- Tap-weight adjustment of NLMS: w[n+1] – w[n] in the direction of u[n].
- Movement of tap-weight adjustment a function of correlation between u[n] and u[n-1].
- Larger correlation indicates u[n] and u[n-1] produces a smaller tap-weight movement.
- Need to pre-whiten observations to attain a faster uniform rate of convergence.

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- Modified Cost function:

$$J(n) = ||\mathbf{w}[n+1]-\mathbf{w}[n]||^2+\sum_{k=0}^{N-1}\lambda_k(d[n-k]-\mathbf{w}^T[n+1]\mathbf{u}[n-k]).$$

- Some definitions:

$$\begin{aligned}\mathbf{A}^T[n] &= [\mathbf{u}[n], \mathbf{u}[n-1], \ldots, \mathbf{u}[n-N+1]],\\ \Lambda^T[n] &= [\lambda_o, \lambda_1, \ldots, \lambda_{N-1}],\\ \mathbf{d}^T[n] &= [d[n], d[n-1], \ldots, d[n-N+1].\end{aligned}$$

- Matrix form:

$$J(n) = ||\mathbf{w}[n+1]-\mathbf{w}[n]||^2+(\mathbf{d}[n]-\mathbf{A}[n]\mathbf{w}[n+1])^T\Lambda.$$

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- Derivative w.r.s.t. Λ:

$$\mathbf{d}[n] = \mathbf{A}[n]\mathbf{w}[n+1].$$

- Derivative w.r.s.t. w[n+1]:

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \frac{1}{2}\mathbf{A}^T[n]\Lambda$$

- Tap-weight update:

$$\begin{aligned}\mathbf{w}[n+1] &= \mathbf{w}[n] + \tilde{\mu}\mathbf{A}_R^{\dagger}[n](\mathbf{d}[n] - \mathbf{A}[n]\mathbf{w}[n])\\ \tilde{\mu} &= \frac{\beta}{||\mathbf{u}[n]||^2+\delta}\end{aligned}$$

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- Alternative form of tap-weight update:

$$\mathbf{w}[n+1] = \left(\mathbf{I} - \tilde{\mu}\mathbf{A}_R^{\dagger}[n]\mathbf{A}[n]\right)\mathbf{w}[n]+\tilde{\mu}\mathbf{A}_R^{\dagger}[n]\mathbf{d}[n].$$

- Regularization for pseudo-inverse:

$$\tilde{\mathbf{A}}_R^{\dagger}[n] = \mathbf{A}^T[n](\mathbf{A}[n]\mathbf{A}^T[n]+\gamma\mathbf{I})^{-1}$$

- Better convergence characteristics comes at the price of computational complexity.