Applications of Adaptive Filtering

- Communication Systems : (a) channel equalization for dispersive channels, (b) multiple access interference mitigation in CDMA systems.
- Speech processing: (a) echo cancellation, speaker separation, (b) noise cancellation
- Biomedical applications: (i) ECG power-line interference removal, (ii) maternal-fetal ECG separation, donor heart-beat suppression.

Applications of Adaptive Filtering

- Radar: (a) multiple target tracking, (b) target clutter suppression.
- Image processing: (a) image restoration, (b) facial motion tracking.
- Pattern recognition : (a) neuron, (b) back-propagation.
- Array processing : (a) adaptive beam-forming, (b) generalized side-lobe canceller.

Optimal Filtering Problem

- Estimate d[n] from u[n] as:

$$\hat{d}[n] = \sum_{k=-\infty}^{\infty} w[k]u[n-k].$$

- Estimation error:

$$e[n] = d[n] - \sum_{k=-\infty}^{\infty} w[k]u[n-k].$$

- Optimal filter satisfies orthogonality principle:

$$E\{e[n]u^*[n-k]\} = 0, \quad k \in \pm \mathrm{I}$$

FIR Optimal Filtering Problem

- Cost function:

$$J(\mathbf{w}) = \sigma_d^2 - \mathbf{w}^H \mathbf{r}_{du} - \mathbf{r}_{du}^H \mathbf{w} + \mathbf{w}^H \mathbf{R}_{uu} \mathbf{w}.$$

- Optimal FIR filter:

$$\mathbf{R}_{uu}\mathbf{w}_{\text{opt}} = \mathbf{r}_{du}$$

- MMSE of optimal FIR filter:

$$\text{MMSE} = \sigma_d^2 - \mathbf{w}_{\text{opt}}^H \mathbf{r}_{du}.$$

Optimal FIR Filter

- Error Performance Surface:

$$J(\mathbf{w}) = MMSE + (\mathbf{w} - \mathbf{w}_o)^H \mathbf{R}_{uu} (\mathbf{w} - \mathbf{w}_o).$$

- Unique global minimum provided :

$$\mathbf{a}^H \mathbf{R}_{uu} \mathbf{a} > 0, \quad \mathbf{a} \in \mathbf{R}^n.$$

- Each tap-weight component contributes to MSE:

$$\mathbf{q} = \mathbf{V}^H(\mathbf{w} - \mathbf{w}_o), \ \mathbf{R}_{uu} = \mathbf{V}\Lambda\mathbf{V}^H, \ J(\mathbf{w}) = MMSE + \sum_{i=1}^{n} \lambda_i |q_i|^2$$

Principle of Orthognonality

- Estimation error orthogonal to observations:

$$e_o[n] \perp u[n-k], \quad k \in I.$$

- Optimal filter : statistical closest point projection of d[n] onto space spanned by observations

$$u[n-k], \quad k \in \ \pm I$$

- Optimal filter output orthogonal to estimation error

$$E\{y_o[n] e_o^*[n-k]\} = 0$$

Properties of Correlation Matrices

- The correlation matrix associated with WSS random processes is symmetric Toeplitz :

$$\{\mathbf{R}_{xx}\}_{ij} = \{\mathbf{R}_{xx}\}_{i-j}, \quad \mathbf{R}_{xx}^T = \mathbf{R}_{xx}.$$

- The correlation matrix associated with a WSS random process is positive semi-definite:

$$\mathbf{a} \in \mathbf{R}^n \longrightarrow \mathbf{a}^T \mathbf{R}_{xx} \mathbf{a} \geq 0.$$

Correlation Matrices Contd.

- The correlation matrix R_{xx} can be expanded as:

$$\mathbf{R}_{xx} = \mathbf{V}\Lambda\mathbf{V}^T = \sum_{i=1}^{n} \lambda_i \vec{v}_i \vec{v}_i^T.$$

- Eigen-values are real and positive:

$$\mathrm{Imag}(\lambda_i) = 0, \quad \lambda_i \geq 0, \quad i = 1, 2, \ldots, n.$$

- Rayleigh quotient bounded by smallest/largest eigenvalue:

$$\lambda_{\min} \leq \frac{\mathbf{a}^T \mathbf{R}_{xx} \mathbf{a}}{\mathbf{a}^T \mathbf{a}} \leq \lambda_{\max}.$$

- Condition number : ratio of largest to smallest eigenvalue

$$\chi(\mathbf{R}_{xx}) = \frac{\lambda_{\max}}{\lambda_{\min}}.$$

- Eigenvalues bounded by power spectrum extrema:

$$\chi(\mathbf{R}_{xx}) \leq \frac{S_{\max}}{S_{\min}}.$$

- Discrete—time random signal x[n] can be expanded in terms of the eigenvectors of R_{xx}:

$$x[n] = \sum_{i=1}^{M} c_i[n] v_i.$$

- Eigenfilter optimal filter in the output SNR sense and is the stochastic counterpart of matched filtering.

$$\mathrm{w}_{\mathrm{opt}} = \arg_{\mathbf{w}} \left\{ \min \left(\frac{\mathbf{w}^T \mathbf{R}_{xx} \mathbf{w}}{\mathbf{w}^T \mathbf{w}} \right) \right\}.$$

- For real random signals, R_{xx} is centro-symmetric, i.e., entries symmetric about main diagonal.
- R_{xx} can be partitioned into blocks as:

$$\mathbf{R}_{M+1} = \begin{pmatrix} R_{xx}(0) & \mathbf{r}^H \\ \mathbf{r} & \mathbf{R}_M \end{pmatrix}.$$

- Nonsingular due to the presence of additive noise:

$$\tilde{\mathbf{R}}_{xx} = \mathbf{R}_{xx} + \sigma^2 \mathbf{I}.$$