Stability and Finite-Time Stability Analysis of Discrete-Time Nonlinear Networked Control Systems

S. Mastellone, C. T. Abdallah, and P. Dorato

Abstract— **In this paper we present an approach to model networked control systems with a discrete-time nonlinear plant, operating in the presence of arbitrary but finite data dropout of state observations. Sufficient conditions for stability of the global system and finite-time stability over transmission intervals are provided.**

I. INTRODUCTION

In several recent works, the problem of networked control systems (NCS) has been posed and partially investigated [8], [12], [13], [19], [20], [21], [22], [23], [24]. This new problem deals with the possibility of controlling a system remotely via a communication network and as such, instantaneous and perfect signals between controller and plant are not achievable.

In [1] an iterative approach is proposed to model networked control linear systems with arbitrary but finite data packet dropout as a switching linear systems. The network is modelled as a switch placed between the sensor and the controller, while the controller is assumed to be directly connected to the plant. The controller uses data from a register to compute the control input which will be applied to the plant. When a sensor data, containing the system state, is successfully sent to the controller through the communication link, it is put in the register to replace the old data.

In this paper, we present an extension of the result in [1] into a nonlinear setting. With this model setting we give a sufficient condition for uniform asymptotic stability, using a similar argument to the one in [26].

Also we consider finite-time stability for the sequence of transmission interval and provide sufficient conditions for it. With the above analysis, we are then able to completely characterize system behavior in term of asymptotic and finite-time stability. The paper is organized as follows: In Section II, we reformulate the networked control problem in the nonlinear setting with arbitrary but finite packet dropouts.

Section III provides sufficient condition for uniform asymptotic stability of the networked control system. Section IV briefly describe finite-time stability and some related

analysis results. It also consider finite-time stability over the transmission intervals, extending the concept of asymptotic stability with further conditions over the transmission intervals. Section V then provide sufficient condition for the " asymptotic stability with decreasing increments". In order to illustrate the result an example is presented in Section VI. Some concluding remarks are given in section VII.

II. PROBLEM FORMULATION AND NONLINEAR NCS MODEL

In [1] a model for networked control systems with observation dropouts is proposed for linear discrete-time systems. Our objective in this paper is to derive a similar framework in the case of nonlinear systems, and to study the stability of the closed-loop system. As depicted in Figure 1, the system is comprised of a plant with the network residing between the sensors of the plant and the actuators. The network is modelled as a switch, where a measurement is dropped if the switch is disconnected, and a measurement is received when the switch is connected. Due to our inability to receive an update of the plant's state at each discrete instant of time, we use a buffer on the controllers side that store the last received sensor measurement. Such a model is given by

$$
\begin{aligned}
x(k+1) &= f(x(k)) + g(x(k))u(k) \qquad (1)\\
u(k) &= K(\bar{x}(k))
\end{aligned}
$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ are the plant state and plant input respectively. $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \to \mathbb{R}^n$, $K : \mathbb{R}^n \to \mathbb{R}^m$ are smooth vector functions. $\bar{x}(k) \in \mathbb{R}^n$ is the state measurement that is successfully transmitted dover the network, and defined as follows

$$
\bar{x}(k) = \begin{cases} x(k) & Data\ Transmitted \\ x(i) & Data\ Dropped \end{cases} \qquad (2)
$$

where $x(i)$, $i < k$ is the last packet transmitted successfully and stored in the buffer. The controller uses data from a register to compute the control input which will be applied to the plant. When a sensor data, containing the system state, is successfully sent to the controller through the communication link, it is put in the register to replace the old data. We assume the first transmission is always successful, i.e. $\bar{x}(0) = x(0)$, and therefore

$$
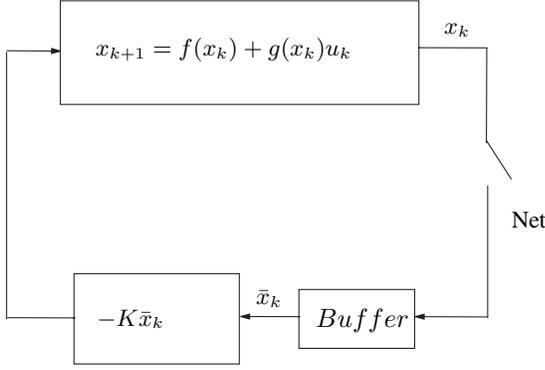x(1) = f(x(0)) + g(x(0))K(\bar{x}(0)) \qquad (3)
$$

Fig. 1. Networked Control System with packet dropout and buffer based state feedback

Define the following operator

$$[f]_k = \underbrace{f \circ \cdots \circ f}_{k-times} \qquad (4)$$

Then from $k = 2$ and later the system configuration can be described as follows

$$x(k) = \begin{cases} f(x_{k-1}) + g(x_{k-1})K(x_{k-1}) & \bar{x}_k = x_k \\ [f + gK(x_i)]_{k-i}(x_i) & \bar{x}_k = x_i \end{cases}$$

where $x(i)$ is the last packet transmitted successfully and stored in the buffer. Call the sequence of update instants $U = \{0, k_1, \ldots, k_i, \ldots\}$, then we can define for it the following system

$$x_{k_j} = F_{k_j}(x(k_{j-1})), \ i = 1, 2, \ldots \qquad (5)$$

where

$$F_{k_j}(x(k_{j-1})) = [f + gK(x(k_{j-1}))]_{k_j - k_{j-1}}(x(k_{j-1})) \quad (6)$$

From U define a new system with $z(0) = x(0)$, $z(j) = x(k_j)$, where $k_j \in U$.

$$z(j) = F_j(z(j-1)) \qquad (7)$$

$$F_j(.) = [f + gK(.)]_{k_j - k_{j-1}}(.) \qquad (8)$$

Assuming that the maximum transmission period is τ_M, then we have that the upper bound of dropped data packet is $\tau_M - 1$. With this assumption we can then say that $F(j) \in \mathfrak{F}$ where

$$\mathfrak{F} = \{\bar{F}_1, \ldots \bar{F}_{\tau_M}\} \qquad (9)$$

and

$$\bar{F}_i = [(f + gK)]_i \qquad (10)$$

Consequently, the NCS (1) at the transmission instant can be described by the following switched system

$$z(k+1) = \bar{F}_i(z(k)), \ k = 1, 2, \ldots, \ \bar{F}_i(.) \in \mathfrak{F} \quad (11)$$

for arbitrary switching, where i is the switching variable. In the system constructed as described, whenever there is no

state measurement transmission, i.e. a data dropping occurs in the network, the control law uses data previously received to generate the control input to the plant. Based on this model we next study stability behavior of NCS (1) using the new model (11).

III. STABILITY ANALYSIS

Consider the networked control system described in (1) and the discrete time switching system (11) that represents (1) at the transmission instants. Let $x = 0$ be an equilibrium point for the system (1). In what follows asymptotic stability of (1), is deduced by stability of the system (11), and the result in [25], [26] and the fact that the state is bounded during transmission intervals.

Theorem 1: Consider the system (1) and (11), also consider a continuous differentiable, locally positive definite function $V : \mathbb{R}^n \to \mathbb{R}^+$, and class K functions α, β, γ. Then the system (1) is uniformly asymptotically stable if the following holds

$$\alpha(||x||) \leq V(x) \leq \beta(||x||) \qquad (12)$$

$$\Delta V_j \triangleq V(z(j)) - V(z(j-1)) \leq -\gamma(||z(j)||) \qquad (13)$$

or equivalently

$$\Delta V_j \triangleq V(x(k_j)) - V(x(k_{j-1})) \leq -\gamma(||x(k_j)||) \quad (14)$$

for all $x \in B_r$ where B_r is the ball of radius r, $B_r = \{x : ||x|| \leq r\}$

Proof. Consider the system (11), state between two transmission is bounded $\forall j$, $k_j \leq k < k_{j+1}$, with transmission interval $[k_j, k_{j+1}]$, since F_j is a superposition of continuously differentiable function,

$$||x(k)|| = ||F_j(x_{k_j})|| \leq ||F_M(x_{k_j})|| \qquad (15)$$

where $F_M = \max_{i=1,\ldots,\tau_M}\{F_i\}$. Since F_M is a superposition of continuously differentiable functions, and (1) is a discrete-time system, then the solution $x(k)$ cannot have finite escape time and therefore $F_M(x(k_j))$ is bounded whenever $x(k_j)$ is bounded, which is for all δ such that $||x(k_j)|| \leq \delta$ there exist δ_2 such that $||F_M(x(k_j))|| \leq \delta_2$. For any $\epsilon > 0$, we define $\bar{\epsilon} = \min(\epsilon, r)$. Since V is continuous, $V(0) = 0$ and by (12), there exist $\delta(\epsilon) > 0$ such that

$$\bar{\beta}(\delta) = \sup_{||x(0)|| \leq \delta} V(x) < \alpha(\bar{\epsilon}) \leq \alpha(\epsilon) \qquad (16)$$

In order to prove that $\forall ||x(0)|| \leq \delta \ ||x(k)|| \leq \epsilon$, $\forall k \in \mathbb{N}$, we first assume that the statement is not true, and therefore $\exists k_1 > 0$, $k_1 \in [k_j, k_{j+1})$ such that

$$||x_1|| \geq \epsilon \qquad (17)$$

Since the state $x(k)$, $k \in [k_j, k_{j+1})$ will be bounded as a consequence of $x(k_j)$ bounded we assume that $k_1 = k_j$, and therefore using the fact that α is a class K function it follows that

$$V(x(k_j)) \geq \alpha(||x(k_j)||) \geq \alpha(\epsilon) \qquad (18)$$

On the other hand, since V is decreasing along k_j and from (16) it follows

$$V(x(k_j)) \leq V(x(0)) \leq \bar{\beta}(\delta) < \alpha(\epsilon) \tag{19}$$

the two last statements provide a contradiction, from which we have that the assumption (17) is wrong, and therefore uniformly stability of (1) follows. Asymptotic stability can then be proved considering $V(x(k_j)), (j = 1, 2, \dots)$ is a positive decreasing sequence, therefore there exist a nonnegative limit $L \geq 0$, and we have

$$\lim_{j \to \infty} \Delta V(x(k_j)) = \tag{20}$$

$$\lim_{j \to \infty} [V(x(k_{j+1})) - V(x(k_j))] = \tag{21}$$

$$\lim_{j \to \infty} V(x(k_{j+1})) - \lim_{j \to \infty} V(x(k_j)) = 0 \tag{22}$$

$$\Delta V(x(k_j)) = [V(x(k_{j+1})) - V(x(k_j))] \leq \tag{23}$$
$$-\gamma(||x(k_j)||) \leq 0$$

$$\lim_{k \to \infty} \gamma(||x(k_j)||) = 0 \tag{24}$$

$$\lim_{k \to \infty} ||x(k_j)|| = 0 \tag{25}$$

$$\lim_{k \to \infty} ||x(k)|| = 0 \tag{26}$$

∎

IV. Finite-Time Stability and Global Properties

The result presented in the previous section provide us with sufficient conditions for asymptotic stability of the system 1, such result does not give any information on how the state grow during transmission intervals. In this section we deal with such issue by providing sufficient conditions to have the system state decreasing in between transmission intervals. In particular at first we will define finite-time stability, in which we are interested in the system's transient behaviors, and then study the global behaviors connecting the finite-time and asymptotic stability concepts. In this section we focus on general discrete-time dynamical systems described by

$$x_{k+1} = f(x_k), \, x \in \mathbb{R}^n, \, x(0) = x_0 \tag{27}$$

Where x is the system state, and $f : \mathbb{R}^n \to \mathbb{R}^n$ is a vector function. For notational simplicity, we use $x_k = x(k)$. Also from now on we will denote $||.|| \equiv ||.||_2^2$. We are interested in studying the state trajectory of the system in a finite time interval.

Definition 1: **Finite-Time Stability** [2], [5] The system (27) is finite-time stable (FTS) with respect to the 4-tuple $(\alpha, \beta, N, ||.||)$, $\alpha \leq \beta$ if every trajectory x_k starting in $||x_0|| \leq \alpha$ satisfies the bound $||x_k|| \leq \beta$ for all $k = 1, \dots, N$.
Some extensions of the FTS concept are presented in [3],[4]. Next we present a new analysis result for FTS of nonlinear

discrete-time systems. Alternative result in discrete-time deterministic finite-time stability can be found in [16].

We consider three classes of systems described in Figure (2): a) systems for which the state trajectories always increase in the norm, b) systems for which states always decrease in the norm, and c) systems whose state trajectories behavior's is mixed. The first step consists of exploring the state trajectories using a discrete version of the continuous-time Bellman-Gronwall inequality [6]. If the state trajectory is always increasing (in the norm) during the time interval of interest, then it is enough to verify that the state at the last time of the interval does not exceed the bound. In the case where the trajectory is always decreasing and it starts inside the bound, the FTS is guaranteed. In the case of a mixed behavior, it is necessary to explore if the trajectory is bounded at each time step. In the next theorem we formulate the conditions for finite-time stability of the system (27).

Theorem 2: The system (27) is finite-time stable with respect to $(\alpha, \beta, N, ||.||)$, $\alpha \leq \beta$, if for a function $V(x_k, k) = V_k \geq 0$ such that $\delta_1 ||x_k|| \leq V_k \leq \delta_2 ||x_k||$, where $\delta_1 > 0$, $\delta_2 > 0$, $\gamma = \delta_1 \beta$, $\gamma_0 = \delta_2 \alpha$, $V_0 \leq \gamma_0$ and $S_\beta = \{x_k : ||x_k|| \leq \beta\}$ we have $\forall k = 0, \dots, N$, $\forall x_k \in S_\beta$

$$\Delta V_k \leq \rho_k V_k \tag{28}$$

and one of the following three conditions occur:

- **Case 1:** $\rho_k \geq 0$

$$\frac{\gamma}{\gamma_0} \geq \prod_{i=0}^{N-1} (1 + \rho_i) \tag{29}$$

  The value of $\rho_k \geq 0$ implies that the bounds on the increments of V_k are as a worse case always greater than one, which is the case of monotonically increasing functions.

- **Case 2:** $0 \geq \rho_k > -1$ No additional conditions are required. The condition $0 \geq \rho_k > -1$ restricts the bounds on the increments of V_k to be always between zero and one, which constrains the function to be monotonically decreasing.

- **Case 3:** $\rho_k > -1$

$$\frac{\gamma}{\gamma_0} \geq \sup_k \prod_{i=0}^{k-1} (1 + \rho_i) \tag{30}$$

  The case $\rho_k > -1$ contains the two previous cases, that is the function V_k may be increasing and decreasing.

Proof: The proof is available in [14], [15] ∎

Now we aim to investigate the overall stability behavior for the system, we not only want to know how that the state will asymptotically converge to zero, but also that during the convergence period, it will not exceed a given bound. The idea of finite-time stability, is to specify precise bound on the state system norm, over an interval of time. Based on this definition of stability we next consider an extension to the concept of asymptotic stability which includes an additional condition over the state.
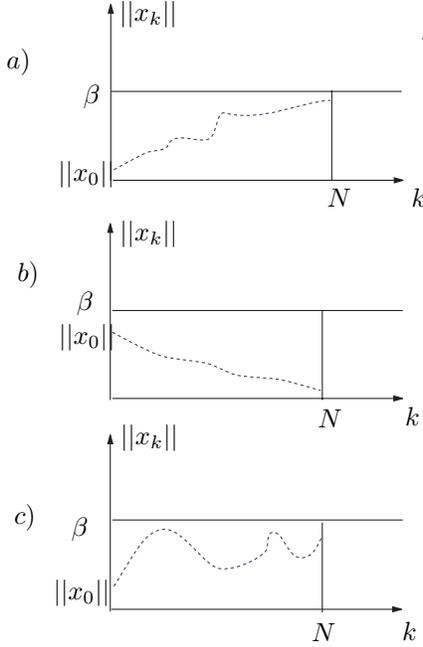
Fig. 2. *a*) Increasing dynamics. *b*) Decreasing dynamics. *c*) Mixed dynamics.

Definition 2: A discrete-time system (27) is said to be decreasing at interval with respect to the interval $I_j \in \mathbb{N}^+, j = 1, \ldots, m$ if, $\max_{j \in I_k} ||x(j)|| \leq \max_{i \in I_{k+1}} ||x(i)||, \forall k \in \mathbb{N}$.

The main idea beyond this definition is that if we partition the state in intervals of varying size $I_j, j = 1, \ldots, m$, with maximum size I_m, then the supremum of the state norm of each interval form a decreasing sequence. This not only guarantee convergence asymptotically, but also gives a bound on how the system state for every interval, moreover those bounds decrease in time. Sufficient condition for such property will be given in the next section directly applied in a networked system framework.

V. ASYMPTOTIC STABILITY WITH DECREASING INCREMENTS ANALYSIS

Next we want to provide sufficient condition on the asymptotic stability with decreasing increments for the system by splitting the system into two parts, one characterized by the system state at transmission steps, and a second characterized by the system in between transmission steps. Then if the first subsystem is asymptotically stable, and the second one is finite-time stable for each interval, i.e. is decreasing at interval, then asymptotic stability with decreasing increments is guarantee.

Definition 3: Consider the Networked-Control System (1), then the system is said to be asymptotic stable with decreasing increments with parameters $(\tau_m, \alpha_j, \Lambda_j)$, $\tau_m = \max_j (k_{j+1} - k_j)$ if the system is uniformly asymptotically stable at the transmission time, and finite-time stable with

respect to $(\alpha_j, \Lambda_j =, ||.||, d_j)$, $d_j = (k_{j+1} - k_j)$ during each transmission period $[k_j, k_{j+1})$, i.e. decreasing at intervals with respect of transmission intervals.

Theorem 3: Consider the Networked-Control System (1), assume same conditions from theorem 1, if moreover we have $\forall k \in [k_j, k_{j+1})$, $\exists \rho > -1$ $\delta_1, \delta_2 \geq 0$ and a positive smooth function V^* such that $\delta_1 ||x(k_j)||^2 \leq V^*(x(k)) \leq \delta_2 ||x(k_j)||^2$

$$\Delta V^*(x(k)) \leq \rho V^*(x(k)), \ k \in [k_j, k_{j+1}) \qquad (31)$$

and defining $x(0) = \alpha_0$, $V^*(x(0)) = \gamma_0 = \frac{\alpha_0}{\delta_2}$, $\Delta_j^* = \delta_2 \Lambda_j$

$$\sup_k \prod_{i=0}^{k}(1 + \rho_i) \leq \frac{\Delta_k^*}{\gamma_0}, \ k \in [k_j, k_{j+1}) \qquad (32)$$

then for all transmission time $||x(k_j)|| \leq \alpha_0$, $j = 1, 2, \ldots$. we have that

$$||x(k)|| \leq \Lambda_j \forall k \in [k_j, k_{j+1}) \qquad (33)$$

then the system is asymptotic stability with decreasing increments with parameters $(\tau_m, x(k_j)), x(k_{j-1}))$.

Proof. Asymptotic stability at transmission instant follows from theorem 1. From (31) we have

$$V^*(x(k+1)) \leq (1 + \rho_k)V^*(x(k)) \qquad (34)$$

iterating the inequality for all the value up to k we obtain

$$\begin{aligned} V^*(x(k)) &\leq \prod_{i=k_j}^{k}(1 + \rho_i)V^*(x(k_j)) \leq \\ &\prod_{i=1}^{k}(1 + \rho_i)V^*(x(0)) \end{aligned} \qquad (35)$$

then from (32)

$$V^*(x(k)) \leq \sup_k \prod_{i=1}^{k}(1 + \rho_i)V^*(x(0)) \leq \Delta_j^* \qquad (36)$$

∎

VI. EXAMPLE

Example 1: Consider the system

$$x(k+1) = 3x(k)^2 + x(k)u(k). \qquad (37)$$

We aim to stabilize such a system across a network by using a control law

$$u(k) = -3.5x(k) \qquad (38)$$

The network we are using guarantees a bound on the transmission period $\tau_m = 3$. Then we can define the set of transmission instants $U = \{0, k_1, \ldots, k_i \ldots\}$, $max\{k_i - k_{i-1}\} = \tau_m$. Trivially the system is unstable in open loop, while is asymptotically stable in closed loop if full information is available.

We rewrite the system at the transmission time

$$\begin{aligned} z(j+1) &= 3z(j)^2 + z(j)u_z(j) \\ u_z(j) &= -3.5z(j) \end{aligned} \qquad (39)$$

in which $z(j) = x(k_j)$, $k_j \in U$. The system (39) is asymptotically stable, then since we can define the following bound

$$
\begin{aligned}
||F_j(x_{k_j})|| &= ||-0.5x_{k_j}|| \leq \qquad (40) \\
&\quad ||-0.5(-0.5(-0.5(-0.5x_{k_{j-1}})))|| = F_m
\end{aligned}
$$

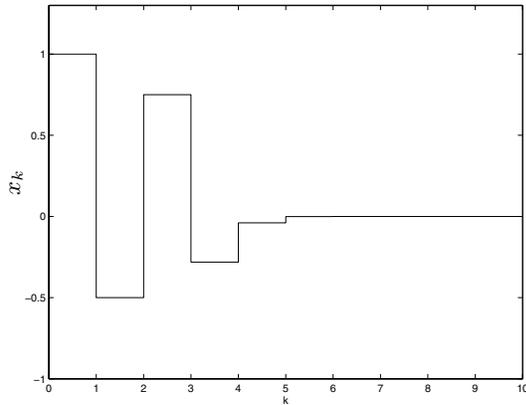Then by theorem 1 we can infer that system (38) is asymptotically stable.

Fig. 3. Closed loop system with maximum transmission interval $\tau_m = 3$

With the first part of our analysis we showed how the system is asymptotically stable, since is asymptotically stable at the transmission time and has increasing but bounded values in between transmission time. In order to verify whether the system is asymptotically stable with decreasing increments we need to check whether the maximum value attained by the system in between transmission times is also decreasing. It is enough to observe that conditions of theorem (3) are satisfied considering a quadratic function $V^* = x(k)^2$

$$
\begin{aligned}
V_j(x_{k_j}) &= ||-0.5x_{k_j}|| \leq \qquad (41) \\
&\quad ||-0.5(-0.5(-0.5(-0.5x_{k_{j-1}})))|| \leq \\
&\quad V_j(x_{k_{j-1}})
\end{aligned}
$$

Hence system (37) with control law (38) can be stabilized across a network with maximum transmission time $\tau_m = 3$, in such a way that the system is asymptotically stable, and moreover the state bound in the transmission intervals is decreasing. The simulation results is depicted in figure 3. For larger values of the transmission interval, conditions of theorem 1 are no longer satisfied, and the closed loop system might not preserve stability.

VII. Conclusions

The problem of networked control system affected by lost data is addressed in a nonlinear discrete-time setting. We presented a recursive approach to model discrete-time nonlinear NCS, in the presence of arbitrary but finite data packet dropout. Based on such a model we investigated stability, whether the state system is bounded between transmission steps, and asymptotic stability of the system

at the transmission times. We also considered an extension of the classical definition of stability involving the finite-time stability concept. The main development focused on the iteration of finite-time stability between transmission periods, in addition to the asymptotic stability of the system at the transmission time instants. We introduced such a concept to deal with the case when state measurements are not received. Therefore, using old information may not guarantee convergence, and in that interval the system state might exceed it allowable bound. It is therefore reasonable to consider a bound on how much the system is allowed to increase whenever data is lost.

References

[1] M. Yu, L.Wang, T. Chu, G. Xie *Stabilization of networked control systems with data packet dropout and network delays via switching system approach* CDC2004

[2] F. Amato, M. Ariola, C. Abdallah, and C. Cosentino, *Application of finite-time stability concepts to control of ATM networks.* In 40th Allerton Conf. on Communication, Control and Computers, Allerton, IL, 2002.

[3] F. Amato, M. Ariola, C. Abdallah, and P. Dorato, *Finite-time control for uncertain linear systems with disturbance inputs.* In Proc. American Control Conf., pp. 1776-1780, San Diego, CA, 1999.

[4] L. Weiss, E.F. Infante, *On the Stability of Systems Defined over a Finite Time Interval* Proc. of the National Academy of Sciences Vol. 54 No. 1 pp.44-48. July 1965.

[5] G. Kamenkov, *On stability of motion over a finite interval of time* [in Russian]. Journal of Applied Math. and Mechanics (PMM), 17:529-540, 1953.

[6] Vidyasagar, *Nonlinear Sysytems Analysis*, Prentice-Hall, 1978.

[7] P.F.Al-Hokayem *Stability Analysis of Networked Control Systems* Ms. Thesis Electrical Engineering The University of New Mexico, Albuquerque, New Mexico August 2003.

[8] B. Azimi-Sadjadi, *Stability of Networked Control Systems in the Presence of Packet Losses*, IEEE Conference on Decision and Control (Maui, Hawaii) December 2003.

[9] S.N.Ethier, T.G.Kurtz *Markov Processes Characterization and Convergence* Wiley Series in Probability and Statistics. Canada 1986.

[10] Y.Fang *Stability Analysis of Linear Control Systems with Uncertain Parameters* Phd Thesis, Case Western Reserve University, Cleveland, Ohio, 1994.

[11] Y. Fang *A New General Sufficient Condition for Almost Sure Stability of Jump Linear Systems* IEEE Transactions on Automatic Control, VOL.42, No.3 March 1997

[12] J. Hespanha, A. Ortega and L. Vasudevan, *Towards The Control of Linear Systems with Minimum Bit-Rate*, Proceedings of the International Symposium on the Mathematical Theory of Networks and Systems, August 2002 ($http : //sipi.usc.edu/ ortega/Papers/HespanhaOrtVas02.pdf$).

[13] D. Liberzon, *Stabilizing a Nonlinear System with Limited Information Feedback*, IEEE Conference on Decision and Control (Maui, Hawaii) December 2003

[14] S. Mastellone *Finite-Time Stability of Nonlinear Networked-Control Systems* M.s. Thesis, University of New Mexico, April 2004.

[15] S. Mastellone, P. Dorato, C.T.Abdallah *Finite-Time Stability of Discrete-Time Nonlinear Systems: Analysis and Design* $43rd$ IEEE Conference on Decision and Control, December 14-17, 2004 Atlantis, Paradise Island, Bahamas.

[16] A. Michel and S.Wu, *Stability of discrete-time systems over a finite interval of time.* Int. J. Control, 9, pp.679-694, 1969.

[17] L.A. Montestruque and P.J. Antsaklis, *Model-Based Networked Control System - Stability,* ISIS Technical Report ISIS-2002-001, Notre Dame, IN, January 2001. {http://www.nd.edu/ pantsakl/elecpubs/298-ACC03.pdf}

[18] L.A. Montestruque and P.J. Antsaklis, Stochastic stability for model-based networked control systems, *Proceedings of the 2003 American Control Conference,* vol. 2, pp. 4119-4124, June 2003.

[19] A.R. Teel and D. Nešić, *Input-Output Stability Properties of Networked Control Systems*, IEEE Conference on Decision and Control (Maui, Hawaii) December 2003.

[20] G. C. Walsh, Octavian Beldiman and Linda G. Bushnell *Asymptotic Behavior of Nonlinear Networked Control Systems* IEEE Trans. Automatic Control, Vol. 46, No. 7, pp. 1093-1097 July 2001.

[21] G.C. Walsh, O. Beldiman and L. Bushnell, *Asymptotic Behavior of Networked Control Systems*, Proceedings of the IEEE International Conference on Control Applications, pp. 1448 - 1453 vol. 2, Kohala Coast-Island of Hawai'i, Hawai'i, USA, August 22-27, 1999. ($http : //www.enme.umd.edu/ice_lab/papers/ps/CCA99.ps$)

[22] G.C. Walsh, O. Beldiman and L. Bushnell, *Error Encoding Algorithms for Networked Control Systems,* Proceedings of the IEEE Conference on Decision and Control, Phoenix, Arizona, USA, pp. 4933 -4938, December 1999.

[23] G.C. Walsh, H. Ye and L. Bushnell, *Stability Analysis of Networked Control Systems,* IEEE Transactions on Control Systems Technology, vol. 10, no. 3, pp. 438-446, May 2002.

[24] G.C. Walsh, H. Ye and L. Bushnell, *Stability Analysis of Networked Control Systems*, Proceedings of the IEEE American Control Conference, San Diego, CA, pp. 2876-2880, June 1999.

[25] M.S. Branicky *Multiple Lyapunov Functions and other analysis tools for switched and Hybrid systems*, IEEE Transactions on Automatic Control, vol.43(4),1998, pp.475-482.

[26] W.Zhang and Branicky,*Stability of networked control systems with time-varying transmission period.* Allerton Conference Communication, Control, and Computing, Urbana-Champaign, IL, Oct.2001.