DEMODULATION OF DISCRETE MULTICOMPONENT AM–FM SIGNALS USING PERIODIC ALGEBRAIC SEPARATION AND ENERGY DEMODULATION

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ABSTRACT

Existing multicomponent AM–FM demodulation algorithms either assume spectrally distinct components or components separable via linear filtering and break down when the components overlap spectrally or if one of the components is stronger than the other. In this paper, we present a nonlinear algorithm for multicomponent AM–FM demodulation which avoids the above shortcomings and works well even for extremely small spectral separation of the components. The proposed algorithm separates the multicomponent demodulation problem into two tasks: periodicity-based algebraic separation of the components and then monocomponent demodulation via energy–based methods.

1. INTRODUCTION

Discrete–time (DT) multicomponent AM–FM signals are mathematically defined as signals of the form

$$x[n] \equiv \sum_{i=1}^{M} A_i[n] \cos\left(\int_0^n \Omega_i[k] dk + \theta_i \right) \ , \ M \geq 2, \qquad (1)$$

where the term "component" refers to a single term in the sum and $A_i[n]$ and $\Omega_i[n]$ are the instantaneous amplitude (IA) and frequency (IF) signals corresponding to the i^{th} component. These information signals are assumed to be slowly time-varying (lowpass) quantities.

Demodulation of these signals can be accomplished via the LMS algorithm [1], the multiband–ESA [2], the HRR algorithm [3] or the EDM algorithm [4]. These techniques, however, develop singularity problems when the components overlap spectrally or when one of the components is stronger than the other.

Algebraic separation of spectrally overlapping periodic signals has recently been studied in [5, 6]. In this paper, we present a solution to the multicomponent AM–FM demodulation problem that extends the algebraic separation approach in [5, 6] to multicomponent AM–FM signals with periodic information signals and combines it with the energy-based monocomponent demodulation method of [7]. Our new approach is called the *periodic algebraic separation and energy demodulation* (PASED) algorithm and can deal with extremely small spectral separations, overlapping IF's, and wide range of amplitude ratios.

2. TWO–COMPONENT ALGORITHM

The PASED, whose block diagram is shown in Fig. 1 can be divided into two sections:

- Model each component as a quasiperiodic signal and separate the components using algebraic separation techniques described in [5, 6].
- Demodulate the separated components into IF and IA information signals for each component using the *energy separation algorithm* (ESA) [7].

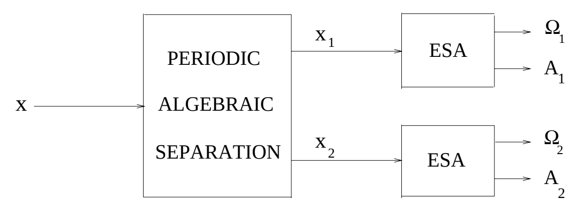

Figure 1. Block diagram of the PASED

Consider a two–component periodic signal of the form $x[n] = x_1[n] + x_2[n]$, where the components $x_1[n]$ and $x_2[n]$ are periodic with fundamental periods N_1 and N_2. Relating the samples of the composite signal to the components yields

$$\overbrace{\begin{pmatrix} x[0] \\ x[1] \\ \vdots \\ x(N-1) \end{pmatrix}}^{\mathbf{x}} = \underbrace{\begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{I}_{N_2} \\ \mathbf{I}_{N_1} & \mathbf{I}_{N_2} \\ \vdots & \vdots \end{pmatrix}}_{\mathbf{S}} \overbrace{\begin{pmatrix} x_1[0] \\ \vdots \\ x_1[N_1-1] \\ x_2[0] \\ \vdots \\ x_2[N_2-1] \end{pmatrix}}^{\mathbf{z}}, \qquad (2)$$

- $\mathbf{I}_{N_1}$ is the identity matrix of order N_1.
- The separation system outlined above is rank deficient with rank $r = N_1 + N_2 - R$, where $R = \gcd(N_1, N_2)$ [5, 6].

Additional constraints need to be appended to this separation system to complete and solve the separation system

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for the components. If the periods of the components are mutually prime the extra constraint needed is obtained as a zero dc spectral value constraint on one of the narrowband components. If the periods of the components are not coprime the additional R constraints are obtained as zero dc conditions of the subsampled components [5, 6].

$$\sum_{j=0}^{\left(\frac{N_1}{R}\right)-1} x_1[Rj+i] = 0 \quad , \quad i = 0, 1, \ldots, R-1. \tag{3}$$

The solution to the component separation problem is then reformulated as the solution to the augmented least–squares problem

$$\underbrace{\begin{pmatrix} \mathbf{S} \\ \mathbf{C} \end{pmatrix}}_{\tilde{\mathbf{S}}} \mathbf{z} = \overbrace{\begin{pmatrix} \mathbf{x} \\ \mathbf{0} \end{pmatrix}}^{\tilde{\mathbf{x}}}, \tag{4}$$

where the dc-value constraints (homogeneous) at the scale of R form the constraint matrix $\mathbf{C}$.

The solution to this problem is of the form

$$\mathbf{z} \;=\; \left(\tilde{\mathbf{S}}^T\tilde{\mathbf{S}}\right)^{-1}\left(\tilde{\mathbf{S}}^T\tilde{\mathbf{x}}\right) \;=\; \left(\mathbf{S}^T\mathbf{S}+\mathbf{C}^T\mathbf{C}\right)^{-1}\left(\mathbf{S}^T\mathbf{x}\right). \tag{5}$$

Each separated component is then modeled as monocomponent AM–FM signal $x_i[n] = A_i[n]\cos(\int_0^n \Omega_i[k]dk)$ and demodulated into IF and IA information signals using the DT ESA algorithm [7], where the DT Teager–Kaiser energy operator $\Psi(x[n]) = x^2[n] - x[n+1]x[n-1]$ is applied to the component $x_i[n]$ and its DT derivative $y_i[n] = x_i[n] - x_i[n-1]$:

$$\begin{aligned} \text{Freq}: \Omega_i[n] &\approx \arccos\left(1 - \frac{\Psi(y_i[n]) + \Psi(y_i[n+1])}{4\Psi(x_i[n])}\right) \\ \text{Env}: |A_i[n]| &\approx \sqrt{\frac{\Psi[x_i[n]]}{\sin^2(\Omega_i[n])}}, \end{aligned} \tag{6}$$

3. EXAMPLES

3.1. Sinusoidal Information Signals

Consider two–component AM–FM signals described by Eq. 1 with $M = 2$, where the IF and IA signals are sinusoidal and of the form

$$\begin{aligned} \Omega_i[n] &= \Omega_{ci} + \Omega_{mi}\cos(\Omega_{fi}n + \psi_i) \\ A_i[n] &= A_{ci}(1 + \kappa_i\cos(\Omega_{ai}n + \zeta_i)) \quad , \quad i = 1\ ,\ 2. \end{aligned}$$

The quantities $L_i = \frac{2\pi}{\Omega_{fi}}$ play the roles of the periods N_1 and N_2. The carrier-to-information bandwidth (CR/IB), the normalized carrier separation NCS (measures spectral separation), and the mean amplitude ratio MAR (measures relative strength) parameters are defined by

$$\begin{aligned} \text{CBW} &= \sum_{i=1,2} (\Omega_{mi} + \Omega_{fi} + \Omega_{ai}) \qquad \text{(Bandwidth)} \\ (\text{CR/IB})_i &= \frac{\Omega_{ci}}{\Omega_{fi}}\ ,\ \text{MAR} = \frac{A_{c1}}{A_{c2}}\ ,\ \text{NCS} = \frac{|\Omega_{c2} - \Omega_{c1}|}{\text{CBW}} \end{aligned}$$

Figure 2 shows an example of a two–component sinusoidally modulated AM–FM signal and the IF and IA estimates of the PASED. The demodulation lengths of the two components were $L_1 = 200$ and $L_2 = 201$.

3.2. Linear–FM Information Signals

In general, we assume here that the IF signal can be expressed as a finite Fourier series of the form [7]

$$\Omega[n] = \Omega_c + \sum_{k=1}^{K} \Omega_{mk}\cos(\Omega_{fk}n + \theta_k). \tag{7}$$

For aperiodic IF signals, this is accomplished by periodic extension of the IF signals [7]. The quantities $\Omega_{fi} = \frac{2\pi}{L_i}$, where L_i are the extension lengths, play the role of information bandwidths.

As an example, consider two–component AM–FM signals with linear FM modulation. The IF signals of the example are

$$\begin{aligned} \Omega_1[n] &= \frac{\pi}{2} + \frac{\pi}{400}\left(\frac{2n}{399} - 1\right) \quad , \quad 0 \le n \le 399 \\ \Omega_2[n] &= \frac{\pi}{2.005} - \frac{\pi}{400}\left(\frac{2n}{400} - 1\right) \quad , \quad 0 \le n \le 400 \end{aligned}$$

The IA signals of the example are sinusoidal with 6% AM, with a CR/IB of 50 and a MAR of 1. The demodulation lengths were $L_1 = 400$ and $L_2 = 401$. The composite signal of the example and the IF and IA estimates of the PASED are shown in Fig. 3.

4. MULTICOMPONENT PASED

The M-component PASED algorithm is based on the same philosophy as that of the two–component problem. The separation matrix $\mathbf{S}_M$ has M circulant blocks

$$\underbrace{\begin{pmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{I}_{N_2} & \ldots \mathbf{I}_{N_M} \\ \mathbf{I}_{N_1} & \mathbf{I}_{N_2} & \ldots \mathbf{I}_{N_M} \\ \vdots & \vdots & \vdots \end{pmatrix}}_{\mathbf{S}_M} \overbrace{\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_M \end{pmatrix}}^{\mathbf{z}}, \tag{8}$$

The rank of the separation system in this case is

$$\begin{aligned} r(\mathbf{S}_M) &= \sum_{i=1}^{M} N_i - (M-1) \quad , \quad \prod_{i,j\ ,\ i\neq j} \gcd(N_i, N_j) = 1 \\ r(\mathbf{S}_M) &\geq \sum_{i=1}^{M} N_i - \sum_{i,j\ \ni\ i<j} \gcd(N_i, N_j)\ ,\ \text{otherwise} \end{aligned}$$

For the first case, $(M-1)$ extra equations are needed to solve the separation system and are obtained as dc value constraints on $(M-1)$ components at their original scale. For the second case, the extra constraints needed are obtained from the dc value constraints applied to the pairwise interactions using the information on the pairwise gcd's.

The first case deals with pairwise coprime periods and the second case deals with the case where the periods are

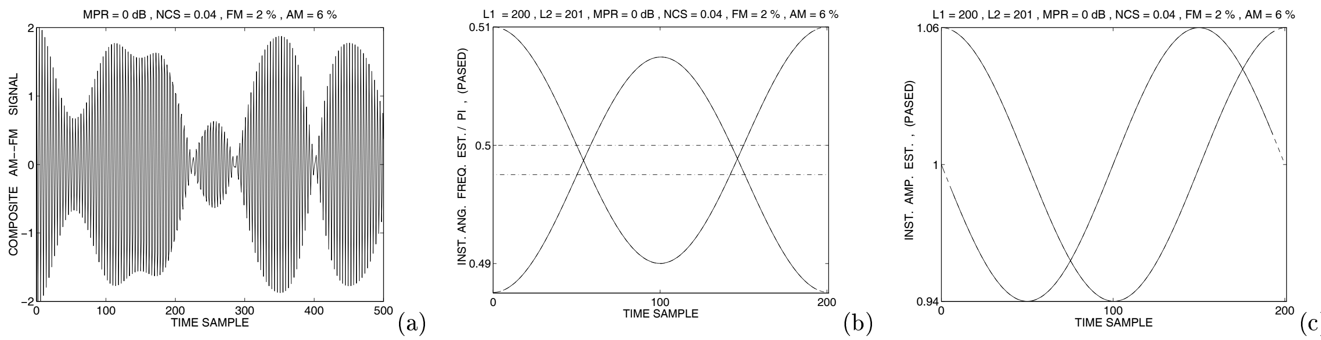

Figure 2. Sinusoidal AM–FM : (a) composite signal, (b,c) angular frequency, carrier estimates (dashed–dotted) of the PASED (as fractions of π) and IA estimates of the PASED, where solid lines indicate estimates and dashed lines indicate actual quantities. The IF signals are sinusoidal with coprime periods and 2% FM modulation. The IA signals are also sinusoidal with 6% AM modulation.

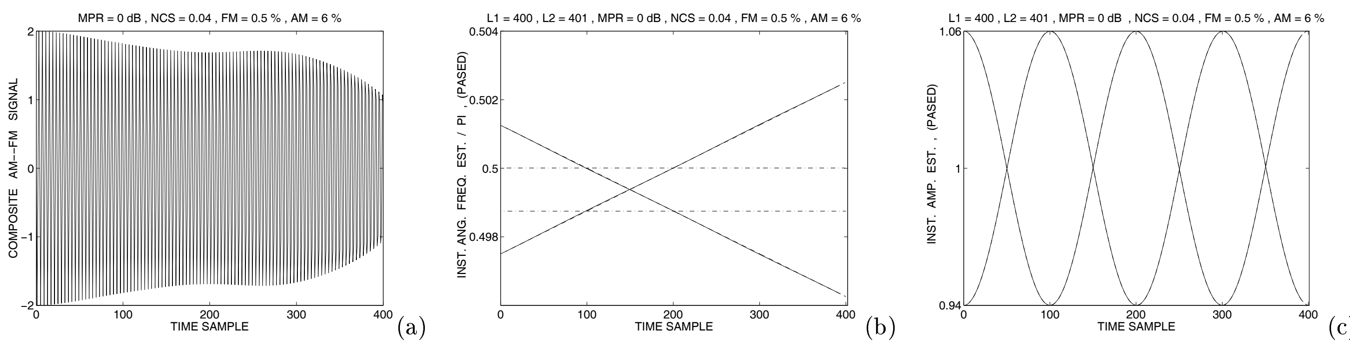

Figure 3. Chirp example (a) composite signal, (b) angular frequency estimates of the PASED (as fractions of π) and (c) IA estimates of the PASED. Solid lines indicate estimates, dashed lines indicate actual quantities and dashed-dotted lines are carrier frequency estimates. The component IA signals are sinusoidal with 6% AM modulation while the IF signals are linear with mutually prime periods and 0.5% FM modulation.

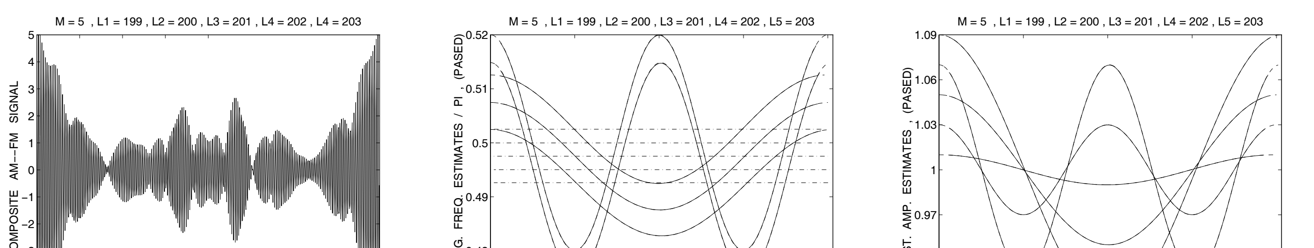

Figure 4. Five-component example: (a) composite signal, (b) IF estimates of the components via the multicomponent PASED, (c) IA estimates via the multicomponent PASED. Solid lines are estimates (almost identical to the true quantities in dash), and dashed–dotted lines are carrier frequency estimates. The periods of the component IF signals $N_i \in \{199, 200, 201, 202, 203\}$, are coprime, but pairwise non-coprime with $\approx 0.5\%$ FM. The IA signals are sinusoidal with $\kappa_i \in \{1, 3, 5, 7, 9\}\%$ AM

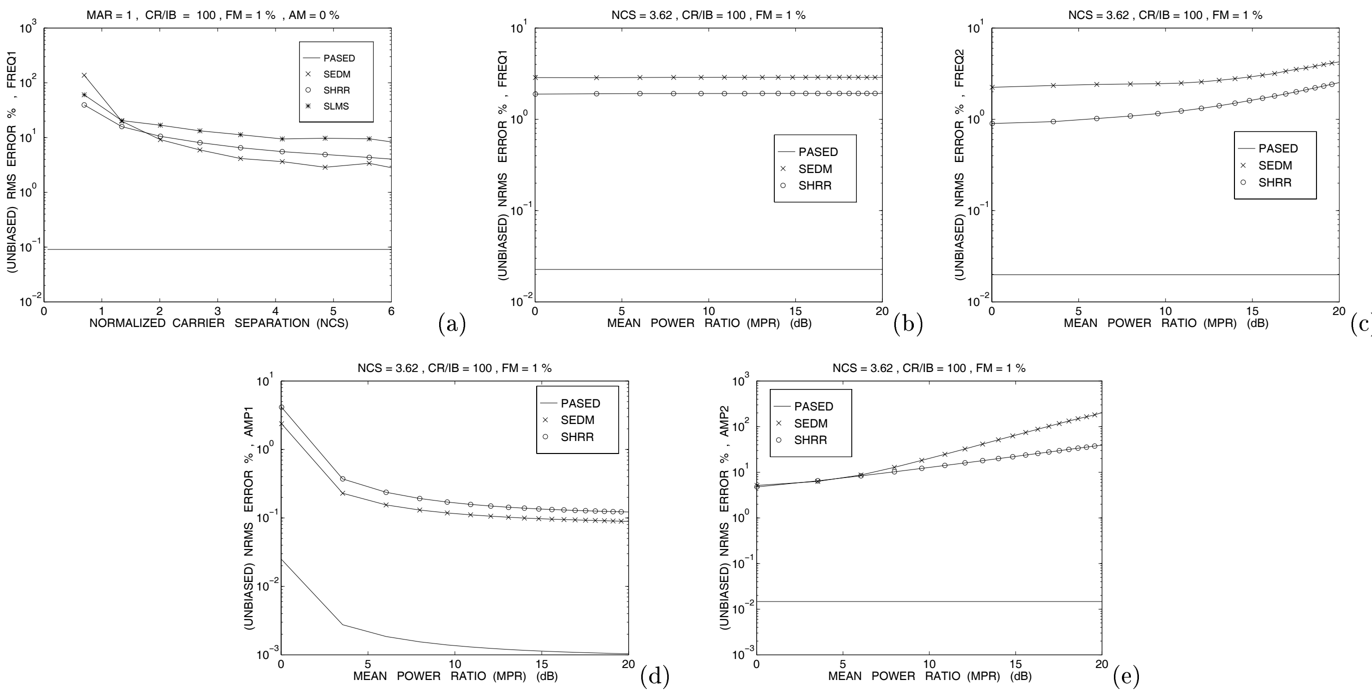

Figure 5. Effect of normalized carrier separation (NCS) and mean amplitude ratio (MAR) parameters on the performance of the PASED, the SEDM, the SHRR and the SLMS algorithms, where SX refers to the algorithm X with post–smoothing using a 9-pt median filter to remove spikes in the estimates followed by a moving average filter to remove interference.

pairwise non–coprime. Fig. 4 describes a multicomponent example ($M = 5$), where the periods are not pairwise coprime.

5. PERFORMANCE OF THE PASED

Existing techniques for multicomponent AM–FM demodulation either assume that the components are distinct or that the components are separable via linear filtering techniques. These techniques work over a narrow range of NCS or MAR parameters but do not provide a solution to the general multicomponent demodulation problem. These assumptions are in particular not valid in the co–channel range, where the signal components overlap significantly and these techniques develop singularity problems as NCS parameter decreases or if the MAR parameter increases [4].

The proposed PASED has the following advantages:

- It does not assume that the components are distinct and its performance is independent of spectral separation as evident from Fig. 5(a).
- Performance of the PASED is independent of the relative amplitude ratio of the components as evident from Fig. 5(b-e).
- It provides separability between the component separation and demodulation tasks.
- The least–squares system in the separation section of the PASED accomplishes simultaneous component separation and noise suppression.

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