Solution to Problem Set # 4.0
ECE-595, Fall 2006
Spatial Array Processing
University of New Mexico, Albuquerque

In this exercise we compare the performance of the conventional beamformer with that of the *minimum variance* (MVDR) beamformer for different scenarios. The CB is the solution to a optimization problem of the form:

$$\mathbf{w}_{\text{opt}} = \max_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_y \mathbf{w}, \text{ subject to } ||\mathbf{w}|| = 1,$$

where $\mathbf{R}_y$ is the spatio-temporal correlation matrix that takes the form:

$$\mathbf{R}_y = A^2 \mathbf{a}(\phi_o)\mathbf{a}^H(\phi_o) + \sigma_n^2 \mathbf{I},$$

where ϕ_o is the *angle of incidence* of the incident plane wave on the ULA. The solution to this problem is of the form $\mathbf{w}_{\text{opt}} = \mathbf{a}(\phi_o)$ assuming that there is only one signal present. The corresponding average power estimate is of the form:

$$P_{\text{ave}}(\phi) = \mathbf{w}_{\text{opt}}^H \mathbf{R}_y \mathbf{w}_{\text{opt}} = \mathbf{a}(\phi)^H \mathbf{R}_y \mathbf{a}(\phi).$$

The MVDR on the other hand is the solution to an optimization problem of the form:

$$\mathbf{w}_{\text{opt}} = \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_y \mathbf{w}, \text{ subject to } \mathbf{w}^H \mathbf{a}(\phi) = 1.$$

The solution to this problem is of the form:

$$\mathbf{w}_{\text{opt}} = \frac{\mathbf{R}_y^{-1}\mathbf{a}(\phi)}{\mathbf{a}^H(\phi)\mathbf{R}_y^{-1}\mathbf{a}(\phi)}$$

and the corresponding PSD estimate is of the form:

$$P_{\text{ave}}(\phi) = \frac{1}{\mathbf{a}^H(\phi)\mathbf{R}_y^{-1}\mathbf{a}(\phi)}$$

These two methods will be equivalent when:

1. there is only one signal impinging on the array and it is the ideal signal vector, i.e., no distortion of the plane wave.
2. the noise corrupting the observations is spatially white.
3. we are looking directly at the signal, i.e., $\mathbf{a}(\phi) = \mathbf{a}(\phi_o)$.

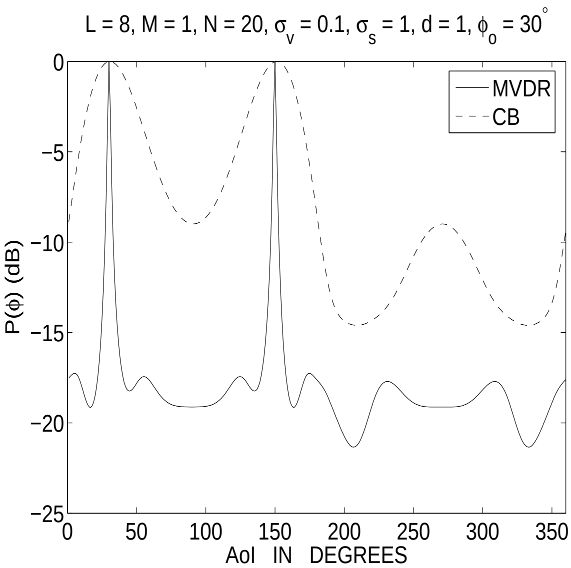

Figure 1: Performance of the MVDR beamformer for the case where one ideal signal is incident on a ULA with spatially white noise. Notice that the peaks of the MVDR approach as sharper and narrower than the conventional beamformer. This can be attributed to the fact that MVDR beamformer is the solution to a constrained minimization problem where the signal of interest is passed without distortion. The CB is the solution to a constrained maximization problem.

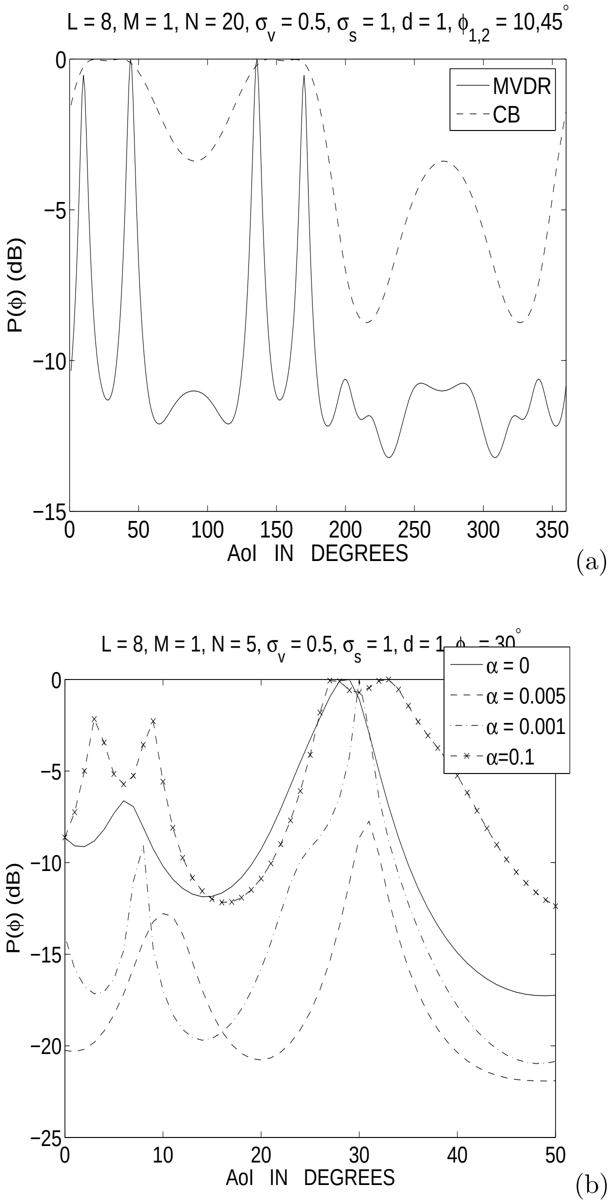

Figure 2: (a) Performance of the MVDR beamformer when there are two signals impinging on a ULA. Note that the MVDR method is able to resolve the peaks while the CB is unable to do so. Also note the presence of the steep nulls in the MVDR spectrum that are absent in the CB spectrum, (b) effect of diagonal loading on the performance of the MVDR beamformer. Note the effect that the various value have on the MVDR solution and the location of the peaks.