Performance Analysis on Synthetic Aperture Radar-based Vibration Estimation in Clutter

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***Abstract*—Recently, a time-frequency method based on the discrete fractional Fourier transform (DFrFT) was proposed for estimating target vibrations using synthetic aperture radar (SAR). Later on, a subspace method was incorporated into the DFrFT-based method. It is shown that the subspace method provides better performance than the direct DFrFT-based method in noise. However, the performance of these two methods has not been studied in clutter that cause strong interference with signals from vibrating targets in real-world applications. In this paper, the performance of the two vibration estimation methods in clutter is characterized and compared via simulations. Simulation results demonstrate that the DFrFT-based method, that yielded reliable results when signal-to-clutter ratios (SCR) exceeds 18 dB, now yields reliable results when SCR exceeds 8 dB with the incorporation of the subspace method. Experimental results show that the subspace method correctly estimates the vibration frequency of a 7 Hz vibration from actual SAR data at an estimated SCR of 14 dB.**

***Index Terms*—synthetic aperture radar, micro-Doppler effect, fractional Fourier transform, subspace methods, vibration estimation, clutter**

I. INTRODUCTION

Vibration signatures associated with various structures bear vital information about these structures; therefore, it is key to have the capability of estimating the vibration signatures. A lack of physical access to these structures typically makes the problem of detecting such activities challenging by current means. Synthetic aperture radar (SAR) has already been proven as a highly effective remote-imaging technique [1]. Moreover, it is inherently capable of sensing Doppler shifts in the electromagnetic returns from objects, thereby allowing us to detect vibrations [2].

The target vibration can be estimated through successive chirp-rate estimations using the discrete fractional Fourier transform (DFrFT), which is inherently geared toward chirp-rate estimation[3], [4], [5], [6], [7]. Later on, a subspace method was incorporated into the DFrFT-based method and it provides better performance than the direct DFrFT-based method in noise [8], [9].

In real-world applications, clutter prevents the DFrFT-based vibration estimation method from yielding reliable results. The performance of the DFrFT-based vibration estimation method is left un-investigated. In this paper, we investigate and compare the performance of both the DFrFT-based method and the subspace method in simulated Gamma-distributed clutter signals. Simulation results demonstrate that the DFrFT-based method yields reliable results with signal-to-clutter ratios (SCRs) $>$ 18 dB and the subspace method yields reliable results with SCRs $>$ 8 dB. Experimental results demonstrate that the subspace method correctly estimates the vibration frequency of a 7 Hz vibration from actual SAR data with an estimated SCR of 14 dB.

II. THEORY

A. Signal Model

In synthetic aperture radar (SAR), low-level target vibrations can be estimated by estimating the instantaneous vibration displacements (in range), Δx, from [4]

$$s[n] \approx \sigma \exp\left[-j\left(k_y y n + k_v \Delta x[n] + \phi\right)\right] + c[n] + w[n], \quad (1)$$

where y and σ are the cross-range position and the reflectivity of the vibrating target, respectively. We define $s[n]$ as the signal of interest (SoI). The noise, denoted by $w[n]$, is assumed to be white and Gaussian. The scaling parameters (k_y, k_v) are known. Signals from other static targets at the same range locations are all represented by $c[n]$ and

$$c[n] = \sum_i \sigma_i \exp\left[-j\left(k_y y_i n + \phi_i\right)\right]. \quad (2)$$

Conventionally $c[n]$ is referred to as the "clutter signal" or simply "clutter". The signal-to-clutter ratio (SCR) is defined as

$$\mathrm{SCR} = 20 \log \frac{|\sigma_v|}{|\sigma_c|}, \quad (3)$$

where σ_v is the reflectance of the vibrating target and σ_c is the average reflectance of 1 m^2 of clutter. We assume that the reflectance of the clutter pixel is Gamma-distributed with the shape parameter $k_{gam} = \sigma_c$. A circle-shaped averaging filter with radius of 1 meter is applied to the clutter to simulate

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the correlations in neighbor pixels. The SNR is defined with respect to the SoI as

$$\mathrm{SNR} = 10 \log \frac{E_s}{E_w}, \tag{4}$$

where E_s is the energy of the signal (including both vibrating targets and clutter) and E_w is the energy of the noise.

B. Methods

1) The DFrFT-based method: The DFrFT-based vibration estimation method is directly applied to the SoI in (1). The SoI is first approximated by chirp signals in successive small time windows (also called sub-apertures). In each sub-aperture the chirp rate of the signal is linearly proportional to the instantaneous vibration acceleration. Then, the DFrFT is applied to the signals in each sub-aperture to estimate their chirp rates in sliding time windows. As a result, the instantaneous vibration accelerations are estimated and the vibration frequencies are estimated from the spectrum of estimated instantaneous accelerations. The reader may refer to [4] for more details.

2) The subspace method: In the subspace method, the chirp rate is estimated after separating the signal into the signal subspace and the noise subspace [8]. It first takes the discrete Fourier transform (DFT) of the row projection of the DFrFT spectrum (with respect to the chirp rate) to obtain a vector such as

$$x_{cr}[r] = \mathbf{DFT}^{-1}\left(\sum_{k=0}^{N-1} |\mathbf{X}_k[r]|^p\right)^{1/p}, \tag{5}$$

where $\mathbf{X}_k[r]$ is the DFRFT spectrum of the SoI. The covariance matrices $\mathbf{R}_{cr}$ for $x_{cr}[r]$ is estimated in order to use the subspace methods to estimate the chirp rate. Virtually the covariance matrix is estimated from noisy observations using sample covariance matrix. Biased covariance estimates are preferred over unbiased estimates for obtaining positive definite matrix [10]. Eigenvalue decomposition of the covariance matrix yields the desired pseudo-subspace decomposition as

$$\mathbf{R}_{cr} = \mathbf{V}_{crs}\mathbf{\Lambda}_{crs}\mathbf{V}_{crs}^T + \mathbf{V}_{crn}\mathbf{\Lambda}_{crn}\mathbf{V}_{crn}^T, \tag{6}$$

where the subscripts s and n denote the signal and noise subspaces, respectively. Unlike the case of sinusoids in noise, the signal and noise subspaces in this case do not completely separate [11], [12]. Subspace methods such as multiple signal classification (MUSIC), minimum-norm and minimum-variance can be applied to the covariance matrices to estimate the chirp rate of the signal [9]. Results in [9] shows that in general the minimum-norm method in combination of the 3-norm has better performance than the MUSIC-based methods. However, in the problem of vibration estimation, the vibration-induced chirp rates are very small. In this case, the MUSIC/root-MUSIC algorithm in combination with the 1-norm yields more reliable estimate then the minimum-norm method. As such, we use MUSIC/root-MUSIC algorithm to estimate the vibration accelerations.

III. SIMULATIONS AND PERFORMANCE ANALYSIS

We analyze the performance of both the DFrFT-based method and the subspace method in clutter by means of simulation. An airborne spotlight-mode SAR working in the K_u band is simulated and its nominal resolution is 0.33 m in both directions. In the simulation, the SNR is fixed at 30 dB. Simulation results demonstrate that the direct DFrFT-based method, in general, yields reliable vibration spectra with SCRs $>$ 18 dB. The subspace method, on the other hand, generally yields reliable vibration spectra with SCRs $>$ 8 dB. The estimated accelerations and vibration spectra of a 1-mm, 5-Hz vibration using both the DFrFT-based method and the subspace method with SCR = 15 dB are shown in Figs. 1-4. The subspace method successfully estimates the 5-Hz vibration frequency while the DFrFT-based method fails to do so according to the estimated vibration spectra shown in Fig. 3 and Fig. 4.

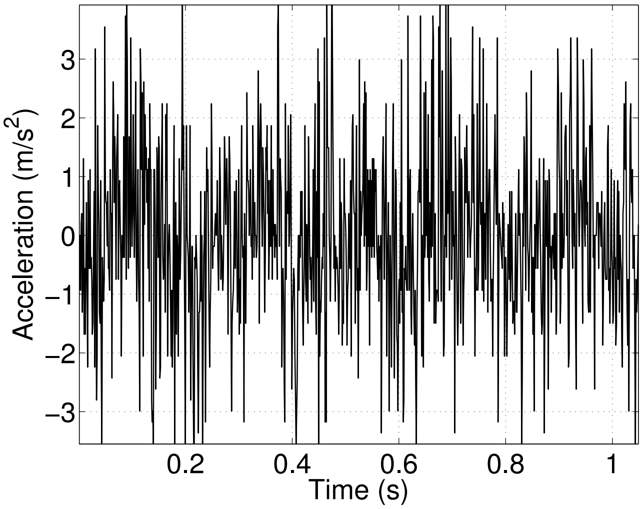

Fig. 1: Estimated accelerations of a 5 Hz vibration with an amplitude of 1 mm by using the direct DFrFT-based method with SCR = 15 dB.

IV. EXPERIMENTAL RESULTS

An experiment was conducted in collaboration with General Atomics Aeronautics Systems, Inc (GA-ASI) to investigate the performance of both the DFrFT-based method and the subspace method in actual SAR data. The Lynx SAR system was used to reconstruct the SAR image [13]. In this experiment, the vibrating target is an aluminum triangular trihedral with lateral length of 12 inches, as shown in Fig. 5. The vibration in this experiment were induced by the rotation of an unbalanced mass that was driven by a fan. The vibration's actual amplitude and frequency were 0.5 mm and 7 Hz, respectively. A SAR image containing the vibrating target reconstructed by the Lynx SAR system is shown in Fig. 6. The vibrating target is at the bottom right portion of the image. Several ghost targets appear along the azimuth direction. The nominal resolution of the reconstructed SAR image is 0.3 m in each direction. The

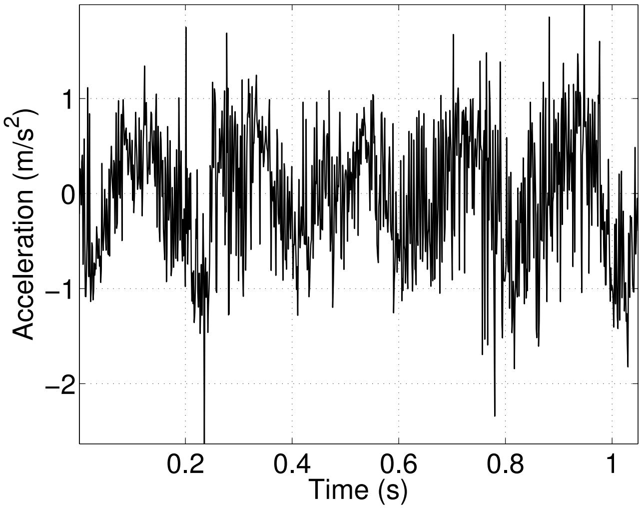

Fig. 2: Estimated accelerations of a 5 Hz vibration with an amplitude of 1 mm by using the subspace method with SCR = 15 dB.

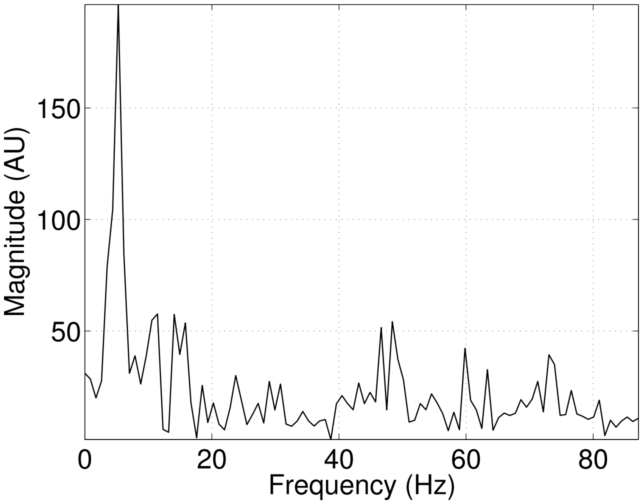

Fig. 4: Estimated vibration spectrum of a 5 Hz vibration with an amplitude of 1 mm by using the subspace method with SCR = 15 dB.

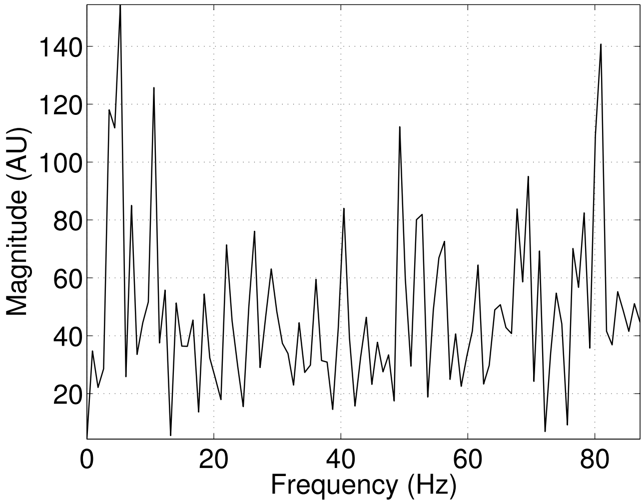

Fig. 3: Estimated vibration spectrum of a 5 Hz vibration with an amplitude of 1 mm by using the DFrFT-based method with SCR = 15 dB.

SCR is estimated to be 14 dB and the SNR is estimated to be above 15 dB. In this experiment, the carrier frequency was 15 GHz and the PRF was 230 Hz. Due to limited SNR and SCR, we selected the total observation time of this target to be 1.2 s, centered at the time closest to target broadside. The length of each time window was chosen to be 0.1 s.

The estimated vibration accelerations of both the DFrFT-based method and the subspace method are shown in Fig. 7 and Fig. 9, respectively. The estimated vibration spectra of both the DFrFT-based method and the subspace method are shown in Fig. 8 and Fig. 10, respectively. For the DFrFT-based method, the estimated vibration accelerations appear to be noisy so that we cannot identify the 7 Hz vibration from its spectrum. The vibration spectrum estimated by the subspace method clearly shows that the vibration is at 7 Hz.

Fig. 5: Vibrating target in a machine shop. It is an aluminum triangular trihedral with lateral length of 12 inches and it vibrates at 7 Hz with an amplitude of 0.5 mm.

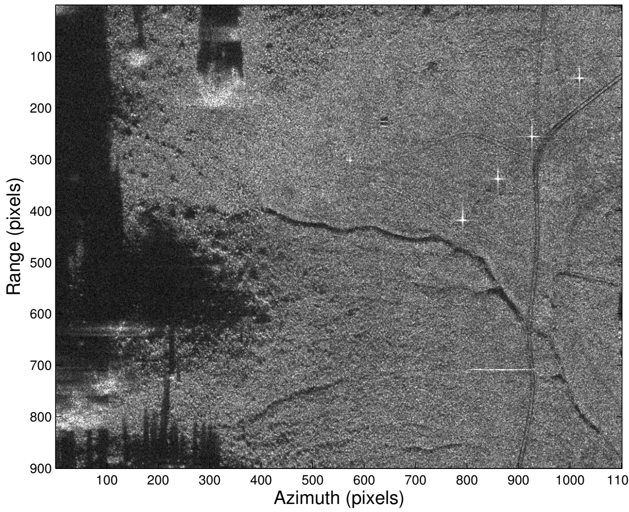

Fig. 6: SAR image containing the vibrating target. The vibration target appears as a strip of white line in the bottom right portion of the image . There are a few static targets above the vibration target which are not part of the vibration-estimation experiment.

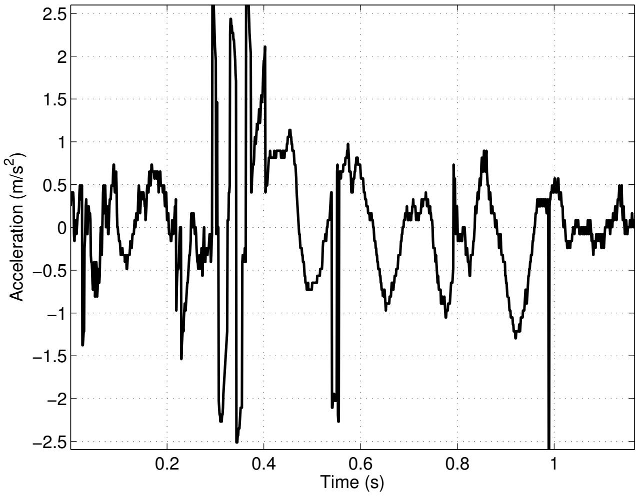

Fig. 7: Estimated spectrum of a 7 Hz vibration with an amplitude of 0.5 mm by using the direct DFrFT-based method with SCR = 14 dB.

V. CONCLUSIONS

In this paper, the performance of both the DFrFT-based method and the subspace method in clutter signals is investigated. Simulation results demonstrate that the DFrFT-based method yields reliable results with SCRs $>$ 18 dB and the subspace method yields reliable results with SCRs $>$ 8 dB. Experimental results demonstrate that the subspace method correctly estimates the vibration frequency of a 7 Hz vibration from actual SAR data with an estimated SCR of 14 dB.

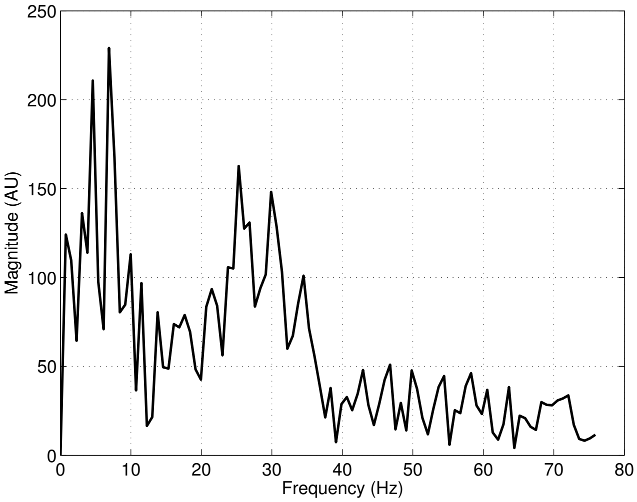

Fig. 8: Estimated vibration spectrum of a 7 Hz vibration with an amplitude of 0.5 mm by using the direct DFrFT-based method with SCR = 14 dB.

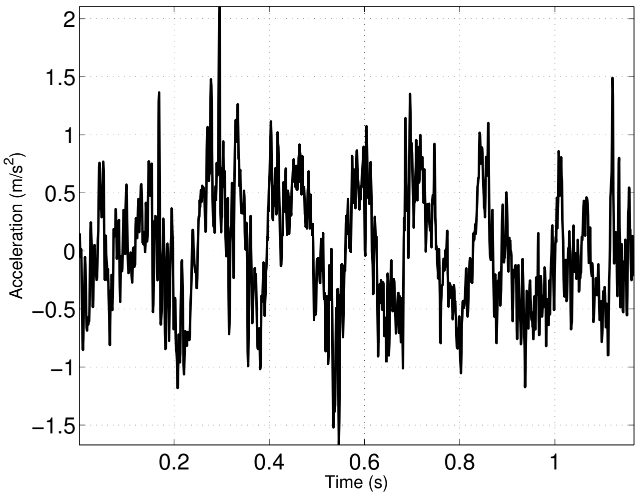

Fig. 9: Estimated accelerations of a 7 Hz vibration with an amplitude of 0.5 mm by using the subspace method with SCR = 14 dB.

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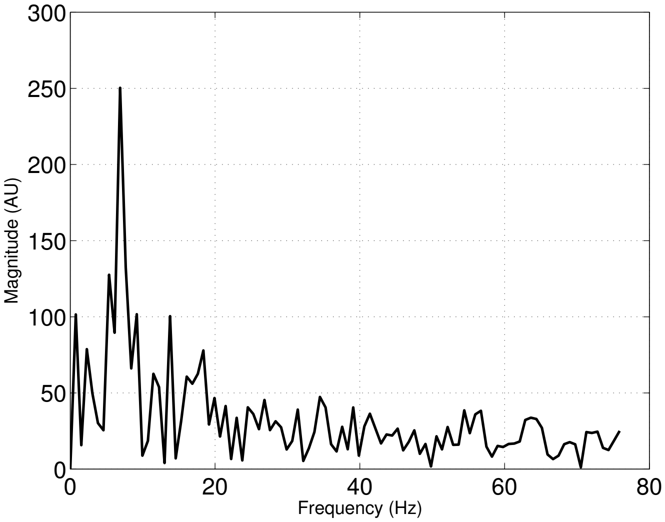

Fig. 10: Estimated vibration spectrum of a 7 Hz vibration with an amplitude of 0.5 mm by using the subspace method with SCR = 14 dB.

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