned}$$

DKF and RLS Filters

- Further correspondence relations:

$$\mathbf{x}[n] = \lambda^{-n/2}\mathbf{w}_{\mathrm{mod}}, \quad y[n] = \lambda^{-n/2}d[n]$$

- RLS unforced state model:

$$\begin{aligned} \mathbf{x}[n+1] &= \lambda^{-1/2}\mathbf{x}[n] \\ y[n] &= \mathbf{u}^T[n]\mathbf{x}[n] + v[n]. \end{aligned}$$

- Correspondence relations : equivalence of RLS deterministic and DKF stochastic framework.