MULTICOMPONENT AM–FM ENERGY DEMODULATION WITH APPLICATIONS TO SIGNAL PROCESSING AND COMMUNICATIONS

A Doctoral Thesis Defence

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Nov 24 , 1997

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Thesis Goal:

Multicomponent AM–FM signal separation and demodulation.

Contributions:

(1) Comparison of existing multicomponent AM–FM demodulation approaches with simulations.

(2) Development of the *energy demodulation of mixtures* (EDM) algorithm and application to co-channel FM–voice separation and demodulation problem.

(3) Development of the *periodic algebraic separation energy demodulation* (PASED) algorithm and application to the co–channel FM–voice separation and demodulation problem.

(4) Harmonic analysis and restoration of separation methods for periodic mixture separation with application to the vowel separation problem.

Monocomponent AM–FM Signal Model:

- Monocomponent AM–FM Signals:

$$s[n] \;=\; A[n] \cos\left(\int_0^n \Omega[m] dm \;+\; \theta\right)$$

  * $\Omega[n]$: Instantaneous Frequency (IF) signal
  * $A[n]$: Instantaneous Amplitude (IA) signal

- IF signal general form: $\Omega[n] = \Omega_c \;+\; \Omega_m q_f[n]$
  * Ω_c: Mean or carrier frequency
  * Ω_m: Maximum frequency deviation
  * $q_f[n]$: Normalized frequency message signal

- IA signal general form: $A[n] = A_c \left(1 \;+\; \kappa q_a[n]\right)$
  * A_c: Mean amplitude
  * κ: AM modulation depth
  * $q_a[n]$: Normalized amplitude message signal

Applications for Monocomponent Model:

- Transmission of information over communication channels.
- Modeling of single speech resonances.
- Modeling target information in radar signals.
- Often present in signals created and processed by biological systems.

Demodulation:

Process of extracting $\Omega[n]$ and $A[n]$ from $s[n]$.

Existing Methods:

- Hilbert Transform Demodulation (HTD).
- Phase–Locked Loops (PLL).
- Extended Kalman filter (EKF).
- Energy Separation Algorithm (ESA).
- Instantaneous Exact Prony (IEP).

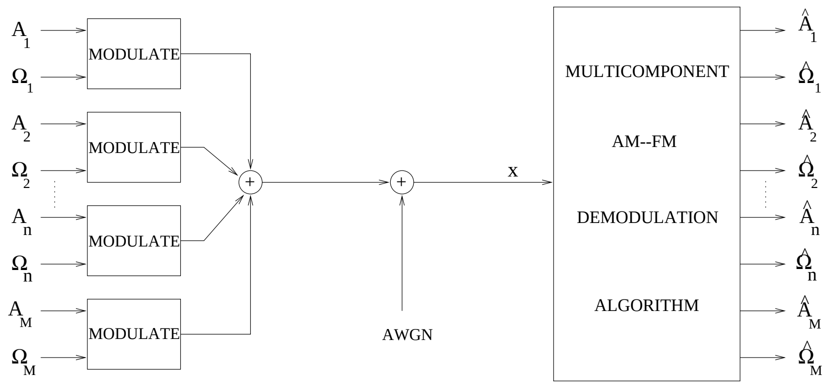
Multicomponent AM–FM Signal Model:

- **Multicomponent AM–FM Signals:**

$$x[n] \equiv \sum_{i=1}^{M} \underbrace{A_i[n] \cos\left(\int_0^n \Omega_i[k]dk + \theta_i\right)}_{x_i[n]}, \quad M \geq 2.$$

- **Applications for multicomponent model:**
  - Modeling co–channel and adjacent channel interference over communication channels
  - Modeling of speech signals as superposition of monocomponent AM–FM resonances
  - Modeling of clutter in high frequency radar.
  - Modeling for multiple target tracking applications
  - Modeling of nonstationary signals as a sum of modulated sines
- **Problems in the multicomponent case:**
  - Spectral overlap of components
  - Large relative power ratio
  - Component IF track crossover

Multicomponent AM–FM Demodulation:



Existing Methods:

- **Energy Demodulation methods**
  - ∗ (MESA) Multiband–ESA [BMQ93].
- **Adaptive Linear Prediction**
  - ∗ LMS Algorithm [Grif75].
  - ∗ Exponentially Windowed RLS Algorithm [Hayes96].
- **State Space methods**
  - ∗ (CC-DPLL) Cross-Coupled Digital Phase-Lock Loops [BrKi93].
  - ∗ (EKF) Extended Kalman Filtering [BrKi96,LuDo96].
- **Hankel and Toeplitz Matrix methods**
  - ∗ (HRR) Hankel Rank Reduction [DiAr90].
  - ∗ (ITD) Instantaneous Toeplitz Determinants [Kum92].
- **Maximum Likelihood (ML) Estimation** [FrFr95]

Parameters and Performance Measures

- **Two–component Sinusoidal AM–FM Signals:**

$$x[n] = \overbrace{A_1[n]\cos\left(\int_0^n \Omega_1[m]dm + \theta_1\right)}^{x_1} + \overbrace{A_2[n]\cos\left(\int_0^n \Omega_2[m]dm + \theta_2\right)}^{x_2}$$

$$A_i[n] = A_{ci}\left[1 + \kappa_i \cos\left(\Omega_{ai} n + \alpha_i\right)\right]$$

$$\Omega_i[n] = \Omega_{ci} + \Omega_{mi}\cos\left(\Omega_{fi} n + \gamma_i\right)$$

- **CR/IB:** (carrier-to-information bandwidth)

$$(\mathsf{CR/IB})_i = \frac{\Omega_{ci}}{\max(\Omega_{ai}, \Omega_{fi})}$$

- **Average Carson Bandwidth =** $\sum_{i=1}^{2} (\Omega_{mi} + \Omega_{fi} + \Omega_{ai})$

- **NCS**: (Normalized Carrier Separation)

$$\mathsf{NCS} = |\Omega_{c2} - \Omega_{c1}| \; / \; \sum_{i=1}^{i=2} (\Omega_{mi} + \Omega_{fi} + \Omega_{ai})$$

- **MPR:** (Mean Power Ratio)

$$\mathsf{MPR\ (dB)} = 20\log\left(\frac{(x_1)_{\mathsf{rms}}}{(x_2)_{\mathsf{rms}}}\right)$$

- **Normalized Errors** (carrier-biased)**:**

$$\mathsf{NRMSE} = \frac{||\Theta - \hat{\Theta}||_2}{||\Theta||_2} \quad , \quad \mathsf{NMAE} = \frac{||\Theta - \hat{\Theta}||_1}{||\Theta||_1}$$

Limitations of Existing Algorithms

- Existing methods work only over a narrow range of NCS and MPR parameters.
- Assumptions regarding distinctness and separability via linear filtering become invalid as NCS decreases.
- As the NCS parameter decreases, interaction between components manifests as beating in the estimates.
- Median and moving average filtering (post–smoothing) removes this beating to a certain extent.
- NCS < 0.1 (co–channel range): all of the existing algorithms develop singularities.
- Exhibit flipping of estimates when the component IF tracks cross.

Singularity Effects:

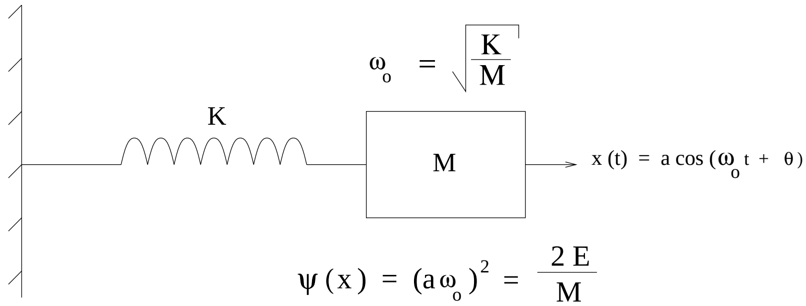
Decrease in Spectral Separation:

- The correlation matrix in the case of the linear predictive techniques becomes singular.
- The Fisher information matrix in the case of the maximum likelihood techniques becomes singular.
- The Toeplitz and Hankel matrix systems of the ITD and the HRR algorithms becomes singular.
- Observability Gramian of the two–component state model for the EKF and the CC–DPLL becomes singular.
- Third–order energy measure in the EDM becomes singular.

Increase in the Relative Power Ratio:

- Correlation matrix in the case of the LMS becomes singular.
- Observability Gramian of the two–component state model becomes singular.

Energy Separation Concept:



- **Energy Operator** (Teager and Kaiser , 1990):

$$\Psi(x) \quad \equiv \quad (\dot{x})^2 - x\ddot{x}.$$

This operator tracks the energy of the source

that produced the signal x.

- **Energy Separation Algorithm** (ESA)

(Maragos , Kaiser and Quatieri , 1991)

The ESA uses the energy operator applied on the signal

$$x(t) \quad = \quad A(t)\cos\left(\int_0^t \omega(\tau)d\tau + \theta\right).$$

and its time derivative to demodulate the signal

into IF and IA signals

$$\omega(t) \;\approx\; \sqrt{\frac{\Psi(\dot{x})}{\Psi(x)}} \quad , \quad |A(t)| \;\approx\; \frac{\Psi(x)}{\sqrt{\Psi(\dot{x})}}.$$

- Related multiband–ESA finds applications in

speech formant and bandwidth tracking.

Higher–order Energy Operators (HOEO's)

- **The k^{th}–order energy operators**:

  (Maragos and Potamianos , 1995)

$$\begin{aligned} \text{Cont.}: \quad \Upsilon_k(x) &\equiv \dot{x}x^{(k-1)} - x x^{(k)}, \\ \text{Discr.}: \quad \Upsilon_k(x_n) &\equiv x_n x_{n+k-2} - x_{n-1} x_{n+k-1}. \end{aligned}$$

- For $k = 2$, we obtain the Teager–Kaiser energy operator

$$\Upsilon_2(x) \equiv \Psi(x) = (\dot{x})^2 - x\ddot{x}.$$

- HOEO's correspond to higher–order energies of a classical oscillator subject to a displacement of x.

- Second,third and fourth–order operators have dimensions of energy, energy–velocity and energy–acceleration.

Energy Demodulation of Mixtures (EDM)

- AM-FM Mixture Signal

$$x(t) = A_1(t)\cos\left(\int_0^t \omega_1(\tau)d\tau + \theta_1\right) + A_2(t)\cos\left(\int_0^t \omega_2(\tau)d\tau + \theta_2\right)$$

- Instantaneous Signal Model

$$x(t) = a_1\cos(\omega_1 t + \theta_1) + a_2\cos(\omega_2 t + \theta_2)$$

- Generating Differential Eqn (GDE)

$$x^{(4)} + c_1 x^{(2)} + c_2 x = 0 \quad , \quad c_1 = \omega_1^2 + \omega_2^2 \ , \ c_2 = \omega_1^2\omega_2^2$$

- Ratios of Higher-order Energies

$$c_1 = -\frac{\Upsilon_5(x)}{\Upsilon_3(x)} \quad , \quad c_2 = \frac{\Upsilon_3(\ddot{x})}{\Upsilon_3(x)}$$

- Frequency estimates

$$\omega_{1,2} = \sqrt{\frac{c_1 \pm \sqrt{c_1^2 - 4c_2}}{2}}$$

- Amplitude estimates

$$|A_{1,2}| = \sqrt{\frac{\omega_{2,1}^4(\Psi(x^{(3)}) - \omega_1^2\omega_2^2\Psi(\dot{x}))}{\omega_1^4\omega_2^4(\omega_1^2 - \omega_2^2)^2} - \frac{\omega_{2,1}^6(\Psi(\ddot{x}) - \omega_1^2\omega_2^2\Psi(x))}{\omega_1^4\omega_2^4(\omega_1^2 - \omega_2^2)^2}}$$

Salient Features of the EDM Algorithm:

- Provides exact amplitudes and frequencies for sinusoidal signals and estimates of the component IF and IA signals for the AM–FM case.

- Non–adaptive, non–iterative and does not assume prior knowledge of carrier frequencies and message bandwidths.

- HOEO's are physically–meaningful, easy to compute and have excellent time resolution.

- Frequency estimation is independent of amplitude estimation and the power of one component relative to the other.

- Performs comparably or better to previous methods for co–channel FM–voice separation and demodulation for $\mathrm{NCS} \geq 1/4$ at a lower complexity.

Performance of the EDM

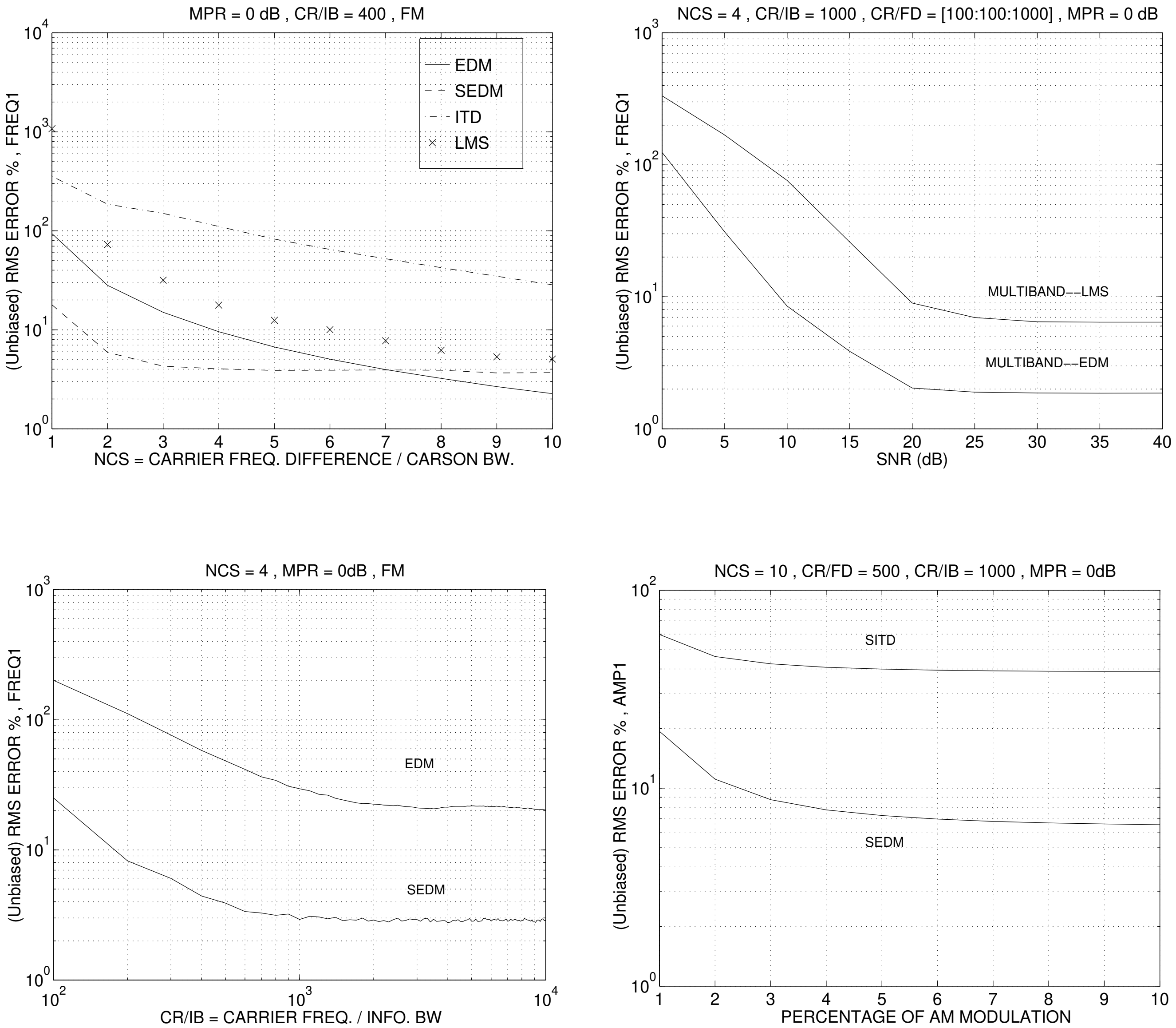
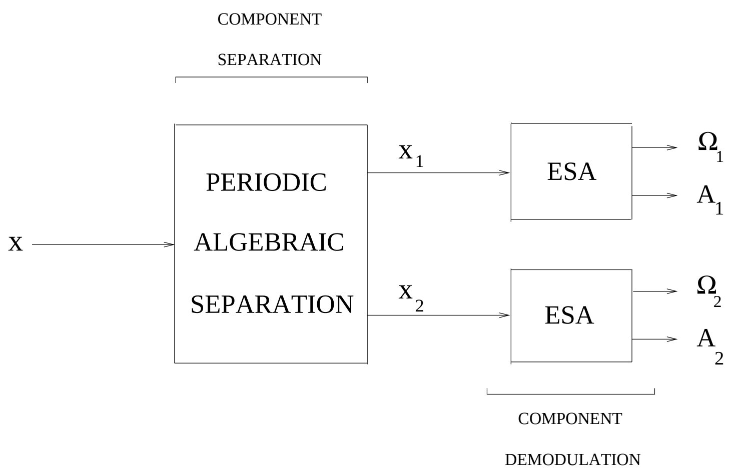


Figure 1: Effect of NCS parameter, SNR, CR/IB parameter and AM modulation percentage on the performance of the EDM algorithm.

PASED Algorithm: $M = 2$



- Separate multicomponent AM–FM demodulation problem into two separate tasks of component separation and demodulation.
- Separation of components accomplished through periodicity based modeling and algebraic separation of the components.
- Demodulation of separated components based on the *energy separation algorithm* (ESA).

Matrix Algebraic Separation (MAS) (1)

- Model components $x_1[n]$ and $x_2[n]$ as periodic signals with fundamental periods N_1 and N_2

$$x_1[n] \;=\; x_1[n+N_1]\;,\; x_2[n+N_2] \;=\; x_2[n].$$

- Relate composite signal samples to components via the *basic separation system*:

$$\underbrace{\begin{pmatrix} x[0] \\ x[1] \\ \vdots \\ x(N-1) \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{I}_{N_2} \\ \mathbf{I}_{N_1} & \mathbf{I}_{N_2} \\ \vdots & \vdots \end{pmatrix}}_{\mathbf{S}} \underbrace{\begin{pmatrix} x_1[0] \\ \vdots \\ x_1[N_1-1] \\ x_2[0] \\ \vdots \\ x_2[N_2-1] \end{pmatrix}}_{\mathbf{z}} .$$

- The separation matrix $\mathbf{S}$ is rank deficient with

$$r(\mathbf{S}) = N_1 + N_2 - \mathsf{gcd}(N_1, N_2)\;,\; r(N(\mathbf{S})) = \mathsf{gcd}\,(N_1, N_2)\,.$$

Matrix Algebraic Separation (MAS) (2)

- Complete basic system with bandpass signal constraints of the form : $\mathbf{C}\mathbf{z} = \mathbf{0}$.
  - Coprime periods : use dc value constraint

$$\sum_{n=0}^{N_1-1} x_1[n] = 0.$$

  - Non–coprime : $R = \gcd(N_1, N_2)$

    generate constraints from subsampling:

$$\sum_{n=0}^{\left(\frac{N_1}{R}\right)-1} x_1[Rn+p] = 0 \quad , \quad p = 0, 1, \ldots, R-1.$$

- Form the *augmented separation system*:

$$\underbrace{\begin{pmatrix} \mathbf{S} \\ \mathbf{C} \end{pmatrix}}_{\tilde{\mathbf{S}}} \mathbf{z} = \underbrace{\begin{pmatrix} \mathbf{x} \\ \mathbf{0} \end{pmatrix}}_{\tilde{\mathbf{x}}}.$$

- Solve augmented separation system:

$$\mathbf{z} = \left(\tilde{\mathbf{S}}^T \tilde{\mathbf{S}}\right)^{-1} \left(\tilde{\mathbf{S}}^T \tilde{\mathbf{x}}\right) = \left(\mathbf{S}^T\mathbf{S} + \mathbf{C}^T\mathbf{C}\right)^{-1} \left(\mathbf{S}^T\mathbf{x}\right).$$

- For aperiodic IF signals, assume a finite Fourier series for each IF signal and periodically extend them with extension periods $L_i = N_i$.

PASED Algorithm: Energy Demodulation

- Model separated components as AM–FM signals:

$$x_i[n] \;=\; A_i[n]\cos\left(\int_0^n \Omega_i[k]dk + \theta_i\right)\;,\; i = 1, ..., M.$$

- Track source energies via energy operator:

$$\Psi(x[n]) \;=\; x^2[n] \;-\; x[n-1]x[n+1]$$

- Apply Ψ to $x_i[n]$ and $y_i[n] = x_i[n] - x_i[n-1]$ to estimate the IF and IA signals using the energy separation algorithm (ESA):

$$\hat{\Omega}_i[n] \;=\; \arccos\left(1 - \frac{\Psi(y_i[n]) + \Psi(y_i[n+1])}{4\Psi(x_i[n])}\right)$$

$$|\hat{A}_i[n]| \;=\; \sqrt{\frac{\Psi[x_i[n]]}{\sin^2(\hat{\Omega}_i)}}$$

- Smoothing the lowpass energy signals can further reduce the demodulation error.

- Carrier frequencies and mean amplitudes are computed as the mean of the IF and IA estimates.

- Why ESA?: simple, efficient, physically–meaningful and has excellent time resolution.

Multicomponent PASED (1)

- Model the components of the signal as periodic signals with fundamental periods N_i , $i = 1, 2, \ldots, M$.

- Generalize separation system to M components:

$$\underbrace{\begin{pmatrix} x[0] \\ x[1] \\ \vdots \\ x(N-1) \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{I}_{N_2} & \ldots \mathbf{I}_{N_M} \\ \mathbf{I}_{N_1} & \mathbf{I}_{N_2} & \ldots \mathbf{I}_{N_M} \\ \vdots & \vdots & \vdots \end{pmatrix}}_{\mathbf{S}_M} \underbrace{\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_M \end{pmatrix}}_{\mathbf{z}}.$$

- Rank of separation matrix:

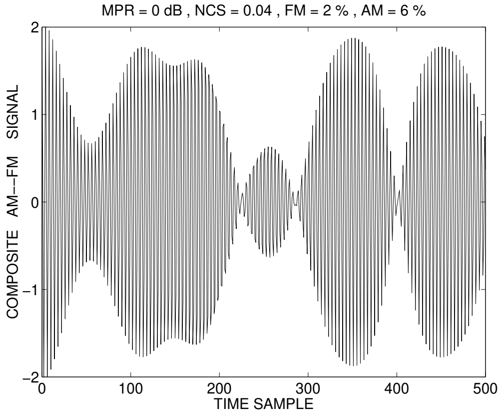
$$\text{If } M = 2 \text{ or } \prod_{i,j \ , \ i \neq j} \gcd(N_i, N_j) = 1$$

$$\text{then } r(\mathbf{S}_M) = \sum_{i=1}^{M} N_i - (M-1)\text{gcd}(N_1, N_2, \ldots, N_M)$$

$$\text{else , } r(\mathbf{S}_M) \geq \sum_{i=1}^{M} N_i - \sum_{i,j \ , \ i<j} \text{gcd}(N_i, N_j)$$

- In either case: $\mathbf{S}_M(\mathbf{z}) = \mathbf{x}$, is rank deficient and extra constraints are needed to complete it.

$M = 2$ components : Sinusoidal AM–FM (coprime)



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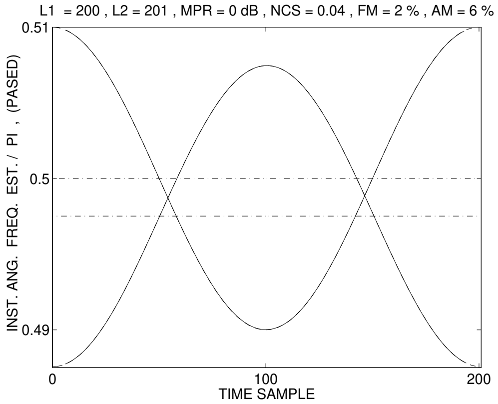

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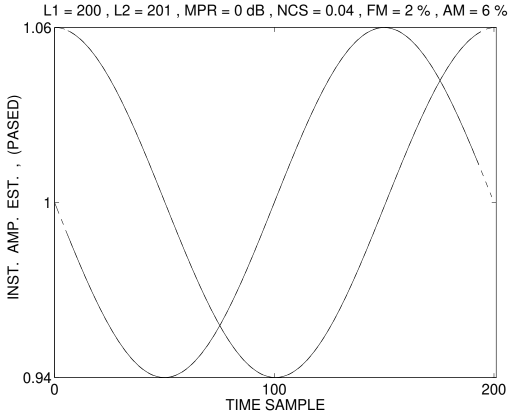

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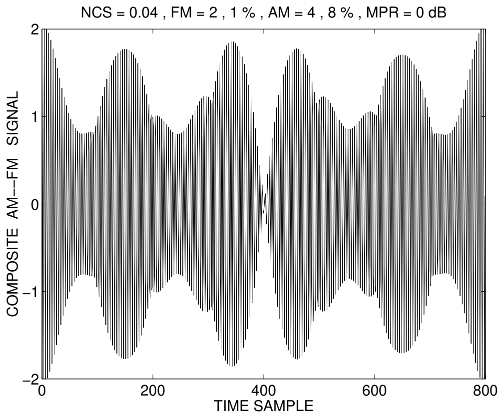
Coprime periods: $L_1 = 200$, $L_2 = 201$.

Sinusoidal IF: 2 % FM , 4 % NCS.

Sinusoidal IA: 6 % AM and MPR of 0 dB

$M = 2$ components : Sinusoidal AM–FM (non-coprime)



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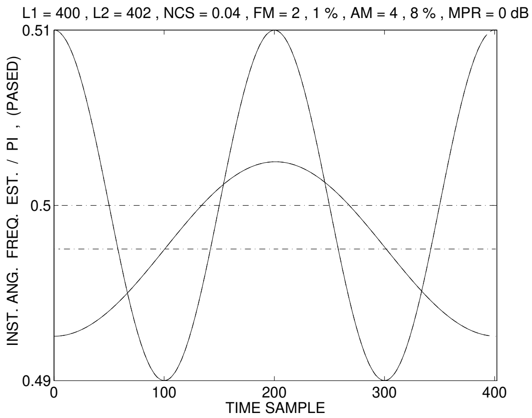

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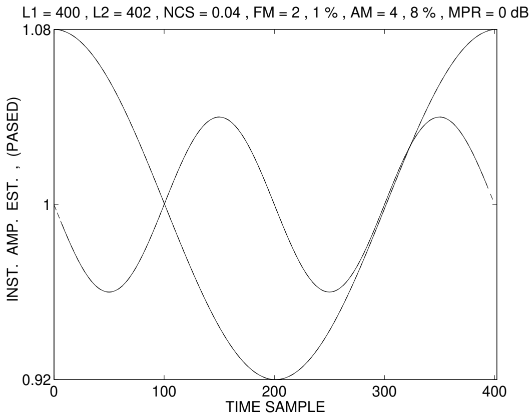

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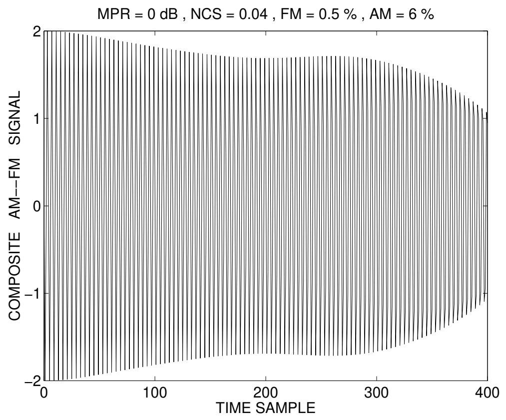
Non coprime periods: $L_1 = 200$, $L_2 = 202$.

Sinusoidal IF: 2 , 1 % FM , 4 % NCS.

Sinusoidal IA: 4 , 8 % AM and MPR of 0 dB

$M = 2$ components : Chirp Example



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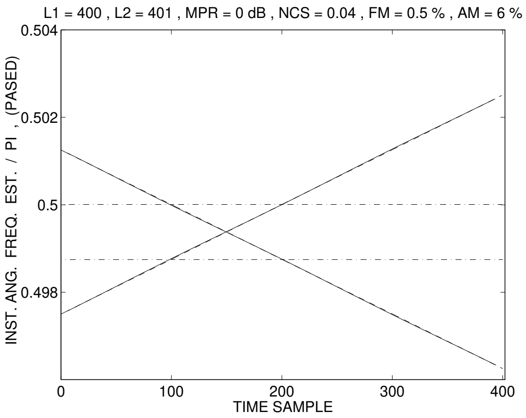

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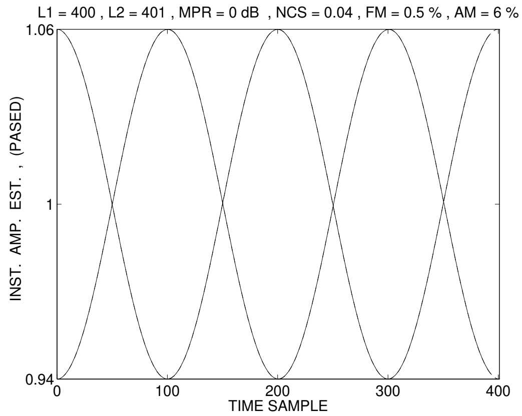

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Coprime periods: $L_1 = 400$, $L_2 = 401$.

Linear FM: 0.5% FM , NCS of ≈ 4 %.

Sinusoidal IA : MPR of 0 dB and 6% AM.

Multicomponent PASED (2)

- **Pairwise Coprime Periods**: Apply dc value constraints on $(M-1)$ of the components

$$\sum_{n=0}^{N_i-1} x_i[n] \quad = \quad 0 \ , \ \ i \ = \ 1, 2, \ldots, (M-1).$$

- **Pairwise Non–coprime**: use dc value constraints on the pairwise interactions at the scale of pairwise gcd $(R_{ij} = \text{gcd}\,(N_i, N_j))$

$$\sum_{n=0}^{\left(\frac{N_i}{R_{ij}}\right)-1} x_i[R_{ij}n+p] \quad + \quad \sum_{n=0}^{\left(\frac{N_j}{R_{ij}}\right)-1} x_j[R_{ij}n+p] \ = \ 0$$
$$\forall i \ , \ j \ , \ p \ \in \mathbf{Z}^+ \ \ni \ 1 \le i < j \le M \ , \ 0 \le p \le R_{ij}-1.$$

- Solve appended separation system for the components and use the ESA to obtain the component IF and IA signals.

- Requires $\sum_{i=1}^{M} N_i$ samples of the composite signal. Two–component seqential separation procedure:

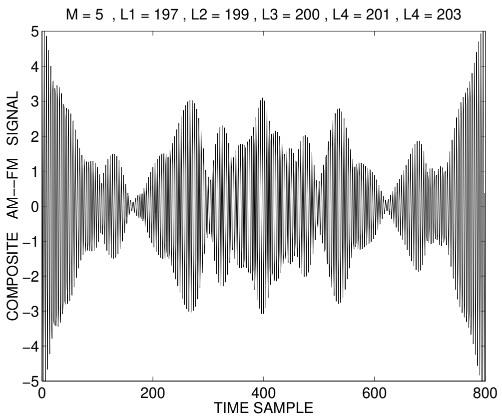
$$x[n] = x_1[n] + \underbrace{\sum_{i=2}^{M} x_i[n]}_{\tilde{x}_2[n]} \ = \ x_1[n] + x_2[n] + \underbrace{\sum_{i=3}^{M} x_i[n]}_{\tilde{x}_3[n]}$$

needs more composite signal samples.

Salient Features of the PASED Algorithm:

- Performance is independent of NCS parameter.
- Performance is independent of MPR parameter.
- Separability between the component separation and demodulation tasks.
- Does not exhibit flipping of the estimates when the IF tracks of the components cross.
- Least–squares smoothing accomplishes simultaneous component separation and smoothing.
- Performance independent of the ordering of periods (roles of the components interchangeable).
- Can be applied to complex valued data as well.

$M = 5$ components : Sinusoidal AM–FM (coprime)



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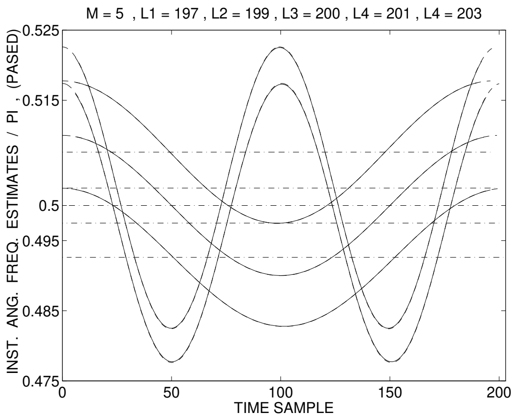

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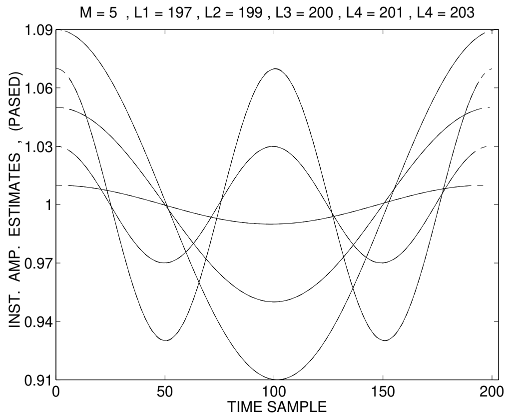

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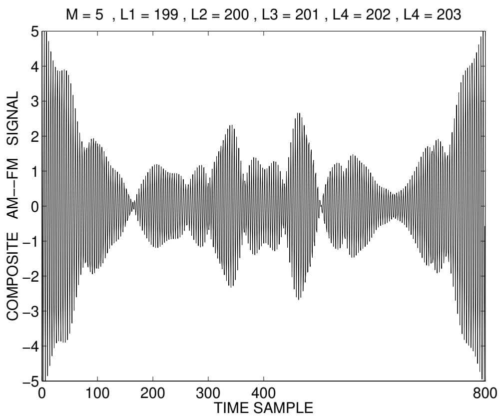
Multicomponent PASED ($M = 5$) : Pairwise Coprime Periods.

Sinusoidal IF: $L_i \in \{197, 199, 200, 201, 203\}$, $\approx 0.5\%$ FM.

Sinusoidal IA: $\kappa_i \in \{1, 3, 5, 7, 9\}\%$ AM and $A_{ci} = 1$, $\forall i$.

$M = 5$ **components : Sinusoidal AM–FM (non-coprime)**



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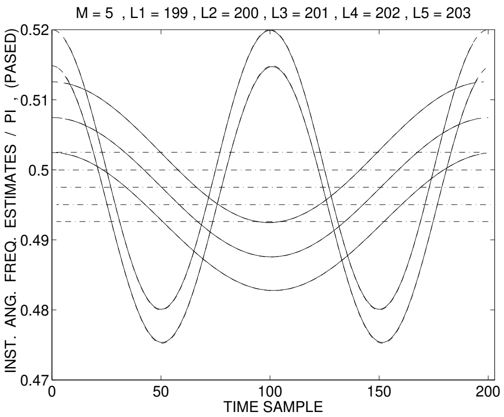

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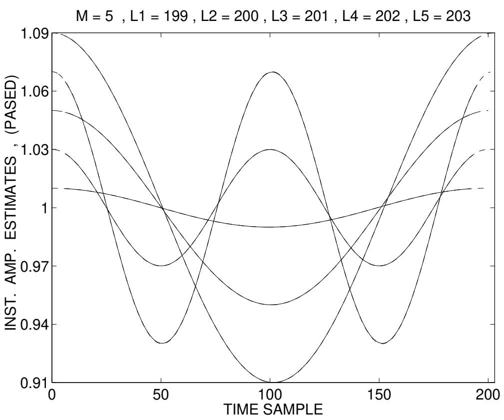

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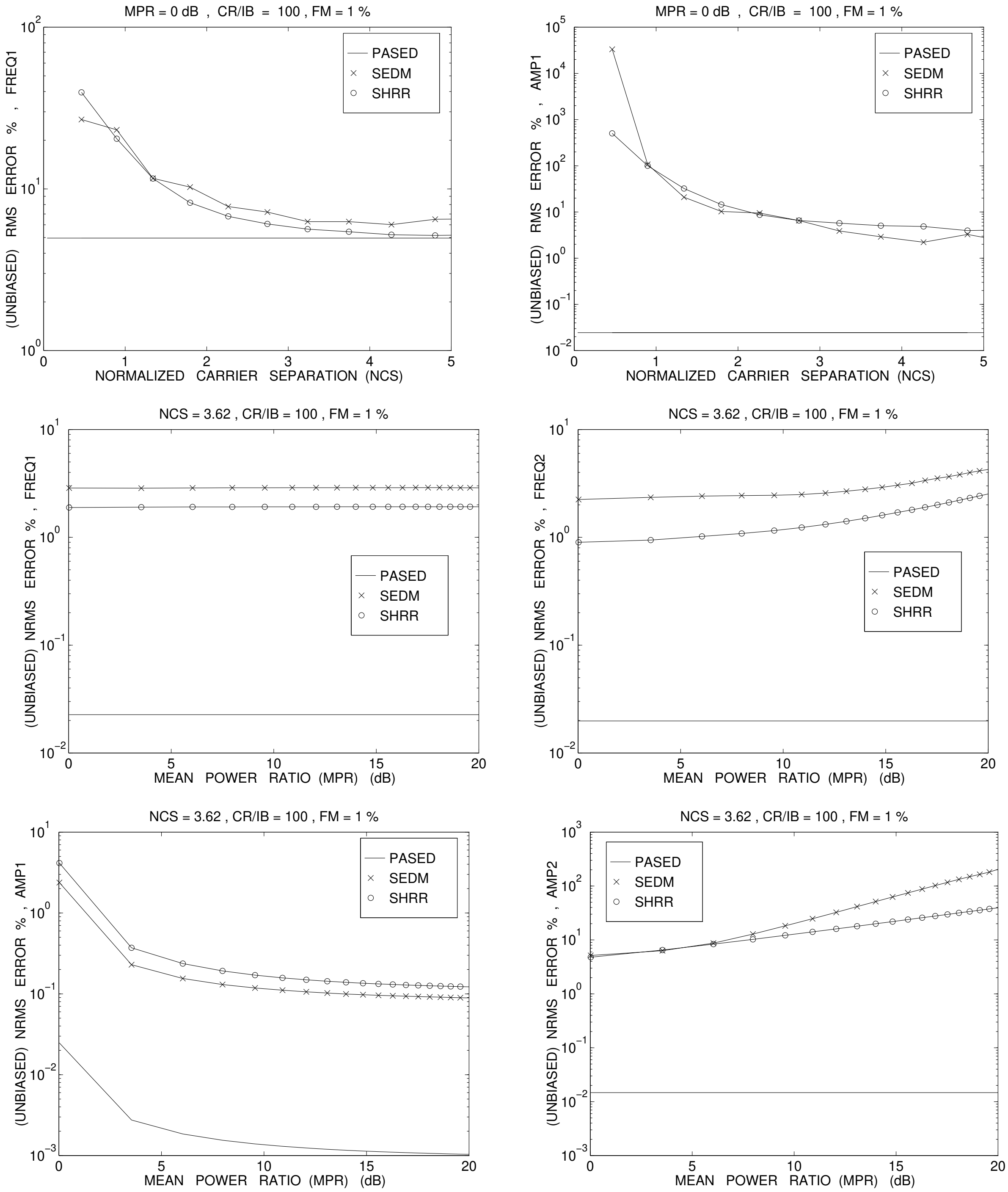
Multicomponent PASED $(M = 5)$: Pairwise Non–coprime Periods.

Sinusoidal IF: $L_i \in \{199, 200, 201, 202, 203\}$, $\approx 0.5\%$ FM.

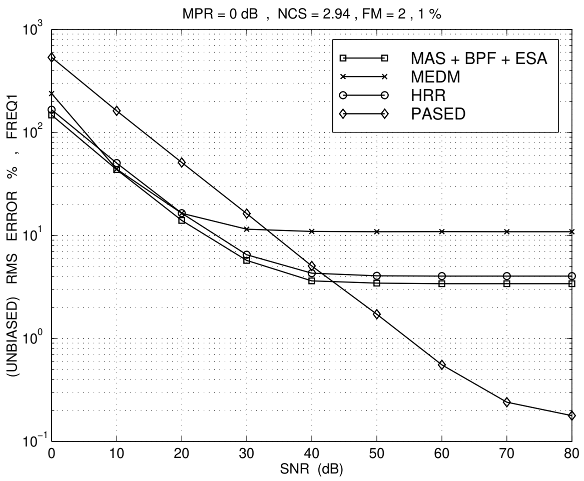
Sinusoidal IA: $\kappa_i \in \{1, 3, 5, 7, 9\}\%$ AM and $A_{ci} = 1$, $\forall\, i$.

Performance of the PASED:



Effect of MPR and NCS parameters on the performance of the PASED algorithm.

Performance of the PASED:



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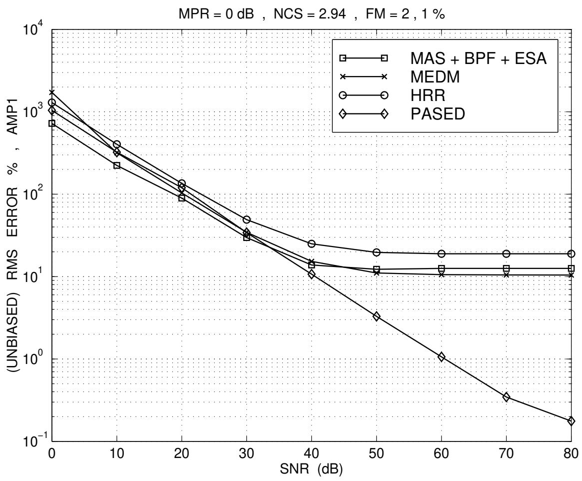

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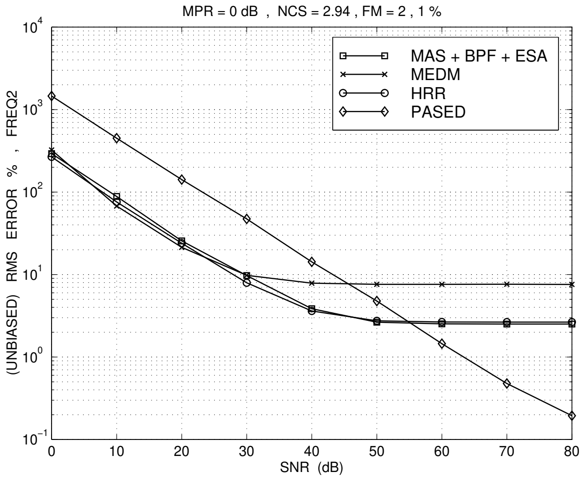

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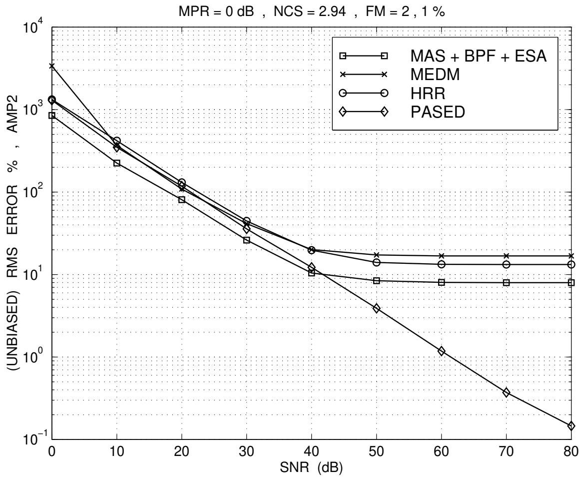

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Effect of SNR on the PASED algorithm.

Component Periodicity Estimation:

- MAS algorithm requires prior knowledge or estimation of component periodicities.

- Search two dimensional lag parameter space (L_1, L_2) of the DDF objective function:

$$\begin{aligned}\mathrm{DDF}[n, L_1, L_2] &= \sum_{m=0}^{L-1} |x[n+m] - x[n+m+L_1] \\ &- x[n+m+L_2] + x[n+m+L_1+L_2]|,\end{aligned}$$

  where L is analysis window duration and n is the analysis point.

- Coordinates of minimum of DDF function furnish estimates of component periodicities.

- Integer search needed for both lag parameters.

- Use symmetry of DDF in the lags to reduce search area to half a quadrant.

- Modular form allows easy extension when $M > 2$.

Harmonic Analysis of SPM:

- Discrete spectral content of $x_i[n]$:

$$X_i(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} a_{im}\delta\left(\Omega - \frac{2\pi m}{N_i}\right) \quad , \quad i = 1, 2.$$

- $x[n]$ also periodic with repetition period $P = \mathrm{lcm}(N_1, N_2)$

$$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} b_m\delta\left(\Omega - \frac{2\pi m}{P}\right).$$

- Component separation: $\{b_m\} \to \{a_{im}\}$.

- Information loss at common harmonics:

$$\Omega_k = \left(\frac{2\pi}{R}\right)k \quad , \quad k = 0\ ,\ 1\ ,\ \ldots\ ,\ (R-1).$$

- $\Omega = 0$ will be a common harmonic for any R.

- Basic options for reassignment of Ω_k:
  - $*$ **OP1** : Assign Ω_k to $x_2[n]$.
  - $*$ **OP2** : Assign Ω_k to $x_1[n]$.
  - $*$ **OP3** : Assign Ω_k to both.
  - $*$ **OP4** : Assign Ω_k to neither.

- **OP4** may violate $X(\Omega_k) = X_1(\Omega_k) + X_2(\Omega_k)$.

Frequency–domain Analysis of MAS:

- Information lost can be supplied via the ratio of spectral impulse amplitudes at Ω_k: $\lambda_k = \frac{X_2(\Omega_k)}{X_1(\Omega_k)}$.
- MAS : dc value constraints on $x_i[n]$ in the time–domain forces spectral nulls at Ω_k

$$\{\mathbf{C}_i\}_{lm} = \delta[(m-l)_{\mathrm{mod}\ N_i}]$$
$$\mathbf{C}_i\mathbf{x}_i = 0 \Longleftrightarrow X_i(\Omega_k) = 0$$

- MAS: Ω_k reassignment options:

$$\mathbf{C}_1\mathbf{x}_1 = \lambda\mathbf{C}_2\mathbf{x}_2 \Longleftrightarrow \mathbf{OP}3$$
$$\mathbf{C}_1\mathbf{x}_1 = 0 \Longleftrightarrow \mathbf{OP}1 \Longleftrightarrow \lambda = 0$$
$$\mathbf{C}_2\mathbf{x}_2 = 0 \Longleftrightarrow \mathbf{OP}2 \Longleftrightarrow \lambda = \infty$$
$$\mathbf{C}_1\mathbf{x}_1 = 0 \ \& \ \mathbf{C}_2\mathbf{x}_2 = 0 \Longleftrightarrow \mathbf{OP}4$$

- $\mathbf{OP}4$ is undesirable because it forces spectral nulls in both components.
- Fill in spectral nulls introduced by $\mathbf{OP}1$ or $\mathbf{OP}2$ using interpolation or the heuristic rule to decide on the reassignment option.

Harmonic Restoration 1:

Spectral Interpolation:

- Interpolate the missing information at Ω_k using spectral information from adjacent harmonics.
- Interpolation schemes that were tried:
  - ∗ Nearest neighbor interpolation.
  - ∗ Spline or polynomial interpolation.
  - ∗ Morphological interpolation (flat closing).
- Obtain missing information for the other component from $X(\Omega_k) \; = \; X_1(\Omega_k) + X_2(\Omega_k)$.
- Time–domain estimates obtained via IFFT.
- Interpolation schemes assume spectral correlation in the local neighborhood of Ω_k.

Harmonic Restoration 2:

Heuristic Rule:

- Critical loss of information: Ω_k falls in the vicinity of the spectral peak for narrowband bandpass signals.

- No information loss constraints for sinusoidally modulated AM–FM signals:

$$\text{Bandwidth constraint}: \overbrace{2\left(\Omega_{mi} + \Omega_{fi} + \Omega_{ai}\right)}^{\text{Carson Bandwidth}} \leq \frac{2\pi}{R}$$

$$\text{Carrier constraint}: \min_k |\Omega_{ci} - \Omega_k| \geq \left(\Omega_{mi} + \Omega_{fi} + \Omega_{ai}\right).$$

- When Ω_k falls in the vicinity of Ω_{ci} use heuristic to minimize damage

$$\begin{aligned} \min_k |\Omega_{c1} - \Omega_k| &> \min_k |\Omega_{c2} - \Omega_k| \implies \lambda = 0 \\ \min_k |\Omega_{c2} - \Omega_k| &> \min_k |\Omega_{c1} - \Omega_k| \implies \lambda = \infty. \end{aligned}$$

- Can also use the heuristic rule in conjunction with spectral interpolation.

Harmonic Restoration for Comb Filtering

- Additive nature of the periodic mixture not incorporated in $H_i(\Omega) = \frac{1}{2}(1 - e^{-j\Omega N_i})$.
- Filters H_i eliminate Ω_k from both components $(\mathbf{OP4})$ and have frequency resolution limitations.
- Incorporating additive nature of mixture:

$$
\begin{aligned}
T_1(\Omega) &= \frac{\zeta H_2(\Omega)}{H_1(\Omega) + \zeta H_2(\Omega)} \ , \ \Omega \neq \Omega_k \\
T_2(\Omega) &= \frac{H_1(\Omega)}{H_1(\Omega) + \zeta H_2(\Omega)} \ , \ \Omega \neq \Omega_k \ , \ \zeta \in \mathbb{R} \\
T_1(\Omega_k) &= \lim_{\Omega \to \frac{2\pi k}{R}} T_1(\Omega) = \frac{\zeta N_2}{N_1 + \zeta N_2} \\
T_2(\Omega_k) &= \lim_{\Omega \to \frac{2\pi k}{R}} T_2(\Omega) = \frac{N_1}{N_1 + \zeta N_2}.
\end{aligned}
$$

- Correspondence with MAS : $\lambda \Longleftrightarrow \frac{\zeta N_2}{N_1}$.
- $T_1(\Omega) + T_2(\Omega) = 1$, but these filters still have frequency resolution limitations.

CONCLUSIONS:

EDM:

- Provides a simpler and physically–meaningful alternative to the existing multicomponent AM–FM demodulation algorithms via the use of the energy operator and its higher–order extensions.
- Yields intelligible message estimates for spectral separations as low as $25\ \%$ of the RF bandwidth of the components when applied to the co–channel FM–voice problem.

PASED:

- Performance independent of spectral separation and relative power ratio and ability to deal with IF track overlap.
- Simplicity, efficiency, and time resolution of the ESA combined with some robustness to noise via least–squares smoothing.

Harmonic Restoration:

- Information lost at the common harmonics can be partially restored via spectral interpolation or the use of the heuristic rule developed for common harmonic reassignment or both.
- Improved MAS algorithm able to separate a mixture of both synthetic vowels and quasiperiodic sections of real vowels.