Overview of Linear Prediction

The Lenvinson-Durbin algorithm is a recursive solution to the symmetric–Toeplitz system of equations:

$$\mathbf{R}_x^{(j)}\mathbf{a}_j = \epsilon_j \mathbf{u}_1.$$

that arises in the context of the linear prediction or the AR-modeling problem:

$$\begin{aligned}
\hat{x}[n] &= \sum_{k=1}^{p} a_p[k]x[n-k] \\
e[n] &= x[n] - \sum_{k=1}^{p} a_p[k]x[n-k] \\
R_{xx}[l] &= \sum_{k=1}^{p} a_p[k]R_{xx}[l-k], \quad l = 1, 2, p \\
\epsilon_p &= R_{xx}[0] + \sum_{k=1}^{p} a_p[k]R_{xx}[k].
\end{aligned}$$

Levinson Durbin Recursion

1. Intialization: $\epsilon_o = R_{xx}[0]$, $a_0[0] = 1$.
2. Reflection coefficient for next stage:

$$\begin{aligned}
\gamma_j &= R_{xx}[j+1] + \sum_{k=1}^{j} a_j[k]R_{xx}[j+1-k] \\
\Gamma_{j+1} &= -\frac{\gamma_j}{\epsilon_j}.
\end{aligned}$$

3. Coefficient update:

$$\mathbf{a}_{j+1} = \begin{pmatrix} 1 \\ a_j[1] \\ a_j[2] \\ \vdots \\ a_j[j] \\ 0 \end{pmatrix} + \Gamma_{j+1} \begin{pmatrix} 0 \\ a_j[j] \\ a_j[j-1] \\ \vdots \\ a_j[1] \\ 1 \end{pmatrix}$$

4. Modeling error update:

$$\epsilon_{j+1} = \epsilon_j \left(1 - \Gamma_{j+1}^2\right).$$

5. Go back to reflection coefficent update.