COCHANNEL FM DEMODULATION VIA THE MULTI ANGLE-CENTERED DISCRETE FRACTIONAL FOURIER TRANSFORM

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ABSTRACT

The problem of demodulating multiple frequency modulated signals that overlap spectrally is seen to pervade communication systems both military and civilian, where bandwidth is a precious resource. In this paper, we extend the use of the recently developed *multiangle discrete fractional Fourier transform* (MA-CDFRFT) for the purposes of cochannel FM demodulation and also investigate its use for the demodulation of cochannel *continuous phase modulation* (CPM) signals used frequently in wireless systems, using rectangular, triangular, or trapezoidal pulse shapes. Simulation results demonstrate that the approach is able to accomplish cochannel signal separation and demodulation, where existing multicomponent AM–FM demodulation algorithms fail. It is further demonstrated that for cochannel CPM demodulation, symbol error probabilities that are comparable to or smaller than that of coherent BPSK detection in additive white Gaussian noise.

Index Terms— cochannel FM demodulation, discrete Fractional Fourier transform, CPM demodulation.

1. INTRODUCTION

The multicomponent linear–FM or chirp signal model finds numerous occurrences in radar systems, biomedical applications, and in communications systems. Demodulating these multicomponent linear chirps is straight forward when the components are distinct. Traditional bandpass filtering followed by monocomponent demodulation would suffice. However, when the components overlap spectrally as in the *cochannel* problem or when one of them is much stronger than the others as in the *near-far problem*, existing demodulation algorithms encounter singularity problems and are unable to effect signal separation and demodulation [6].

Chirps are a particular category of non-stationary signals and exhibit a very specific type of time-frequency coupling. The multi angle-centered discrete fractional Fourier transform (MA-CDFRFT) approach developed in [1] was shown to be a useful time-frequency analysis tool for signals with this form of time-frequency coupling. The associated chirp rate versus frequency representation of the MA-CDFRFT is meaningful in the context of chirps because of its capability to concentrate linear chirps in a few coefficients [1]. Empirical expressions that relate the chirp rate and center frequency of each component to the coordinates of the peaks of the MA-CDFRFT were discussed in [1]. The MA-CDFRFT based analysis technique has been further been successfully applied towards the demodulation of multicomponent FM signals in [5]

In this paper, we extend the application of the MA-CDFRFT demodulation approach presented in [1, 5] to the case of cochannel FM signals with identical carrier frequencies. This was not possible with the PASED algorithm [6] that required distinct periodicities for the components. We further consider the problem of demodulating cochannel CPM signals [7] with rectangular, triangular, or trapezoidal pulse shaping functions, where the underlying multicomponent chirp model directly applies and demonstrate that significantly lower symbol error probabilities than that of BPSK in AWGN are achievable when employing *instantaneous frequency* (IF) based detection.

2. SIGNAL MODEL AND THE MA-CDFRFT

Multicomponent linear chirp signals are signals of the form:

$$x[n] = \sum_{i=1}^{K} A_i \cos\left(\int_0^n \Omega_i[m]dm + \theta_o\right), \qquad (1)$$

where the *instantaneous frequency* (IF) of the i^{th} component is given by:

$$\omega_i[n] = \omega_{ci} + c_{ri}\left(n - \frac{N-1}{2}\right),\ \ 0 \leq n \leq N-1, \qquad (2)$$

and ω_{ci}, c_{ri} are the corresponding carrier frequency and chirp rate of the i^{th} component.

Towards analysis of these signals we employ the *centered discrete fractional Fourier transform* (CDFRFT) [1]:

$$\mathbf{A}_\alpha = \mathbf{W}^{\frac{2\alpha}{\pi}} = \mathbf{V}_G \mathbf{\Lambda}^{\frac{2\alpha}{\pi}} \mathbf{V}_G^T, \qquad (3)$$

where $\mathbf{V_G}$ is the matrix of Grünbaum eigenvectors of the centered DFT matrix $\mathbf{W}$, and $\mathbf{\Lambda}^{\frac{2\alpha}{\pi}}$ is a diagonal matrix with the fractional powers of the eigenvalues of $\mathbf{W}$. The multi-angle version of the CDFRFT [1] is expressed as:

$$\begin{aligned} X_k[r] &= \sum_{p=0}^{N-1} z_k[p] e^{-j\frac{2\pi}{N}pr}, \\ z_k[p] &= v_{kp} \sum_{n=0}^{N-1} x[n] v_{np} \end{aligned} \qquad (4)$$

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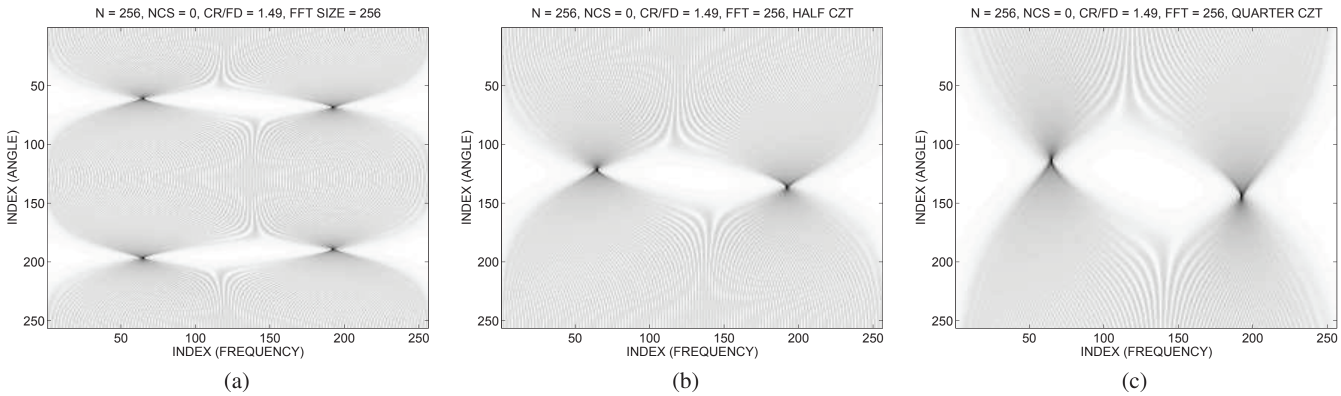


Fig. 1. Angular zooming and the CZT : (a) full spectrum MA-CDFRFT of a two component cochannel chirp signal, (b) corresponding half spectrum MA-CDFRFT, (c) quarter spectrum MA-CDFRFT.

and v_{kp} denotes the p-th component of the k-th eigenvector. It was also shown in [1] that one obtains an impulse-like transform analogous to what the DFT produces for sinusoids. The chirp rate and center-frequency estimates of the MA-CDFRFT are obtained via:

$$\begin{aligned} \mathbf{c_r} &= 2\frac{\tan(\alpha_p - \pi/2)}{N} + 1.41\frac{(\alpha_p - \pi/2)}{N}, \\ \omega_o &= \omega_p + 0.85(\alpha - \pi/2)^3, \end{aligned} \tag{5}$$

α_p and ω_p are the angle and frequency coordinates of the MA-CDFRFT spectral peak. In this paper, the FFT, the key ingredient of the MA-CDFRFT approach, is implemented with the *chirp Z-Transform* (CZT) [4] enabling the MA-CDFRFT matrix to zoom into an angular region of interest. Using the CZT version of the MA-CDFRFT affords us two options: (a) reducing the search area for peaks and the associated computational complexity, (b) or improving the resolution of the underlying carrier frequency and chirp rate estimators of the MA-CDFRFT and a accompanied reduction of the associated IF demodulation error.

To study the performance of the MA-CDFRFT based approach for cochannel FM demodulation, we shall adopt the performance metrics used in [6] for brevity sake. Specifically we use the *normalized carrier separation* (NCS) parameter as a measure of spectral separation and the *mean power ratio* (MPR) parameter to measure the relative power between components. Towards avoiding aliasing, we restrict the component chirp rates to be small in comparison to the carrier frequency, i.e., large CR/IB and CR/FD ratios [6].

3. COCHANNEL FM DEMODULATION

Significant spectral overlap exists between the components of a multicomponent AM–FM signal in the cochannel regime and as described in [6], a situation where existing multicomponent AM–FM demodulation approaches fail. In this regime, the separation between the carrier frequencies are less than 25% of the RF bandwidth. The problem of demodulating cochannel FM signals with periodic instantaneous frequencies was described in [6], where DC value constraints on the components were incorporated into a matrix framework to recover information lost at the common harmonics. Although the PASED approach in [6], is able to demodulate FM signals where the carrier frequencies are very close, it still requires the carrier frequencies to be distinct and further requires the periodic extension of the components. In this paper, we focus on the case where the carrier frequencies of the components are identical.

3.1. Synthetic Signals

The effectiveness of the MA-CDFRFT approach towards cochannel FM demodulation can be illustrated by first looking at a synthetic example in the cochannel regime of parameters [6]. Consider the two component sinusoidal chirp example, where NCS = 0, CR/FD = 2.19, 3.75 and MPR = 0 dB. Fig. 2(a) depicts the composite chirp signal, while Fig. 2(b) depicts the magnitude of the MA-CDFRFT, and Fig. 2(c) describes the estimated IF's using the MA-CDFRFT and the original IF's. Note that even though the carrier frequencies of the components are identical, the chirp rates are different, enabling the MA-CDFRFT approach to accomplish demodulation and is in contrast to the PASED approach in [6] that is unable to deal with this situation. Note further that the demodulation is accomplished in the pass-band rather than shifting the signals to baseband.

The above example is significant in that it illustrates the fact that the proposed approach: (a) is able to deal with the difficult situations where the component IF's overlap or intersect, where all other chirp demodulation approaches develop singularity problems and fail [6], (b) is also able to accomplish harmonic reassignment at the IF cross-over point, a task other multicomponent demodulation algorithms are unable to perform[1]. While we have restricted our investigation to cochannel chirp signals, the MA-CDFRFT can accommodate more general pulse shaping functions also [5].

When one or more components of the composite chirp signal are much stronger than the others then this situation will manifest as a singularity problem in the eventual demodulation of the weaker component's. This situation that occurs in wireless communications is referred to as the *near-far* problem or *capture effect* and translates to a larger value for the MPR parameter. The efficiency of the proposed demodulation approach *w.r.t.* MPR is directly related to the efficiency of the constituent center-frequency and chirp-rate estimators. Fig. 5(a,b) depicts the normalized MSE of the center frequency and chirp rate estimates of the weaker component. Performance analysis of the MA-CDFRFT approach for different MPR parameters indicate that the approach has a 12dB threshold, above which the approach fail's to detect the weaker component.

[1] A survey of existing multicomponent AM–FM demodulation approaches is done in [6], where the complexity of the cochannel problem is described.

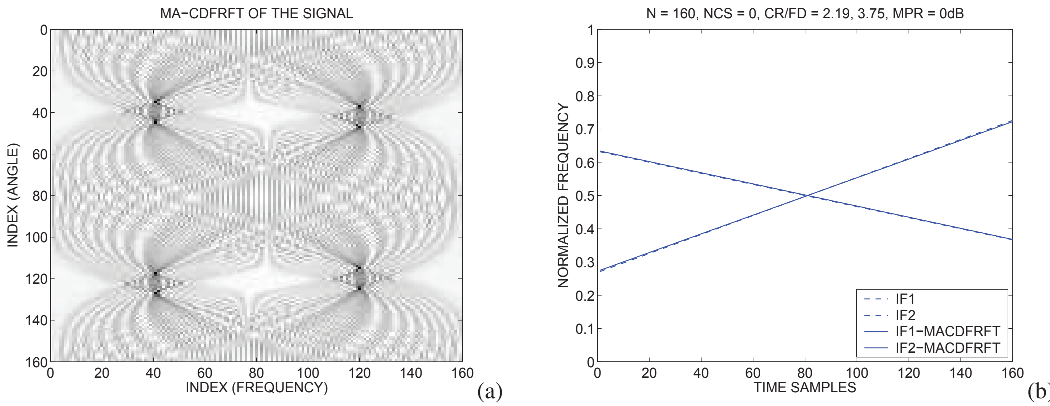


Fig. 2. Co-channel problem: (a) composite signal,(b) magnitude of the corresponding MA-CDFRFT using 160-pt FFT's, and (c) estimated normalized IF's using the MA-CDFRFT approach, where solid lines are estimates and dashed lines are actual quantities.

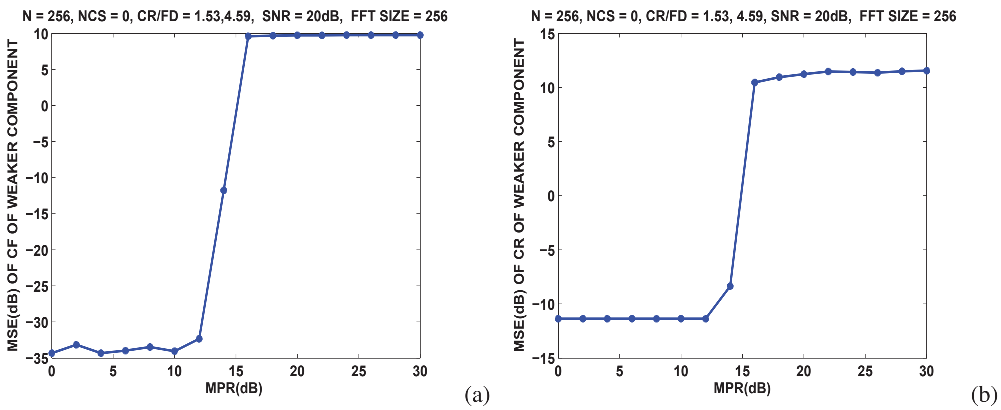


Fig. 3. Effect of MPR: (a,b) comparison of the normalized MSE of the center-frequency and chirp-rate estimates of the weaker component for different MPR parameters in the cochannel range.

3.2. Application to Cochannel CPM Demodulation

Continuous phase modulation (CPM) signals find widespread use in wireless communications due to their spectral efficiency and efficient modulator and demodulator implementations [3]. The demodulation approach that is optimal for CPM signals in AWGN is maximum likelihood sequence detection implemented via the Viterbi algorithm. The complexity of this approach is however, exponential in terms of the number of states, channel memory, and the number of users. Several other sub-optimal variations of this detector and other CPM demodulation algorithms were discussed in [3, 7]. For the sake of analysis, we restrict our analysis to rectangular, triangular, or trapezoidal pulse-shaping functions $p(t)$ with a duration of L symbol periods and binary PAM symbols $a[k] \in \{-1, 1\}$. This is so that the chirp signal model discussed before directly applies and smaller *symbol error probabilities* (SEP) can be attained.

The IF signal in the model takes the form:

$$\omega_i(t) = \omega_c + 2\pi h \sum_{k=-\infty}^{\infty} a[k] p(t - kT_b), \tag{6}$$

where ω_c is the carrier frequency and h is the modulation index of CPM. The phase deviation from the carrier phase is given by:

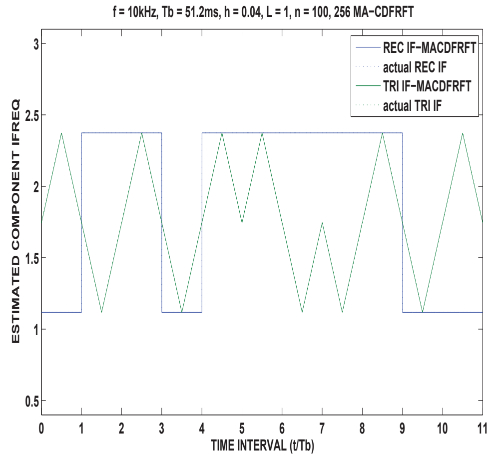
$$\phi_{\text{dev}}(t; \mathbf{a}) = 2\pi h \sum_{k=-\infty}^{\infty} a[k] q(t - kT_b), \tag{7}$$

where $q(t) = \int_0^t p(\tau) d\tau$ corresponds to the phase pulse shaping function. The CPM signal is then obtained via frequency modulation:

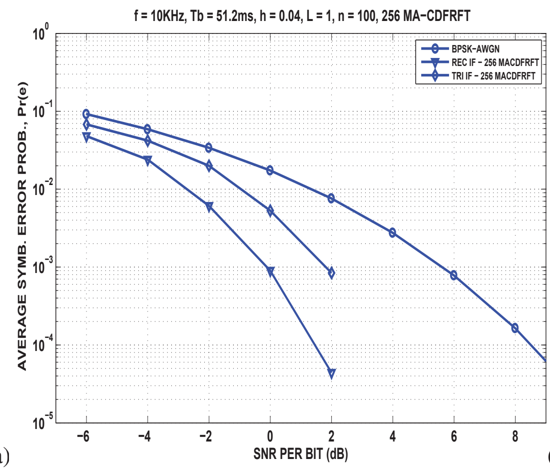
$$r(t) = A \cos\left(\int_{-\infty}^{t} \omega_i(\tau) d\tau \; + \theta_o \right).$$

Using a pulse shaping function of duration larger than a symbol period introduces memory into the modulation scheme (LREC-CPM). In this paper, we will focus our attention on the memoryless case with $L = 1$, i.e., (1REC/TRI-CPM). Specifically CPM with a rectangular pulse of one symbol duration (1-REC-CPM) is equivalent to continuous phase *frequency shift keying* (CPFSK). Another form of CPM, *minimum shift keying* (MSK), is equivalent to 1-REC-CPM with $h = 0.5$, while GMSK can be put into the CPM framework with a Gaussian pulse function [2].

Now let us look at a two-component cochannel example where one of the components has 1-REC-CPM and the other has 1-TRI-CPM with a modulation parameter of $h = 0.04$. The carrier frequencies of both components are identical. Fig. (4)(a) depicts the com-

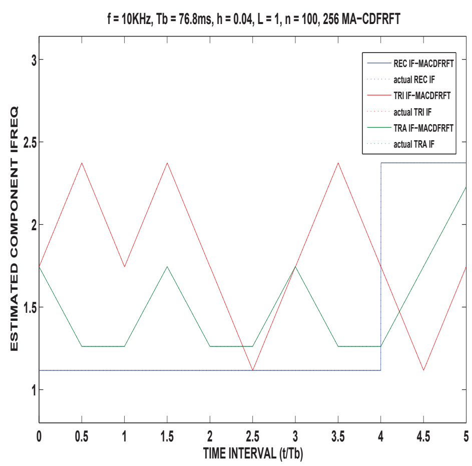


(a)

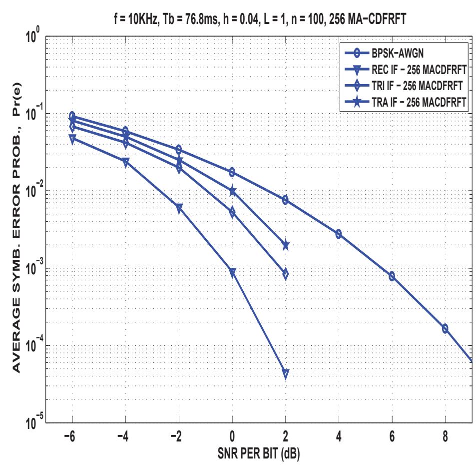


(b)

Fig. 4. Cochannel CP-FSK demodulation: (a) instantaneous frequency estimates of the MA-CDFRFT based approach, where solid lines represent estimates and dashed lines are actual quantities, (b) symbol error probabilities in AWGN for the components obtained after averaging over 100 experiments.



(a)



(b)

Fig. 5. Three component example: (a) composite three-component CPM signal, (b) instantaneous frequency estimates of the MA-CDFRFT based approach, where solid lines are estimates while dashed lines are actual quantities, (c) symbol error probabilities for the components.

posite CPM signal. Fig. (4)(b) describes the IF estimates of the MA-CDFRFT approach for either component, where in the absence of noise, we obtain zero frequency demodulation error. Fig. (4) depicts the SEP's of each component in relation to BPSK-AWGN, obtained by averaging over 100 experiments. The MA-CDFRFT approach is able to attain significantly lower SEP's in comparison to BPSK-AWGN and demonstrates the ability of this approach to accomplish complete demodulation even in the difficult cochannel case where the component spectra completely overlap. The performance of the approach for the case where there are three components[2] with rectangular, triangular, and trapezoidal IF's indicates that there is very little difference in the performance of the algorithm in comparison to the two component case, as illustrated in Fig. (5), as long as the FFT size used to estimate the center-frequencies and the chirp rates is sufficient to resolve their corresponding MA-CDFRFT peaks.

4. CONCLUSION

We have investigated the utility of a recently proposed multicomponent chirp demodulation approach based on the MA-CDFRFT towards the goal of cochannel FM demodulation. The proposed approach was shown to be particularly effective in the cochannel and near-far scenarios where existing multicomponent AM–FM demodulation approaches are known to fail. The MA-CDFRFT approach was also able to handle the difficult case where the carrier frequencies are identical, which the PASED algorithm [6] is unable to handle. Upon application to the problem of demodulating cochannel CPM signals, it was shown to yield significant improvement in the symbol error probability, when a rectangular, triangular, or trapezoidal pulse shaping function was employed.

5. REFERENCES

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