ECE-595, FALL 2011

ADAPTIVE FILTER THEORY

RLS Algorithm: Convergence in Mean

The RLS tap-weights are solutions to

$$\mathbb{R}[n]\, \underline{w}_{RLS}[n] = \underline{z}[n],$$

$$\mathbb{R}[n] = \sum_{i=1}^{n} \lambda^{n-i}\, \underline{u}[i]\, \underline{u}^T[i]$$

$$\underline{z}[n] = \sum_{i=1}^{n} \lambda^{n-i}\, d[i]\, \underline{u}[i]$$

In recursive form:

$$\mathbb{R}[n] = \lambda\, \mathbb{R}[n-1] + \underline{u}[n]\, \underline{u}^T[n]$$

$$\underline{z}[n] = \lambda\, \underline{z}[n-1] + d[n]\, \underline{u}[n]$$

With $\lambda = 1$

$$\mathbb{R}[n] = \mathbb{R}[n-1] + \underline{u}[n]\, \underline{u}^T[n]$$

$$\underline{z}[n] = \underline{z}[n-1] + d[n]\, \underline{u}[n]$$

Solving the recursion:

$$\mathbb{R}[n] = \mathbb{R}[0] + \sum_{i=1}^{n} \underline{u}[i]\,\underline{u}^T[i]$$

$$z[n] = \cancelto{0}{z[0]} + \sum_{i=1}^{n} d[i]\,\underline{u}[i]$$

Incorporating the regression model for the SOI $d[n]$:

$$d[n] = w_o^T u[n] + e_o[n],$$

$$z[n] = \sum_{i=1}^{n} \left(w_o^T \underline{u}[i] + e_o[i] \right) \underline{u}[i]$$

$$z[n] = \sum_{i=1}^{n} u[i]\,u^T[i]\,w_o + \sum_{i=1}^{n} e_o[i]\,u[i]$$

$$= (\mathbb{R}[n] - \mathbb{R}[0])\,w_o + \sum_{i=1}^{n} e_o[i]\,\underline{u}[i]$$

$$w_{RLS}[n] = \mathbb{R}^{-1}[n]\,(\mathbb{R}[n] - \mathbb{R}[0])\,w_o + \mathbb{R}^{-1}[n] \sum_{i=1}^{n} e_o[i]\,\underline{u}[i]$$

$$\omega_{RLS}[n] = \omega_o - \mathbb{R}^{-1}[n]\,\mathbb{R}[0]\,\omega_o + \mathbb{R}^{-1}[n] \sum_{i=1}^{n} e_o[i]\,\underline{u}[i]$$

Assuming $e_o[n]$, $\underline{u}[n]$ are uncorrelated:

$$E\{\omega_{RLS}[n]\} = \omega_o - \mathbb{R}^{-1}[n]\,\mathbb{R}[0]\,\omega_o$$

$$\mathbb{R}[0] = \delta\,\mathbb{I}$$

$$E\{\omega_{RLS}[n]\} = \omega_o - \mathbb{R}^{-1}[n]\cdot\delta\,\omega_o$$

Assuming correlation ergodicity

$$\mathbb{R}^{-1}[n] = \frac{1}{n}\mathbb{R}_{uu}^{-1}, \quad n > M$$

$$E\{\omega_{RLS}[n]\} = \omega_o - \frac{\delta}{n}\mathbb{R}_{uu}^{-1}\,\omega_o$$

Note that $\mathbb{R}_{uu}^{-1}\,\omega_o \neq p$

$$\lim_{n\to\infty} E\{\omega_{RLS}[n]\} = \omega_o - \lim_{n\to\infty}\frac{\delta}{n}\mathbb{R}_{uu}^{-1}\,\omega_o$$

$$= \omega_o$$

$\longrightarrow$ RLS tap-weight bias is non-zero for finite n

$\longrightarrow$ Bias becomes zero asymptotically.