WIDEBAND-FM DEMODULATION FOR LARGE WIDEBAND TO NARROWBAND CONVERSION FACTORS VIA MULTIRATE FREQUENCY TRANSFORMATIONS

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***Abstract*—Existing approaches for wideband FM demodulation are based on negative feedback, frequency tracking or multirate signal processing and heterodyning. Prior work that utilizes multirate frequency transformations for wideband-FM demodulation is impractical for large wideband to narrowband conversion factors such as those needed in DRFM systems.**

In this paper, we present a frequency transformation approach to wideband FM demodulation, using multirate systems, that can accommodate large conversion factors.

***Index Terms*—Wideband signal, frequency demodulation, multirate systems, heterodying, adaptive linear prediction**

I. Introduction

Frequency modulation signals are time-varying sinusoids of the form:

$$s\left(t\right) = A\cos\left(\int_{-\infty}^{t} \omega_i\left(\tau\right) d\tau + \theta_1\right), \tag{1}$$

where the *instantaneous amplitude* (IA) remains constant while the *instantaneous frequency* (IF) is given by

$$\omega_i\left(t\right) = \omega_c + \omega_m q_i(t), \tag{2}$$

where $q_i(t)$ is the normalized baseband modulated signal, and for sinusoidal FM it becomes

$$\omega_i\left(t\right) = \omega_c + \omega_m cos(\omega_f t + \theta_2). \tag{3}$$

Sinusoidal FM signals can be expressed via Bessel function as

$$s\left(t\right) = A\sum_{n=-\infty}^{+\infty} J_n\left(\beta\right)\cos\left(\omega_c t + n\omega_m t\right), \tag{4}$$

where J_n is the n^{th} order cylindrical Bessel function of the first kind. The modulation index of sinusoidal FM is defined as the ratio $\beta = \omega_m/\omega_f$ and the associated carson's bandwidth is given by

$$B = 2(\beta + 1)\omega_f. \tag{5}$$

If $\beta \gg 1$, then it corresponds to the traditional wideband FM setting, where the *carrier-to-information-bandwidth ratio* (CR/IB) and the *carrier-to-frequency deviation ratio* (CR/FD) are defined respectively as

$$\frac{CR}{IB} = \frac{\omega_c}{\omega_f} \quad \frac{CR}{FD} = \frac{\omega_c}{\omega_m}. \tag{6}$$

Signals in this category are widely used in applications such as speech formant tracking and analysis [1], satellite communication [2] and DRFM system [3]. However, most conventional demodulation techniques only perform well under the narrowband assumptions of the signal where the two parameters CR/IB and CR/FD are usually large, and fail in the wideband setting. For example, the Hilbert transform requires a relatively large carrier frequency of the signal to form an accurate analytic signal approximation. Larger ratios of CR/IB and CR/FD are beneficial for the analytic approximation and hence help to improve the demodulation. In this paper, a general approach that involves multirate systems as well as heterodyning is proposed for wideband FM demodulation that usually has a: 1) $\beta > 2$; 2) large information bandwidth. The adaptive linear predictive IF tracking technique as described in [4] is chosen as the demodulation method for implementing the proposed approach. In fact, it can be integrated with other existing FM demodulation methods, such as the zero-crossing approach according to [5], [6] and feedback demodulation as mentioned in [7], serving as a general framework for dealing with wideband FM. In this paper, we demonstrate that large wideband to narrowband conversion factors are feasible using the proposed system with designs that are realizable.

II. Multirate and Heterodyning System

The motivation for incorporating the multirate and heterodying systems into the demodulation framework is to apply *multirate frequency transformations* (MFT) that first compress the spectrum of wideband FM signals and then shift them into an optimal region with large CR/IB and CR/FD ratios, where existing demodulation techniques perform well.

A. Prior Work

Using the scaling property of the Fourier transform, compression in frequency domain is equivalent to expansion in the

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time domain expressed as

$$y(t) = x(at) \Longleftrightarrow Y(\omega) = X\left(\frac{\omega}{a}\right), \tag{7}$$

where $a = 1/R < 1$ is the factor of frequency compression. Then IF of the compressed signal becomes a scaled version of the input IF by a factor R expressed as

$$\tilde{\omega}_i(t) = \frac{\omega_i(t)}{R} = \frac{\omega_c}{R} + \frac{\omega_m}{R} q_i\left(\frac{t}{R}\right). \tag{8}$$

Note that for compressed signal, the carrier frequency is also scaled by the same factor R, which is undesirable since the ratios CR/FD and CR/IB that we wish to increase still remain invariant. Hence the heterodying operator is cascaded right after the compression process in order to upshift the carrier frequency to a higher level where we can attain larger CR/FD and CR/IB ratios. The output of the heterodyne module is given by

$$y_{ush}(t) = [y(t)\cos(\omega_d t)] * h_{BPF}(t), \tag{9}$$

where $*$ denotes the convolution, ω_d refers to the amount of frequency translation and $h_{BPF}(t)$ represents the impulse response of the bandpass filter. Specific for the case of sinusoidal FM, it can be further simplified as

$$y_{ush}(t) \simeq \frac{1}{2} A \cos\left(\phi\left(\frac{t}{R}\right)\right), \tag{10}$$

where $\phi(t)$ denotes the phase of the original FM signal. The resultant signal that has a scaled information bandwidth with a higher CR/IB then passes through the demodulation block for IF extraction. Eventually, the IF estimate of the original signal is evaluated by

$$\omega_i^{\text{out}}(t) = R\left(\tilde{\omega}_i(Rt) - \omega_d\right), \tag{11}$$

where $\tilde{\omega_i}(t)$ is the IF of the compressed signal.

As for discrete-time signals, compression and expansion can be substituted by the corresponding multirate operations of interpolation and decimation as described in [8] with their properties carried over to their discrete counterparts. The block diagram of such a MFT framework in prior work [9] is illustrated by Fig. 1.

Interpolating the input signal will result in the reduction of both the frequency deviation and information bandwidth by a factor of R. Similar to increasing the sampling rate, the IF of the interpolated signal becomes slow-varying and the assumption that the message signal remains constant over the carrier period is more likely to hold, which will boost the performances of conventional demodulation algorithms. Meanwhile, heterodying serves the purpose of increasing the ratios of CR/FD and CR/IB by compensating for the scaled carrier frequency. By passing through the heterodying process, the CR/FD and the CR/IB of $y_{ush}(t)$ are given by

$$\left[\frac{CR}{FD}\right]_{out} = \left[\frac{CR}{FD}\right]_{in} + \frac{R\omega_d}{\omega_m} \tag{12}$$

$$\left[\frac{CR}{IB}\right]_{out} = \left[\frac{CR}{IB}\right]_{in} + \frac{R\omega_d}{\omega_f}. \tag{13}$$

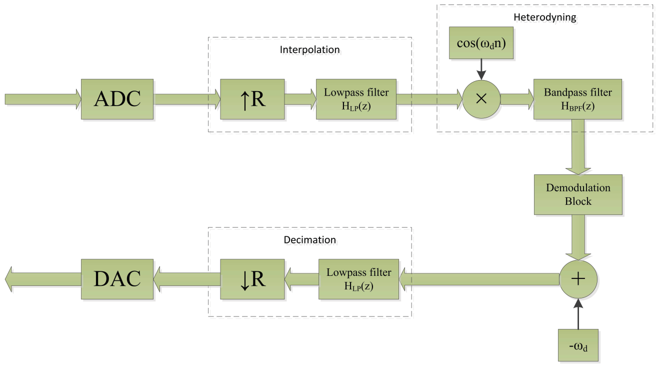


Fig. 1. Block diagram of the prior MFT framework in [9]. The wideband signal is first sampled above the Nyquist rate, interpolated by a factor R and then heterodyned via multiplying $cos(\omega_d n)$, followed by a discrete FIR bandpass filter with a scaling module based on (10) to achieve MFT. Then it goes through a demodulation block to obtain the IF estimation of the compressed heterodyned signal. To estimate IF of the original signal, the compressed heterodyned IF is then shifted back by subtracting ω_d, decimated by R and scaled back approapriately according to (8), followed by the DAC module.

B. Constraints of Prior MFT System

As we look further into this framework, an important question regarding the selection of the conversion factor R arises. Specifically prior work [9] only deals with small multirate compression factors. However, larger factors over hundred or thousand can be supported by current high-speed DSP with large memory, as in the case of DRFM system design. It is intuitive to expect a further reduction in the demodulation error since the gain brought by frequency compression should be extendable through the use of a larger factor. But for really large factors R, the passband of the lowpass filter in the multirate operation and that of the heterodyne-BPF operation will be scaled by R. For example, if $R = 1000$, we require a lowpass filter with cut-off frequency at $\pi/1000$ and a bandpass filter with a passband edge less than or equal to of $\pi/1000$. However, filters with such narrow passbands are unrealistic for direct implementation by any structure[1]. Thus the design of BPF within the previous MFT framework becomes the bottleneck that constrains the use of a very large factor.

C. Proposed System

In order to reduce the burden placed on the practical implementation of the bandpass filter, we first consider a different MFT framework where the order of the interpolation operator and the heterodyning operator are exchanged. Due to the switch of interpolation and heterodying, the CR/FD and CR/IB parameters under this MFT framework are given by

$$\left[\frac{CR}{FD}\right]_{out} = \left[\frac{CR}{FD}\right]_{in} + \frac{\omega_d}{\omega_m} \tag{14}$$

$$\left[\frac{CR}{IB}\right]_{out} = \left[\frac{CR}{IB}\right]_{in} + \frac{\omega_d}{\omega_f}. \tag{15}$$

[1]Narrow passband implies clustered poles and zeros that result in sensitivity and stability issues of digital filters as described in [10].

By comparing these ratios with (12) and (13), note that the upshift frequency ω_d in this case needs to be large enough such that the ratios of CR/FD and CR/IB still stay at high level. However, ω_d cannot be too large such that the resultant carrier frequency after heterodying exceeds one half of the sampling rate, we otherwise will need to interpolate the signal first by an appropriate factor in order to perform discrete-time bandpass filtering after heterodyning. Hence the practical implementation of MFT framework for a large conversion factor is not as simple as just exchanging the order of interpolation and heterodyning. Actually, an interpolation operation is still required prior to the heterodying with an appropriate factor that depends on the upshift frequency ω_d and the sampling frequency of the original wideband FM signal. This implies that the overall interpolation factor can be split into two with the first one prior to the frequency translation and the other one right after. Then upshifting by a frequency ω_d that is not too large would result in a relatively small factor for the first interpolation, lessening the burden of the heterodyne-BPF.

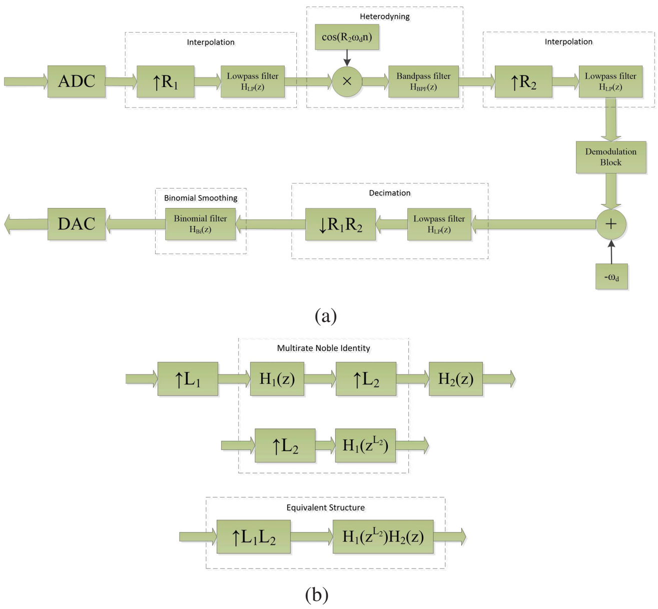


Fig. 2. (a) Block diagram of the proposed MFT framework for large wideband to narrowband conversion factors. The interpolation module of the prior framework is separated into two with one in fornt of the heterodyning module and the other one right after. The first interpolation module has a relatively small upsampling rate of R_1 which is appropriately chosen such that the discrete BPF can be implemented within the range of half the sampling rate after heterodyning the signal with a frequency translation of ω_d. The relatively small R_1 would result in a wider passband for the discrete bandpass filter, thus reducing the its design of complexity. (b) Multistage implementation for interpolation modules in 2(a) based on Noble identity. Note that multistage implementation for the corresponding decimation modules can be realized in a similar fashion.

In this paper, we propose a MFT framework for large conversion factors illustrated in Fig. 2(a) that achieves a practical design of the heterodyne-BPF. As previously discussed, the interpolation module of prior framework is separated into two with one in fornt of the heterodyning module and the other one right after in the proposed framework. The first interpolation module has a relatively small upsampling rate of R_1 which is appropriately chosen such that the discrete BPF can be implemented within the range of half the sampling rate after heterodyning the signal with a frequency translation of ω_d. The relatively small R_1 would result in a wider passband for the discrete bandpass filter, thus reducing the its design of complexity. In general, there is a sacrifice in terms of achievable CR/IB and CR/FD ratios for the proposed MFT framework, however, the system suggested in prior work does not realize large conversion factors, due to the placement of impractical constraints on the BPF design.

To further relax the constraint imposed on filter design, we first note that the lowpass filters in the multirate structure can be implemented efficiently using the *multirate noble identities* mentioned in [8]. As shown by Fig. 2(b), the original one stage multirate structure is equivalent to multistage implementation with R factorized into smaller integer factor corresponding to each stage. As a result, the lowpass filter corresponding to each stage will have much larger cut-off frequency and thus a wider passband. Therefore we can conclude that the use of a large conversion factor is primarily constrained by unrealistic requirements on the heterodyne-BPF, which can not be easily relaxed as in the case of the lowpass filter within the multirate structure.

The heterodyne-BPF also plays a crucial role when we take noise into consideration. Since the spectrum of the wideband FM have an infinite number of spectral components, the passband for the heterodyne-BPF is required to cover the scaled spectrum as much as possible in order to reduce the distortion caused by loss of spectrum when the noise is inconsiderable. In the presense of observable noise, however, the noise introduced by covering wider ranges may significantly corrupt the IF estimate. Due to this fact, the passband width of the heterodyne-BPF needs to be optimized around the Carson bandwidth to appropriately trade-off harmonic distortion and noise related distortion [11].

In addition, a binomial smoothing module is incorporated into the proposed framework as shown in Fig. 2(a), to further reduce the effects of noise. Even though the FM signal is wideband, the IF waveform itself is not necessarily wideband in nature. In many cases, the wideband FM is primarily generated by a large modulation index while the IF still remains in the narrowband range. Under this assumption, by applying the binomial smoothing we can efficiently filter out the high frequency noise in the corrupted IF estimation. When the SNR is high, the improvement becomes extremely evident as we shall see later. Usually we would expect a gain between 5 dB and 10 dB in the scenario of relatively high SNR.

III. Adaptive Linear Predictive IF Tracking

According to the Wiener-Hopf equations [12], the optimal coeffcients of a linear predictor are given by

$$\mathbf{w}^{opt} = \mathbf{R}_{xx}^{-1}\mathbf{r}_{dx}, \qquad (16)$$

where $\mathbf{w}^{opt}$ denotes the optimal tap weight vector, $\mathbf{R}_{xx}$ denotes the input correlation matrix and $\mathbf{r}_{dx}$ denotes the cross-correlation between input vector and desired signal. As summarized in [4], the prediction error filter is given by:

$$E(z) = 1 - \sum_{l=1}^{L} g_l^{opt} z^{-l}, \tag{17}$$

where $\{g_l^{opt}\}_{l=1}^{L}$ are the coefficients of the corresponding optimal predictor. Conditioned on the small prediction error assumption and the further assumption that the message signal remains essentially invariant over the sampling range of the linear prediction filter, we end up with the approximation in [13] through (17) given by

$$\sum_{l=1}^{L} g_l(k) \exp\{-jl[w_c + m(k)]\} \simeq 1, \tag{18}$$

where $g_l(k)$ is the weight coefficient of tap l at time index k and $m(k)$ is the sample of the message signal at time index k. Then the IF of the signal of interest can be estimated by executing the following steps: 1) Compute the coefficients of the prediction error filter; 2) Obtain the roots of the coefficient polynomial; 3) Calculate the phase argument of the complex conjugate pole location of the corresponding roots.

In the previous work of Asilomar [4], adaptive algorithms such as AS-LMS and AF-RLS have been incorporated into the structure of linear predictor and compared with each other based on the demodulation error. However, for both algorithms the step-size or the forgetting factor need to be truncated to remain within certain range . In this paper, we choose the *generalized normalized gradient descent* (GNGD) [14] for coefficients update of the linear predictor, which avoids truncation of the adaptively adjusted step-size. The algorithm for this GNGD linear predictive filter is summrized via

$$e(k) = x(k+1) - \sum_{l=1}^{L} g_l(k)\, x(k-L+1) \tag{19}$$

$$\beta(k) = \beta(k-1) - \rho\alpha \frac{e(k)e(k-1)\mathbf{x}^T(k)\mathbf{x}(k-1)}{(\|\mathbf{x}(k)\|_2^2 + \beta(k))^2} \tag{20}$$

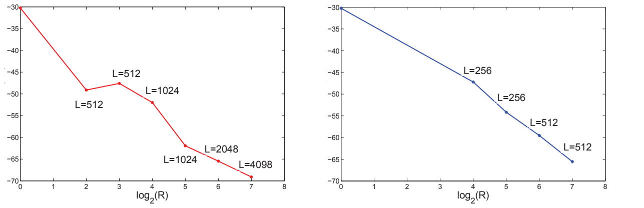
$$\eta(k) = \frac{\alpha}{\|\mathbf{x}(k)\|_2^2 + \beta(k)} \tag{21}$$

$$\mathbf{g}(k+1) = \mathbf{g}(k) + \eta(k)e(k)\mathbf{x}(k), \tag{22}$$

where $\mathbf{x}(k)$ and $\mathbf{g}(k)$ denote the vectors of input and tap weights at time index k respectively, α is the step-size parameter and ρ is the offset learning rate parameter. The merit of the GNGD algorithm lies in that the adaptation of its learning rate provides compensation for the assumptions in the derivation of NLMS. Due to its robustness and stability, the GNGD is well-suited for narrowband nonstationary signal environments.

IV. Simulation Results

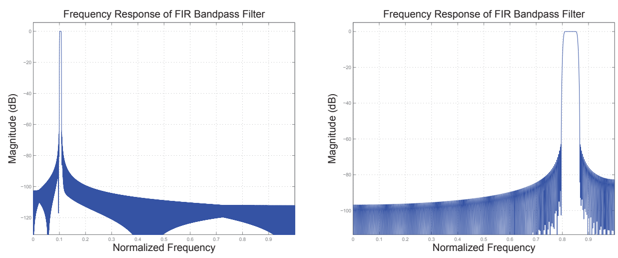
In this section, we present demodulation results using the proposed and prior MFT frameworks respectively under both noise free and noisy environments. Note that the demodulation performance is judged by the *normalized RMS IF demodulation error* (NRMSE) throughout this section.



(a) (b)

Fig. 3. Comparison between performances of both MFT frameworks under noise free environments. (a) Performance of the prior MFT framework with R specifying the multirate factor and L specifying the order of BPF. (b) Performance of the proposed MFT framework with R specifying the multirate factor and L specifying the order of BPF.

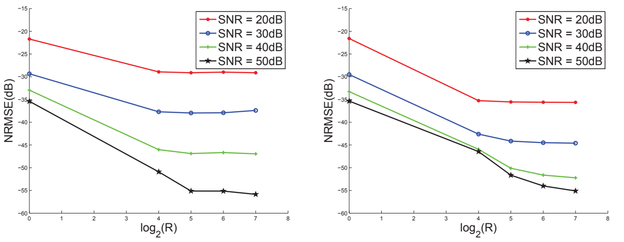
We first look at the example of a wideband sinusoidal FM signal that has a modulation index of 10 and the CR/IB of 20. Under a noise free environment, the performance of the prior MFT framework is illustrated by Fig. 3(a). Note that the performance associated with $R = 1$, i.e, the origin of the performance curve corresponds to direct demodulation by GNGD without MFT, while ω_d is the normalized upshift radian frequency translation in the range of $[0, \pi]$. By applying a large conversion factor of $R = 128$, a reduction of around 40 dB in the demodulation error over direct GNGD demodulation is attained. The result of Fig. 3(a) confirms the claim that a large conversion factor strengthens the effect achieved by frequency compression thus leading to significant reduction in the demodulation error. Moreover, it reflects the fact that the use of a larger factor requires a very high order FIR bandpass filter with a satisfactory frequency response. For example, $R = 128$, demands the order of FIR bandpass filter to be as high as 4096, which results in unrealistic parameters for the narrow passband. This constraint seriously limits the implementation in a practical system for large factors.



(a) (b)

Fig. 4. Comparison between the previous and proposed MFT frameworks in terms of the requirements on the heterodyne-BPF with a conversion factor $R = 128$. (a) Frequency response of BPF in the previous MFT framework with order $L = 4096$. (b) Frequency response of BPF in the previous MFT framework with order $L = 512$. Note that the proposed framework succesfully reduced the complexity of the design of heterodyne-BPF.

To relax the constraint, an alternative method is to apply the proposed MFT framework, whose performance is illustrated by 3(b). In comparison to Fig. 3(a), it can be observed that the required order for the FIR bandpass filter is effectively reduced at the cost of sacrificing a small amount of improvement in demodulation error. For instance, when $R = 128$, the order for the FIR bandpass filter drops significantly from 4096 to 512 with just 4 dB loss in performance, suggesting no observable difference in performance between both MFT frameworks except for the dramatic reduction of the order for the heterodyne-BPF. Note that the frequency response of the heterodyne-BPF in the proposed MFT framework for the case $R = 128$ is illustrated by Fig. 4(b), which has much wider passband compared to the frequency response of the BPF in the previous MFT framework shown in Fig. 4(a) and thus practical for implementation.



(a) (b)

Fig. 5. Comparison between performances of both MFT frameworks in environments corrupted by AGWN. (a) Performance of the prior MFT framework with R specifying the multirate conversion factor. (b) Performance of the proposed MFT framework.

For noisy environments, the performance of the prior MFT framework is illustrated by Fig. 5(a) for the same sinusoidal FM signal corrupted by additive white Gaussian noise of different SNR. Based on the observation, improvement over GNGD demodulation without MFT varies according to the different SNRs. For example, when the SNR is 20 dB, the improvement is only aournd 6 dB, and when the SNR increases to 40 dB, improvement increases to around 20 dB. In addtion, Fig. 5(b) summarizes the results of the proposed MFT framework in the presense of noise, indicating better performance due to postsmoothing of the IF. For the case of 20 dB SNR, the improvement increases to 15 dB compared with prior system. Note that the NRMSE gradually becomes saturated as R increases, due to LTI filtering induced harmonic distortion of the FM signals[2]. In contrast, the proposed MFT approach results in a better performance, and further provides a more practical approach to the BPF design.

In order to analyze the performance achieved by each module of the MFT framework, we can demodulate the output after each module (interpolation, heterodyning and postfiltering) sequentially and compare the corresponding error (see Fig. 6). To illustrate the point that the MFT framework can be combined with other traditional demodualtion methods, here we replace GNGD by *Hilbert transform* (HT) to demodulate the IF. On one hand, the gain through interpolation module is more apparent with high SNR due to its sensitivity to noise. On the other hand, the heterodyning module produces a steady increase in gain, particularly with low SNR. It is the filtering process inherent in the heterodyning module that accounts for

[2]Since they are only approximate eigenfunctions as described in [15].

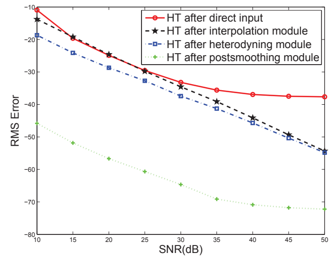
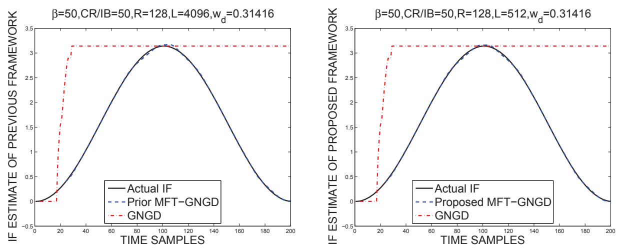


Fig. 6. Performances of demodulation via *Hilbert transform* (HT) after direct input, interpolation module, heterodying module, and postsmoothing module respectively.

its robustness to noise. The postfiltering module significantly reduce the error since the IF spectrum for sinusoidal FM is exactly narrowband for this example.

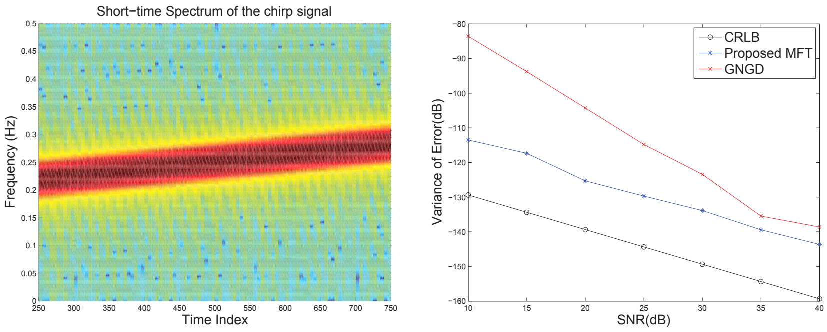


(a) (b)

Fig. 7. Demodulation performances of both MFT frameworks with conversion factor $R = 128$ and normalized radian frequency shift $w_d = 0.1\pi$ under the extreme senario with modulation index $\beta = 50$. (a) IF estimates (dashed line) of GNGD under the prior MFT framework with the heterodyne-BPF order $L = 4096$, (dashed-dotted line) the GNGD, and (solid line) actual IF. (b) IF estimates (dashed line) of GNGD under the proposed MFT framework with the heterodyne-BPF order $L = 512$, (dashed-dotted line) the GNGD, and (solid line) actual IF. Note that the GNGD alone fails under this large deviation FM signal with a modulation index as large as 50 while the GNGD combined with both MFT frameworks maintain tracking with similar performances.

We further investigate an extreme wideband scenario under the noise free environment, where the modulation index β is as large as 50 and the frequency deviation is equal to the carrier frequency with the IF varying over the entire carrier range. For the signal of interest, the IF estimates of both the prior and proposed MFT frameworks are illustrated by Fig. 7(a) and Fig. 7(b) respectively. It can be observed that the GNGD demodulation alone fails in this extreme wideband scenario, while both the prior and proposed frameworks that exploit MFT maintain tracking. The observation implies that both MFT frameworks guarantee the demodulation with acceptable performance even in the worst senarios where conventional algorithms would normally fail.

To quantify the performance of the porposed MFT approach, we explore another scenario where the signal is a wideband linear chirp instead of a sinusoidal FM. The short-time spectrum of the chirp signal is illustrated by Fig. 8(a). To validate the performance of the proposed MFT approach , we can compare the variance of error with respect to the chirp rate estimate with its *Cramér Rao lower bound*



(a) (b)

Fig. 8. Performance of the proposed MFT framework with a multirate conversion factor of 128 in wideband linear chirp scenario. (a) Short-time spectrum of the wideband linear chirp. (b) Comparison of the error variance with respect to the estimate of chirp rate. Note that there is only a small gap less than 20 dB between the error variance estimate of the proposed MFT and the corresponding $Cramér\ Rao\ lower\ bound$ (CRLB).

(CRLB). The chirp rate can be obtained via demodulated IF followed by a least square estimator. In the presence of noise with different SNRs, the result is summarized in Fig. 8(b). Improvement over the GNGD alone is more apprent with respect to lower SNR, indicating satisfactory performance of proposed MFT approach in noisy condition. Also note that the gap between the error variance estimate of the proposed MFT and the corresponding CRLB is nearly a constant, which can be explained directly via the loss of spectrum incurred by filtering the FM signal.

V. Conclusion

A system that combines multirate processing and heterodying to accomplish wideband FM demodulation for large wideband to narrowband factors was proposed. Prior work combining these systems was shown to produce impractical designs for large factors, needing bandpass filters of very high orders and very narrow passbands. Interchanging the order of the heterodyne and multirate modules was shown to reduce the computational burden placed on the bandpass filter for for large conversion factors.

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