

# WIDEBAND FINGERPRINT DEMODULATION VIA BI-DIMENSIONAL MULTIRATE FREQUENCY TRANSFORMATIONS

Wenjing Liu and Balu Santhanam

Department of ECE, University of New Mexico  
Albuquerque, NM: 87131-001  
Email: wenjing,bsanthan@unm.edu

## ABSTRACT

In prior work, the AM-FM signal model coupled with dominant component analysis has been used for fingerprint extraction for eventual fingerprint recognition. In earlier work by the authors, multirate frequency transformations were employed to transform wideband signals into narrowband signals to effect wideband AM-FM demodulation of both 1D and 2D signals. In this paper, we apply the 2D, wideband AM-FM energy-based demodulation approach towards AM-FM feature extraction and recognition. Simulation results are used to demonstrate the efficacy of the proposed approach.

**Keywords:** Wideband image, AM-FM demodulation, multirate frequency transformations, higher order energy operator, fingerprint image analysis

## 1. INTRODUCTION

The amplitude-modulation frequency-modulation (AM-FM) [2] representation model has found various applications with images recently including image analysis, texture processing and fingerprint classification [5, 6, 7]. According to earlier work by Havlicek, Bovik et al., nonstationary images can be modeled as superpositions of multiple AM-FM components [7]:

$$I(x, y) = \sum_{i=1}^n a_i(x, y) \cos(\phi_i(x, y)). \quad (1)$$

The multi-component AM-FM image is first decomposed by a set of bandpass filters such as Gabor filterbanks or the *Bi-dimensional empirical mode decomposition* (BEMD). Each resulting monocomponent AM-FM image is further demodulated into corresponding instantaneous amplitude (IA)  $a(x, y)$  and instantaneous frequency vector (IF),

$$\nabla\phi(x, y) = \left[ \frac{\partial\phi(x, y)}{\partial x}, \frac{\partial\phi(x, y)}{\partial y} \right]^T. \quad (2)$$

In particular, the IA of the image depicts the contrast present in the image while the IF reveals the locally emergent frequency variation. Conventional image demodulation approaches are either based on 2D extension of the analytic signal (AS) or 2D extension of the energy separation algorithm (ESA) with additional processing techniques such as *dominant component analysis* (DCA).

However in both prior approaches, narrowband assumptions were imposed on each AM-FM component of the image. For

example, most literature inexplicitly assumes the AM-FM image to be globally wideband yet each of its component to be locally narrowband. In general, both the IA,  $a(x, y)$  and the IF,  $\nabla\phi(x, y)$  of a single component are assumed to be slowly varying, otherwise the approximations inherent in most demodulation approaches are no longer valid and incur significant error especially under the wideband scenario.

In prior work, we have proposed the novel *bi-directional multirate frequency transformation* (MFT) approach [1, 4] that can be combined with a variety of demodulation techniques to enhance their demodulation performance traditionally limited by the narrowband constraints on the frequency modulation part of the monocomponent AM-FM image [4]. In this paper, we apply the BMFT approach to the fundamental AM-FM component of fingerprint images, typically a wideband image, to demonstrate that significantly improved demodulation results and enhanced IF features with better defined ridges are achievable.

## 2. BMFT PRIMER

Here we briefly review the BMFT approach developed in prior work, assume that the input is a monocomponent wideband FM image of the form:

$$J(x, y) = A \cos(\phi(x, y)). \quad (3)$$

It is first compressed in frequency domain by appropriate factors  $\vec{R} = \text{diag}[R_x, R_y]$ , which corresponds to spatial expansion given by

$$J_1(x, y) = A \cos\left(\phi\left(\frac{x}{R_x}, \frac{y}{R_y}\right)\right). \quad (4)$$

Then we heterodyne the resultant image by a frequency translation vector  $\vec{\Omega} = [\Omega_x, \Omega_y]$  via

$$\begin{aligned} J_2(x, y) &= J(x, y) \cos(\Omega_x x) \cos(\Omega_y y) \\ &= \frac{A}{2} \cos\left(\Omega_x x + \phi\left(\frac{x}{R_x}, \frac{y}{R_y}\right)\right) \cos(\Omega_y y) \\ &\quad + \frac{A}{2} \cos\left(\Omega_x x - \phi\left(\frac{x}{R_x}, \frac{y}{R_y}\right)\right) \cos(\Omega_y y) \\ &= \frac{A}{4} \cos\left(\Omega_x x + \Omega_y y + \phi\left(\frac{x}{R_x}, \frac{y}{R_y}\right)\right) \\ &\quad + \frac{A}{4} \cos\left(\Omega_x x - \Omega_y y + \phi\left(\frac{x}{R_x}, \frac{y}{R_y}\right)\right) \\ &\quad + \frac{A}{4} \cos\left(\Omega_x x + \Omega_y y - \phi\left(\frac{x}{R_x}, \frac{y}{R_y}\right)\right) \\ &\quad + \frac{A}{4} \cos\left(\Omega_x x - \Omega_y y - \phi\left(\frac{x}{R_x}, \frac{y}{R_y}\right)\right). \end{aligned} \quad (5)$$

A 2D separable bandpass filter then extracts the desired high-frequency term through

$$\begin{aligned}\tilde{J}(x, y) &= J_2(x, y) * h_{BP}(x, y) \\ &\approx \frac{A}{4} \cos\left(\Omega_x x + \Omega_y y + \phi\left(\frac{x}{R_x}, \frac{y}{R_y}\right)\right) \\ &= \frac{A}{4} \cos\left(\tilde{\phi}(x, y)\right).\end{aligned}\quad (6)$$

The BMFT approach has two specific goals:

1. Increase the carrier (or mean) frequencies of the modulation in both dimensions via the frequency translation vector  $\tilde{\Omega} = [\Omega_x, \Omega_y]$ .
2. Reduction of the bandwidth of the modulating image by the appropriate conversion factors  $\mathbf{R} = \text{diag}[R_x, R_y]$ .

The motivation behind these two goals is to increase the CR/FD and CR/IB ratios of the image on both directions and enable the application of narrow band AM-FM demodulation [1, 4]. We recover the IF of the input image from the IF estimation of the transformed image  $\tilde{J}(x, y)$ . The IF components of  $J(x, y)$  and  $\tilde{J}(x, y)$  are given by

$$\begin{aligned}\Omega_1(x, y) &= \frac{\partial \phi(x, y)}{\partial x}, \quad \Omega_2(x, y) = \frac{\partial \phi(x, y)}{\partial y}, \\ \tilde{\Omega}_1(x, y) &= \frac{\partial \tilde{\phi}(x, y)}{\partial x}, \quad \tilde{\Omega}_2(x, y) = \frac{\partial \tilde{\phi}(x, y)}{\partial y}.\end{aligned}$$

The IF and IA estimates of the input image are computed via:

$$\Omega_1(x, y) = R_x \left( \tilde{\Omega}_1(R_x x, R_y y) - \Omega_x \right), \quad (7)$$

$$\Omega_2(x, y) = R_y \left( \tilde{\Omega}_2(R_x x, R_y y) - \Omega_y \right), \quad (8)$$

$$A(x, y) = \tilde{A}(R_x x, R_y y), \quad (9)$$

where  $\tilde{\Omega}_1(R_x x, R_y y)$  and  $\tilde{\Omega}_2(R_x x, R_y y)$  represent spatial compression (or frequency expansion) of the IF estimation for the transformed image  $\tilde{J}(x, y)$ . We further assume that the variation of the IA of the image is slow varying in either dimension.

In order to implement the BMFT in discrete time, we replace compression and expansion in frequency domain by their discrete equivalences. Note that the compression in frequency domain corresponds to interpolation while the expansion corresponds to decimation. As a result, the block diagram of the BMFT demodulation is depicted in Fig 1. The BMFT is implemented through discrete-time operations of interpolation, heterodyning, and bandpass filtering.

### 3. DEMODULATION OF FINGERPRINT IMAGES

#### 3.1. Dominant Component Analysis

According to prior work by Pattichis et al., a given image  $I(x_1, x_2)$  can be approximated by

$$I(x_1, x_2) \approx a(x_1, x_2) \sum_n H_n \exp[jn\phi(x_1, x_2)], \quad (10)$$

where  $H_n$  denotes the Fourier series coefficient and the subscript has been dropped from the phase function.

Let  $g$  denote the impulse response of a linear system, the response  $t(x_1, x_2)$  to the given image expressed as a sum of AM-FM harmonics can be approximated by [2, 3]:

$$\begin{aligned}t(x_1, x_2) &\approx a(x_1, x_2) \sum_n |G[n\nabla\phi(x_1, x_2)]| H_n \\ &\quad \times \exp\{jn\nabla\phi(x_1, x_2) + \angle G[n\nabla\phi(x_1, x_2)]\}\end{aligned}\quad (11)$$

where  $G$  denotes the Fourier transform of  $g$  and  $\angle$  denotes the angle argument symbol. A realistic assumption is to require that the local power captured in the fundamental AM-FM component is always higher than any other harmonic. Hence the patterns and features of the fingerprint are likely to be captured by the fundamental AM-FM component.

DCA is then applied to the fingerprint image to obtain the fundamental AM-FM component:

- Filter the fingerprint image via a set of Gabor channel filters with response  $g_i$ , we obtain different channel output images  $t_i = I * g_i$ .
- Estimate the instantaneous amplitude  $a_i(x_1, x_2)$  for each channel using conventional demodulation techniques. For each pixel, we select the channel with the maximum amplitude estimate as the output.
- Smooth the resultant fundamental AM-FM component using 2D smoothing filters to reduce the noise.

#### 3.2. Fundamental AM-FM Component Demodulation

Since the fundamental AM-FM component is determined by outputs of multiple channels, it is in general not as narrowband as each channel output. Hence traditional demodulation techniques based on narrowband assumption may incur significant error. This motivates the application of the combination of the BMFT framework and *higher-order energy operators* (HOEO) to demodulate the fundamental AM-FM component. Salzenstein, Diop and Boudraa recently proposed an extension of the classical ESA using the HOEO and was shown to provide better performance for narrowband AM-FM images than the classical 2D ESA [4]. The  $k$ -order HOEO in 1D for any given signal  $s(t)$  is defined by

$$\Psi_k[s(t)] = \frac{\partial s(t)}{\partial t} \frac{\partial^{k-1} s(t)}{\partial t^{k-1}} - s(t) \frac{\partial^k s(t)}{\partial t^k}, \quad (12)$$

where  $\Psi_2$  refers to the commonly used Teager-Kaiser energy operator. For a given image  $I(k, l)$ , the discrete-time higher order demodulation algorithm (DHODA) can be summarized via:

$$I_1(k, l) = \frac{1}{2} [I(k+1, l) - I(k-1, l)],$$

$$I_2(k, l) = \frac{1}{2} [I(k, l+1) - I(k, l-1)],$$

$$I_{12}(k, l) = \frac{1}{2} [I_2(k+1, l) - I_2(k-1, l)],$$

$$\begin{aligned}\Psi_2[I(k, l)] &= \{2[I(k, l)]^2 - I(k-1, l)I(k+1, l) \\ &\quad - I(k, l-1)I(k, l+1)\} + 2[I_1(k, l)I_2(k, l) \\ &\quad - I(k, l)I_{12}(k, l)],\end{aligned}$$

$$I_{12}^1(k, l) = \frac{1}{2} [I_{12}(k+1, l) - I_{12}(k-1, l)],$$

$$I_{12}^2(k, l) = \frac{1}{2} [I_{12}(k, l+1) - I_{12}(k, l-1)],$$

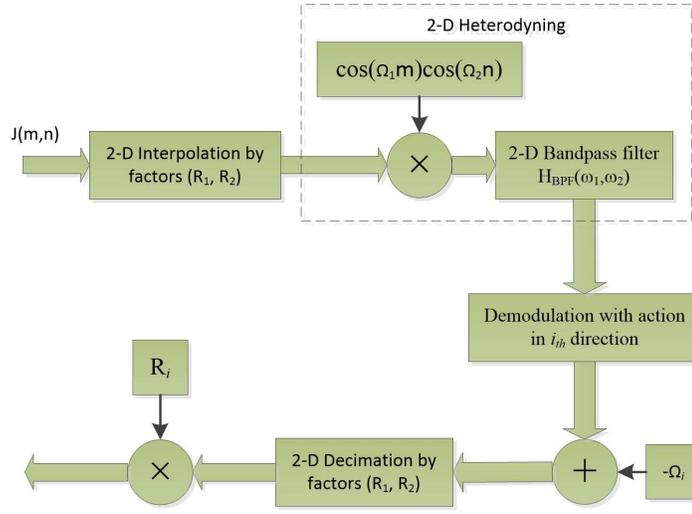


Fig. 1. Block diagram of the BMF based wideband image demodulation approach.

$$|\hat{a}(k, l)| = \left( \frac{\Psi_2[I_1(k, l)]\Psi_2[I_2(k, l)]}{\Psi_2[I_{12}^1(k, l) + I_{12}^2(k, l)]} \right)^{1/2}, \quad (13)$$

$$|\hat{\Omega}_1(k, l)| = \arcsin \left( \left( \frac{\Psi_2[I_{12}(k, l)]}{\Psi_2[I_2(k, l)]} \right)^{1/2} \right), \quad (14)$$

$$|\hat{\Omega}_2(k, l)| = \arcsin \left( \left( \frac{\Psi_2[I_{12}(k, l)]}{\Psi_2[I_1(k, l)]} \right)^{1/2} \right), \quad (15)$$

where  $\hat{a}(k, l)$  is the IA estimation while  $\hat{\Omega}_1(k, l)$  and  $\hat{\Omega}_2(k, l)$  are the IF estimates along the spatial axes of the image<sup>1</sup>.

#### 4. EXPERIMENTAL RESULTS

A separable FIR bandpass filter with 1025 taps in each direction are employed in the BMFT heterodyne module. Separable heterodyning along with separable bandpass filters are chosen to reduce the complexity of the BMFT system. A multichannel Gabor filterbank with eight rays or orientations and nine radial frequencies per ray are used to isolate the fundamental FM component [7]. A  $3 \times 3$  Gaussian filter is used to smooth the DCA image prior to demodulation. Residual low frequencies appear as a background in the DCA image.

Figure 2(a) describes a finger print image and Fig. (2b) describes the fundamental AM–FM component extracted using the Gabor filterbank described before. Figure 3(b,c) depicts the IF needle plot associated with the application of the directional Hilbert transform [7] to the original image and the fundamental FM component. The needle plot of the IF of the dominant AM–FM component shows that the fundamental AM–FM component has better defined ridges in comparison to direct demodulation of the original image.

Figure 4 (a,b) depict the application of the BMFT approach with directional Hilbert transform demodulation of the dominant component for conversion factors of 8 and 16 respectively. The IF needle plots further depict a significant improvement in the IF needle plots in comparison to the results without the BMFT.

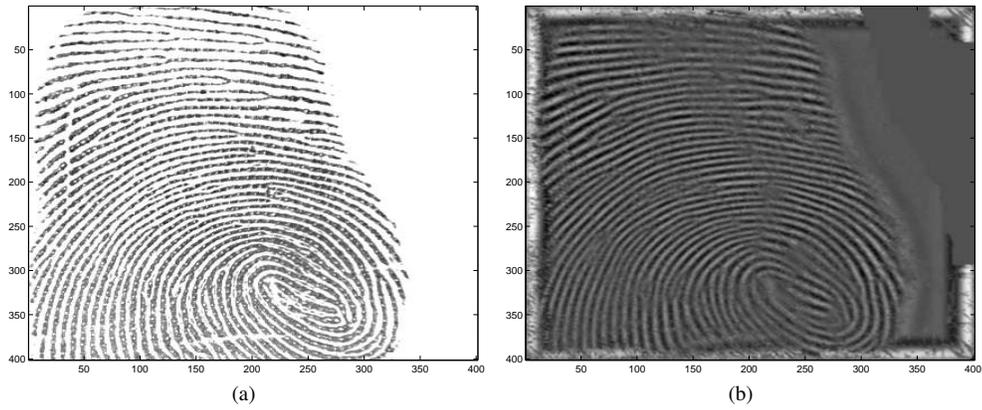
<sup>1</sup>ESA based approaches require IF components ranging between 0 and  $\frac{\pi}{2}$ , or up to a fourth of the sampling frequency due to the inverse sine function.

#### 5. CONCLUSION

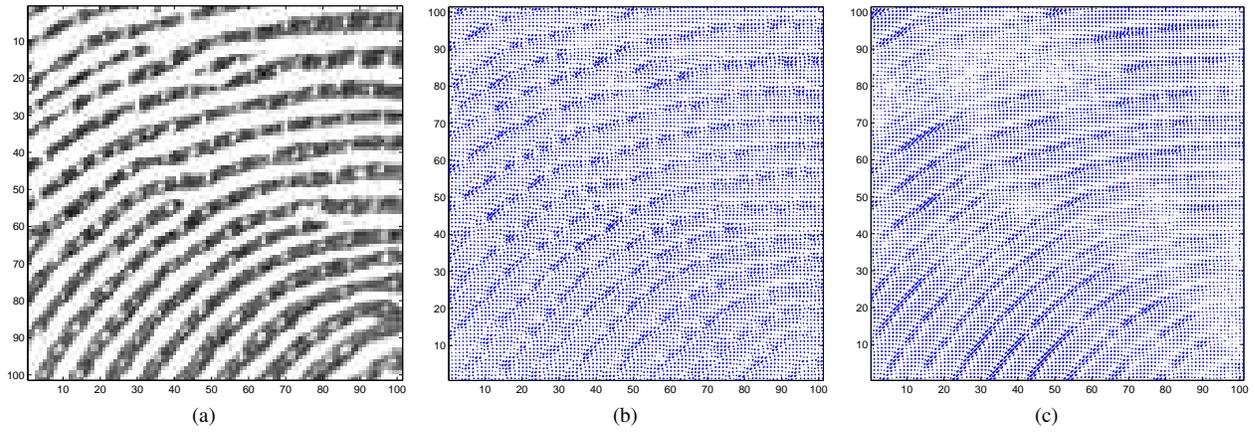
In this paper, we have applied recent work on wideband image demodulation using bi-dimensional multirate frequency transformations and higher-order energy operators to the problem of demodulating wideband fingerprint images. Results indicate clearly that significant reduction in the IF demodulation error and more refined IF derived features with better defined ridges can be attained by combining dominant AM–FM component analysis and the BMFT framework to obtain better recognition performance with latent fingerprint images.

#### 6. REFERENCES

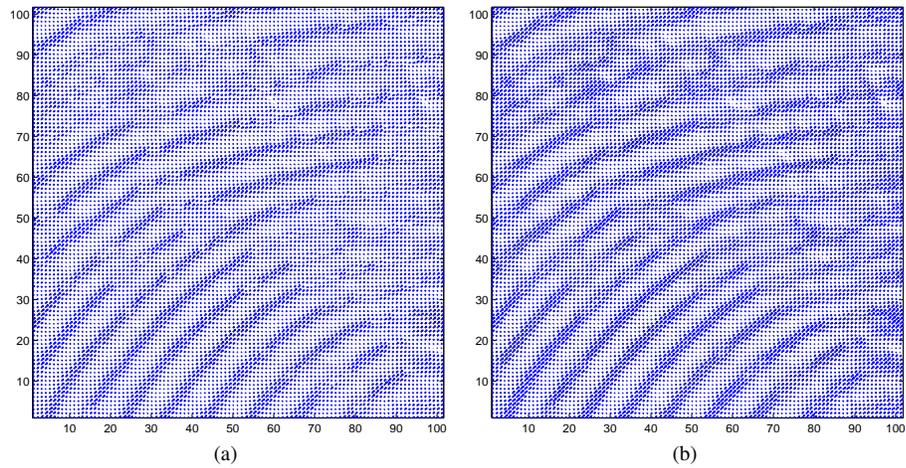
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**Fig. 2.** Fundamental AM-FM component. (a) Original fingerprint image. (b) Fundamental AM-FM component extracted via DCA using the Gabor filterbank approach outlined in [5, 6].



**Fig. 3.** (a) Original zoomed in fingerprint image. (b) IF needle plot of the original fingerprint via the directional Hilbert transform. (c) IF needle plot of the estimated fundamental AM-FM component via the directional Hilbert transform.



**Fig. 4.** (a) IF needle plot of the estimated fundamental AM-FM component via the BMFT with factors [8, 8] and the DHODA. (b) IF needle plot of the estimated fundamental AM-FM component via the BMFT with factors [16, 16] and the DHODA.