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Discussion on: "Continuous Fuzzy Controller Design Subject to Minimizing Control Input Energy with Output Variance Constraints"

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In the paper of Chang and Wu a fuzzy controller is designed to stabilize a nonlinear continuous-time system, while simultaneously minimizing the control input energy and satisfying constraints placed on the output. The design is illustrated on the well-known translational oscillator with rotational actuator (TORA), also known as the rotational/translational proof-mass actuator (RTAC) system, and contrasted with two previously published controllers, namely a passivity-based design and a gain-scheduling design.

The purpose of this discussion note is to point out some related work by various authors to both the idea of using linear matrix inequalities (LMIs) in designing fuzzy controllers and the ability of handling multiple performance objectives. It was already clear by the late 1990's that the usual stability limitations of fuzzy control designs may be alleviated, at least for the Tagaki–Sugeno (T–S) fuzzy models [3], using LMIs. Recall that the T–S models, later elaborated upon by Sugeno and Kang [4] allows the system dynamics to be written as a set of fuzzy implications which characterize local models in the state space. A T–S fuzzy model then expresses the local dynamics of each fuzzy rule by a linear dynamical model, and the overall fuzzy model is achieved by a blending of these rules. This type of modeling was later shown to be able to approximate nonlinear systems [5]. Then, linear control theory was used to design local controllers for each linear model, followed by an "intelligent" blending of the local controllers into a global controller for the original system.

In fact, it was already proven in Ref. [2] that closed-loop stability may be guaranteed for the set of fuzzy plants described by

$$\begin{aligned} &\text{If } x_j(t) \textit{ is } M_{ij}; \quad i=1,\ldots,r;\ j=1,\ldots,n \\ &\text{Then } \dot{x}(t) = A_i x(t) + B_i u(t) + D_i v(t) \\ &\text{and} \quad y(t) = C_i x(t) \end{aligned}$$

using the blended state-feedback controller of the form $u = \sum_{i=1}^{r} h_i(t) G_i x(t)$ if one could find the common positive-definite matrix P that satisfies the Lyapunov inequalities (10) and (11) in the discussed paper, and repeated here as:

$$\begin{aligned} &0 > (A_i + B_i G_i)^{\mathrm{T}} P + P(A_i + B_i G_i), \quad i = 1,\ldots,r \\ &0 > R_{ij}^{\mathrm{T}} P + P R_{ij}, \quad i < j \le r \\ &R_{ij} = \frac{(A_i + B_i G_j) + (A_j + B_j G_i)}{2}, \quad i < j. \end{aligned}$$

However, finding such a common solution was difficult if not impossible until Jadbabaie and others proposed using LMIs to obtain computationally efficient methods to solve the original problem. Such an approach was pursued by Jadbabaie in his Masters thesis [6], where the T–S modeling approach was re-interpreted using polytopic linear differential inclusions (PLDI) in such a way as to allow LMIs to be used for solving the state-feedback control design

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problem. Using duality, Jadbabaie also pursued the idea of designing fuzzy observers, and finally, under some mild conditions was able to show a separation property allowing him to replace the state-feedback controller with a dynamic output-feedback controller. In fact, Jadbabaie was able to use his framework to design guaranteed-cost and non-fragile compensators using the same approach. The reason such designs are interesting in relation to the discussed paper is because effectively Jadbabaie used LMIs to guarantee various objectives in addition to the basic stability result. Not unlike the result of Chang and Wu of Theorem 1, Jadbabaie and various other authors have combined the modeling capabilities of T–S models, with the computational effectiveness of LMIs to design controllers for various nonlinear systems. The fact that the discussed paper focuses on minimizing the input energy as well as satisfying output variance constraints represent an interesting variation on a well-established theme. This is made very explicit in the papers [7–10].

Moreover, it seem that the authors of the current paper are unaware of the special issue [1] which focuses on the TORA example. In that special issue, various authors presented controllers to stabilize TORA while minimizing the effects of external disturbances. Therefore, the comparison of the two designs mentioned in the paper are incomplete.

In short, the discussed paper provides an example in a long series of papers that combine T–S fuzzy models for nonlinear systems with LMIs to design controllers that achieve multiple performance objectives. The book [11] presents a complete theory on the subject as well as many examples.

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