

# REDUCTION OF VIBRATION-INDUCED ARTIFACTS IN SYNTHETIC-APERTURE-RADAR IMAGERY USING THE FRACTIONAL FOURIER TRANSFORM

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## ABSTRACT

In synthetic aperture radar (SAR) images of objects exhibiting low-level vibrations are accompanied by localized artifacts, or ghost targets, caused by the micro-Doppler present in the returned SAR signals. Conventional Fourier-transform-based SAR processing techniques are not fit to remove ghosting effects prior to image formation due to the non-stationary nature of the returned signals from vibrating objects. Recently, a method based on the discrete fractional Fourier transform (DFrFT) has been developed for estimating the instantaneous vibration accelerations and vibrating frequencies from returned SAR signals. Here, a novel image de-ghosting algorithm for vibrating targets in SAR imagery is proposed by employing the DFrFT-based vibration-estimation algorithm. The proposed de-ghosting method is applied to SAR data collected by the Lynx SAR system. Experimental results show a substantial reduction in ghosting caused by a 1.5-cm amplitude, 0.8-Hz vibration present in a test target.

**Index Terms**— synthetic aperture radar, ghost target, image de-noising, vibrating target, fractional Fourier transform

## 1. INTRODUCTION

Synthetic aperture radar (SAR) is a well-established technique for generating high-quality images of the earth's surface through measurement of the electromagnetic reflectivity [1]. SAR is an active, coherent radar imaging technique that operates under the assumption that all the reflecting targets within the scene are static during the SAR data-collection time. However, it has been shown that high-definition SAR systems are sensitive to low-level target vibrations that induce localized artifacts, or ghost targets in the SAR images [2]. For instance, a 1.5-mm, 5-Hz target vibration may cause several ghost targets in the reconstructed SAR images along the azimuth direction [3]. This is because vibrations induce non-stationary frequency modulation in the SAR returned signals

and conventional SAR image-formation methods, which are based on Fourier-transform processing, generate side-lobes for the non-stationary signals causing artifacts in the formed image.

Vibration-induced ghost targets cause ambiguity in the identification of objects in a SAR image; hence, there would be great benefits to having a reliable, automatic method for reducing them. Since ghost targets caused by vibrating objects resemble series of real stationary point-sources in the SAR image, it is impossible to directly remove them via processing the formed image without risking the removal of true objects from the SAR image. Such processing of the image merely attempts to remove the symptoms of vibrations rather than removing the cause. To the best of our knowledge, no de-ghosting algorithm is available that removes the cause of the vibration-induced ghost targets prior to SAR image formation.

Recently, a time-frequency method based on the discrete fractional Fourier transform (DFrFT) has been proposed to estimate the vibration information from the SAR returned signals [3, 4]. This method yields an estimate of the instantaneous vibration accelerations as well as the vibrating frequencies. Simulations and experimental results have shown that this method is capable of estimating low-level vibrations (e.g., 1.5-mm amplitude, 5-Hz) with practical levels of signal-to-noise ratio (SNR) and signal-to-clutter ratio (SCR).

In this paper, a de-ghosting algorithm for vibrating targets in SAR imagery is proposed by exploiting the DFrFT-based estimation algorithm for instantaneous-vibration-acceleration and vibrating frequency reported in [3, 4]. In [3, 4], the authors focused on developing the DFrFT-based vibration estimation method. In this paper, the authors exploit the results of the vibration estimation method and propose a method for reducing the vibration-induced artifacts in SAR imagery. Provided that the target exhibits a single-component harmonic vibration, the instantaneous vibration displacements are first

reconstructed from the estimated instantaneous vibration accelerations and the vibrating frequency. Next, a reference signal whose phase is modulated precisely by the instantaneous vibration displacement is constructed. Finally, the vibration-induced phase modulation is removed from the returned SAR signal by multiplying, in the complex domain, the reference signal with the returned SAR signal from the vibrating target. The proposed method is applied to SAR data collected by the Lynx SAR system and the experimental results show a substantial reduction in ghosting caused by a 1.5-cm amplitude, 0.8-Hz vibration present in a test target.

## 2. METHOD

### 2.1. Signal model

Several different methods exist to form images from the SAR returned signal. In general, the SAR returned signal can be transformed into a 2-D signal of the form [1]

$$r[l, n] \approx \sum_i \sigma_i \exp[-j(k_x x_i l + k_y y_i n + \phi_i)], \quad (1)$$

where  $(x_i, y_i)$  and  $\sigma_i$  are the coordinates and the reflectivity of the  $i$ th target, respectively. The pair of indices  $(l, n)$  corresponds to the range and azimuth direction, respectively. Usually they are constrained by  $0 \leq l < L, 0 \leq n < N$  where  $L$  and  $N$  are determined by the bandwidth of the SAR sent pulse and the size of the synthetic aperture, respectively. The scaling parameters,  $(k_x, k_y)$ , are assumed to be known and they are calculated from the formulas

$$k_x = \frac{4\pi K}{c f_s}, \quad k_y = \frac{4\pi f_c V}{c R_0 f_{prf}}, \quad (2)$$

where  $K$  is the chirp rate of the SAR sent pulse,  $c$  is the propagation speed of the sent pulse,  $f_s$  is the sampling frequency of the returned SAR signal,  $f_c$  is the carrier frequency of the SAR sent pulse,  $V$  is the speed of the antenna carrier,  $R_0$  is the distance from the scene center to the mid-aperture, and  $f_{prf}$  is the radar pulse-repetition frequency (PRF).

In conventional SAR image reconstruction, all the targets are assumed to be static. The 2-D SAR image is obtained by applying a 2-D Fourier transform to  $r[l, n]$  in (1). When non-static targets are present in the scene, however, the Fourier transform-based image-formation techniques cannot focus them well and ghost targets appear in the reconstructed SAR image. It has been shown that the SAR returned signals from a scene containing vibrating targets can be written as [4]

$$r[l, n] \approx \sum_i \sigma_i \exp[-j(k_x \bar{x}_i l + k_y \bar{y}_i n + k_v \Delta x_i[n] + \phi_i)], \quad (3)$$

where  $k_v = 4\pi f_c/c$  and  $\Delta x_i[n]$  is the relative vibration displacement in range direction. Provided that the vibration amplitudes are small, the coordinates  $(\bar{x}_i, \bar{y}_i)$  are used to denote the averaged position of the vibrating target during the data collection time of the SAR system.

### 2.2. Vibration estimation using the fractional Fourier transform

Recently, a time-frequency method based on the DFrFT has been proposed to estimate low-level target vibrations. Simulations and experimental results show that this method is capable of estimating low-level vibrations (e.g., 1.5-mm amplitude, 5-Hz) with both practical SNR and SCR. For completeness, a brief introduction of this method is provided. The reader may find more details in [4, 5].

In this method the range compression (the Fourier transform in the range direction) is first applied to the 2-D signal  $r[l, n]$  in (3). After this procedure, each target is focused on its range position. Assuming that the vibrating targets are well separated in range and both the SNR and SCR are reasonably high, the range-compressed signal on the range line of  $\bar{x}_i$  can be approximated by

$$s[n] \approx \sigma_i \exp[-j(k_y \bar{y}_i n + k_v \Delta x_i[n]) + \phi_i]. \quad (4)$$

The signal  $s[n]$  is defined as the signal of interest (SoI). The SoI is further approximated by chirp signals in successive small time windows (also called sub-apertures). In each sub-aperture the chirp rate of the chirp signal is linearly proportional to the instantaneous vibration acceleration. The DFrFT is then applied to estimate the chirp rates in sliding sub-apertures. The DFrFT is an analysis tool geared specifically toward chirp signals [6]. It introduces an angular parameter  $\alpha$  to estimate the chirp rate. The DFrFT is capable of concentrating a chirp signal into an impulse in the angle-frequency plane, the location of which is determined from the center frequency and chirp rate of the signal. As a result, the history of the instantaneous vibration accelerations can be reconstructed from the estimated chirp rates and the vibration frequencies are estimated from the spectrum of the instantaneous vibration accelerations.

### 2.3. The de-ghosting algorithm

Under the assumption that the vibration is harmonic and single component, we can estimate the instantaneous vibration displacements from the estimated instantaneous vibration accelerations via

$$\Delta \hat{x}_i[n] = -\frac{1}{4\pi^2 \hat{f}_v^2} \hat{a}[n], \quad (5)$$

where  $\hat{f}_v$  and  $\hat{a}[n]$  are the estimated vibrating frequency and estimated vibration accelerations, respectively. A reference signal  $g[n]$  is reconstructed from  $\Delta \hat{x}_i[n]$  and it is given by

$$g[n] = \exp[jk_v \Delta \hat{x}_i[n]]. \quad (6)$$

Thus if we multiply the SoI with the reference signal, we obtain

$$s_c[n] = s[n]g[n] \approx \sigma_i \exp[-jk_y \bar{y}_i n + \phi_i]. \quad (7)$$

This causes the Fourier transform of  $s_c[n]$  to focus the target at its correct azimuth position  $\bar{y}_i$  and the vibration-induced ghost targets are reduced substantially.

The signal  $s[n]$  also contains weak returned signals from clutter and the multiplication in (7) introduces phase modulation into the returned signal from clutter. To overcome this side-effect some pre-processings are usually applied to  $s[n]$  before the multiplication. First, the Fourier transform is applied to  $s[n]$  and we find the smallest region containing all the ghost targets. Then the signal in this region is filtered out by using a spatial window. Third, the inverse Fourier transform is applied to the filtered signal and the aforementioned phase-compensation multiplication is applied. Finally the de-ghosted region is window-filtered again and registered at the known position of the SAR image to form a complete reconstructed SAR image with reduced ghost targets.

### 3. RESULTS

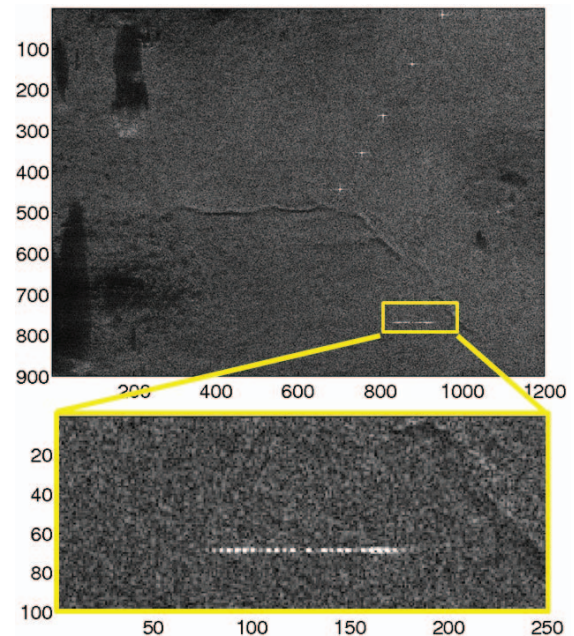
The proposed de-ghosting algorithm was first verified by means of simulations. Simulation results showed that the proposed method is capable of reducing ghosting caused by low-level target vibrations (e.g., 5-mm amplitude, 5-Hz) with practical levels of SNR ( $>5\text{dB}$ ). For brevity, however, the details of the simulations are not shown here; instead we will focus on presenting the experimental results.

An experiment was conducted in collaboration with General Atomics Aeronautics Systems, Inc (GA-ASI) to test the performance of the proposed de-ghosting method. A vibrating retro-reflector with lateral length of 21 inches was constructed in our laboratory and deployed on the test ground (near Julian, CA), as shown in Fig. 1. The target was positioned such that the vibration displacement was in the range direction. The SAR data was collected by the Lynx SAR system developed by GA-ASI. In this experiment, the Lynx system operated in the  $K_u$  band with a PRF of 330 Hz. Figure 2 shows the reconstructed SAR image with a nominal resolution of 0.3 m in each direction. The image of the vibrating target is located at the bottom right portion of the frame (see magnified inset). It appears as a horizontal sequence of points, with the actual object surrounded at each side with a series of ghost targets. The vibration-induced ghost targets span about 100 pixels in the azimuth direction. Note that there are also several well-separated static retro-reflectors extending from the center of the image to the top right corner; as expected they do not exhibit any ghosting.

The vibration was first estimated using the DFrFT-based method. Figure 3 shows the estimated instantaneous vibration acceleration for a duration of 2.6 seconds. The vibrating frequency was estimated to be 0.8 Hz from the peak location in the spectrum of the instantaneous vibration accelerations shown in Fig. 4. By using the relation denoted in (5), the instantaneous vibration displacements were calculated. Figure 5 shows the SAR image after applying the proposed de-ghosting algorithm to the SAR image in Fig. 2. The vibration-induced ghost targets now span only 20 pixels in the azimuth direction. The experimental results demonstrate that the proposed algorithm can substantially reduce the vibration-induced ghost targets in SAR images.



**Fig. 1.** Vibrating retro-reflector on the test ground near Julian, CA. The target is an aluminum triangular trihedral with lateral length of 21 inches. The vibration frequency and amplitude were 0.8 Hz and 1.5 cm, respectively.

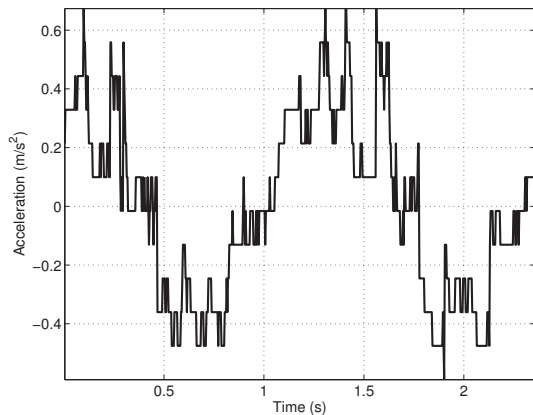


**Fig. 2.** Reconstructed SAR image provided by the GA-ASI Lynx system. The nominal resolution of the SAR image is 0.3 m in each direction. The vibrating test target is in the lower right portion of this image. The vibration-induced ghost targets span about 100 pixels in the azimuth direction. There are a few static targets extending from the center of the image toward the top right corner.

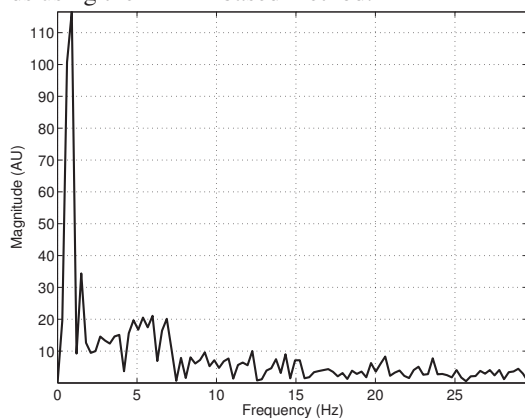
### 4. CONCLUSIONS

In this paper a de-ghosting algorithm for vibrating targets in SAR images is proposed and tested. The algorithm is based on first estimating the instantaneous acceleration of the vibration object using a DFrFT-based approach and then removing the vibration-induced micro-Doppler component from the returned SAR signals. The proposed method is verified by sim-





**Fig. 3.** Estimated instantaneous vibration acceleration for 2.6 seconds using the DFrFT-based method.



**Fig. 4.** Spectrum of the estimated instantaneous vibration acceleration. From the peak location, the vibrating frequency is estimated to be 0.8 Hz.

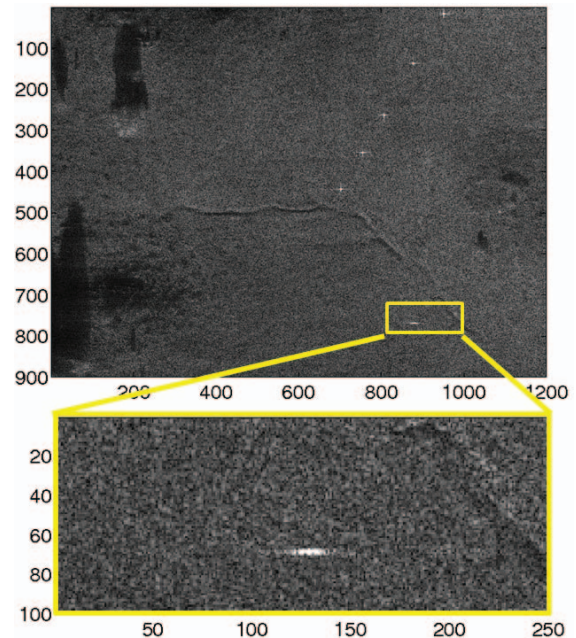
ulations and applied to SAR data collected by the Lynx SAR system showing a substantial reduction in the ghost targets. The de-ghosted SAR image provides a clearer image of the vibrating target than the original SAR image. This will help us identify or classify the vibrating targets using their estimated vibration information.

## 5. ACKNOWLEDGMENTS

This work was supported by the United States Department of Energy (Award No. DE-FG52-08NA28782), the National Science Foundation (Award No. IIS-0813747), National Consortium for MASINT Research, and Sandia National Laboratories. The authors also thank GA-ASI for making the Lynx system available to this project. The authors wish to thank Aleck Wright for valuable comments in the preparation of this manuscript.

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**Fig. 5.** De-ghosted image of the SAR image in Fig. 3 using the proposed method. The vibration-induced ghost targets now span only 20 pixels in the azimuth direction.

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