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Stabilization of Linear and Nonlinear Systems with Time Delay

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Abstract

This paper considers the problem of stabilizing linear and nonlinear continuous-time systems with state and measurement delay. For linear systems we address stabilization via fixed-order dynamic output feedback compensators and present sufficient conditions for stabilization involving a system of modified Riccati equations. For nonlinear systems we provide sufficient conditions for the design of static full-state feedback stabilizing controllers. The controllers obtained are delay-independent and hence apply to systems with infinite delay.

1. Introduction

In dynamical systems such as the control of flexible structures with non-collocated sensors and actuators, teleoperators, biological systems [1], and electrical networks [2], time delay arises frequently and can severely degrade closed-loop system performance and in some cases drive the system to instability. Since controllers designed with the assumption of instantaneous information and power transfer may fail to stabilize dynamic systems with time delay [3] it is of paramount importance that delay system dynamics be accounted for in the control-system design process. There exists an extensive literature devoted to the control of dynamical systems with time delay (see, for example, [4–12] and the numerous references therein). Three main approaches can be distinguished for designing stabilizing controllers for delay systems. Namely:

i) Stabilization independent of delay amount [13,14]: In this approach the delay can be large (even infinite) without destabilizing the closed-loop system. However, the conditions for stabilization are often conservative.

ii) Stabilization dependent on delay amount [15–17]: Such approaches rely on Razumikin-like theorems [18] and provide stabilization conditions if the delay is less than a given amount.

iii) Stabilization based on delay amount [19,20]: In this approach there exist delay windows which allow a stabilizing compensator to exist, while no stabilizing compensators are possible outside these windows. This approach however applies to a restricted class of systems.

In this paper we design feedback controllers which are independent of the delay amount. Furthermore, we address both linear and nonlinear dynamical systems. Specifically, we present a rigorous development of sufficient conditions via fixed-order dynamic compensation and static full-state feedback controllers for stabilization of systems with state and measurement delay. For linear plants these sufficient conditions are in the form of a coupled system of algebraic Riccati equations that explicitly characterize dynamic controllers of fixed dimension while for nonlinear plants our sufficient condition is given by a modified Riccati equation for characterizing static full-state feedback controllers. We emphasize that our approach is constructive in nature rather than existential. In particular, as opposed to the results of [6] which are based on the total stability theorem [21] our sufficient conditions provide explicit formulae for controller gains that guarantee stabilization of systems with time delay. For the linear plant case, in order to account for closed-loop system performance our framework also includes minimization of a given performance functional. Finally, even though for simplicity of exposition we do not address system parametric uncertainty as in [7,22,23] the proposed approach can be merged with the guaranteed cost control approach [24] to provide robust stability and performance in the face of system uncertainty and system delay.

The contents of the paper are as follows. In Section 2 we state the problem of fixed-order dynamic compensation for systems with state and measurement delay. Sufficient conditions for stabilization of systems with time delay are given in Section 3. Section 4 provides design equations for characterizing fixed-order dynamic controllers for linear systems with time delay. In Section 5, we state the full-state feedback control problem for nonlinear systems with time delay and provide design equations for full-state feedback controllers. Section 6 provides two illustrative numerical examples. Finally, Section 7 gives conclusions.

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Nomenclature

$\mathbb{R}, \mathbb{R}^{r\times s}, \mathbb{R}^r$	– real numbers, $r \times s$ real matrices, $\mathbb{R}^{r\times 1}$
$()^T$, $()^{-1}$, tr $()$	– transpose, inverse, trace
$I_r, 0_r$	– $r \times r$ identity matrix, $r \times r$ zero matrix
$\|\cdot\|_2$	– Euclidean vector norm
$\lambda_{\min}(Z)$	– minimum eigenvalue of the symmetric matrix Z
$\alpha, \gamma, \epsilon, \sigma$	– real positive scalars
$n, l, m, n_c, \tilde{n}$	– positive integers; $1 \le n_c \le n$; $\tilde{n} = n + n_c$
$x, u, y, x_c, \tilde{x}$	– $n-, m-, l-, n_c-, \tilde{n}$ – dimensional vectors
A, B, C	– $n \times n, n \times m, l \times n$ matrices
$A_{\rm d}, C_{\rm d}$	– $n \times n, l \times n$ matrices
A_c, B_c, C_c, K	– $n_c \times n_c, n_c \times l, m \times n_c, m \times n$ matrices
V_1, V_2	– $n \times n, l \times l$ matrices
R_1, R_2	– $n \times n, m \times m$ matrices

2. Fixed-Order Controller Synthesis for Systems with Time Delay

In this section we introduce the fixed-order dynamic compensation problem for linear systems with state and measurement delays. Specifically, given the $n^{\rm th}$-order stabilizable and detectable dynamical system, where stabilizability and detectability are defined in the sense of [25], with state and measurement delay

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_{\rm d}x(t-\tau_{\rm d}) + Bu(t),\ t \in [0,\infty), \\ &\tau_{\rm d} > 0,\ x(t) = \phi(t),\ t \in [-\tau_{\rm d}, 0],\ \phi(0) = x_0, \quad (1) \\ y(t) &= Cx(t) + C_{\rm d}x(t-\tau_{\rm d}), \quad (2) \end{aligned}$$

where $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^l$, and $\phi : \mathbb{R}^+ \to \mathbb{R}^n$ is a continuous vector valued function specifying the initial state of the system, determine an $n_c^{\rm th}$-order $(1 \le n_c \le n)$ dynamic compensator

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c y(t), \quad x_c(0) = x_{c0}, \quad (3) \\ u(t) &= C_c x_c(t), \quad (4) \end{aligned}$$

which satisfies the following design criteria:

i) the closed-loop system (1)–(4) is asymptotically stable; and

ii) the performance functional

$$J(x(t), x_c(t), x(t-\tau_{\rm d})) \triangleq \int_0^\infty L(x(t), x_c(t), x(t-\tau_{\rm d}))dt, \quad (5)$$

where $L : \mathbb{R}^n \times \mathbb{R}^{n_c} \times \mathbb{R}^n \to \mathbb{R}$, is minimized. An explicit characterization of $L(x(t), x_c(t), x(t-\tau_{\rm d}))$, $t \ge 0, \tau_{\rm d} > 0$, is given in Section 3.

3. Sufficient Conditions for Stabilization of Systems with Time Delay

In this section we provide a Riccati equation that guarantees that the closed-loop system (1)–(4) consisting of the $n^{\rm th}$-order time-delayed system (1), (2) and the $n_c^{\rm th}$-order dynamic compensator (3), (4) is asymptotically stable. First note that for a given fixed-order controller (A_c, B_c, C_c) the closed-loop system (1)–(4) can be written as

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{A}\tilde{x}(t) + \tilde{A}_{\rm d}\tilde{x}(t-\tau_{\rm d}), \quad \tilde{x}(0) = \tilde{x}_0, \\ &t \in [0,\infty), \quad \tau_{\rm d} > 0, \quad (6) \end{aligned}$$

where

$$\tilde{x}(t) \triangleq \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}, \quad \tilde{A} \triangleq \begin{bmatrix} A & BC_c \\ B_cC & A_c \end{bmatrix},$$
$$\tilde{A}_{\rm d} \triangleq \begin{bmatrix} A_{\rm d} & 0_{n\times n_c} \\ B_cC_{\rm d} & 0_{n_c\times n_c} \end{bmatrix}.$$

For the statement of the next result define

$$\hat{I} \triangleq \begin{bmatrix} I_n & 0_{n\times n_c} \\ 0_{n_c\times n} & 0_{n_c} \end{bmatrix}.$$

Theorem 3.1 [26]. Let (A_c, B_c, C_c) be given. Suppose there exists an $\tilde{n}\times\tilde{n}$ positive-definite matrix $\tilde{P}$ and scalars $\alpha, \epsilon > 0$ such that

$$0 = \tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} + \epsilon\tilde{P} + \alpha^2\hat{I} + \alpha^{-2}\tilde{P}\tilde{A}_{\rm d}\tilde{A}_{\rm d}^T\tilde{P} + \tilde{R}, \quad (7)$$

where $\tilde{R}$ is an $\tilde{n}\times\tilde{n}$ nonnegative-definite matrix. Then the function

$$V(\tilde{x}) = \tilde{x}^T\tilde{P}\tilde{x} + \alpha^2\int_{t-\tau_{\rm d}}^t \tilde{x}^T(s)\hat{I}\tilde{x}(s)ds, \quad (8)$$

is a Lyapunov function that guarantees that the closed-loop system (6) is globally asymptotically stable.

Next, we consider an explicit characterization of $L(x(t), x_c(t), x(t-\tau_{\rm d}))$ in (5). Specifically, let $\tilde{R} = \begin{bmatrix} R_1 & 0 \\ 0 & C_c^TR_2C_c \end{bmatrix}$, where $R_1 \ge 0$ and $R_2 > 0$, and define

$$\begin{aligned} L(x(t), x_c(t), x(t-\tau_{\rm d})) &\triangleq \tilde{x}^T(t)[\epsilon\tilde{P} + \tilde{R}_1 + \alpha^{-2}\tilde{P}\tilde{A}_{\rm d} \\ &\cdot\tilde{A}_{\rm d}^T\tilde{P}]\tilde{x}(t) + u^T(t)R_2u(t) + \alpha^2\tilde{x}^T(t-\tau_{\rm d})\hat{I}\tilde{x}(t-\tau_{\rm d}) \\ &-2\tilde{x}^T(t-\tau_{\rm d})\tilde{A}_{\rm d}^T\tilde{P}\tilde{x}(t), \quad t \ge 0, \quad (9) \end{aligned}$$

where $\tilde{R}_1 \triangleq \begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix}$. Now, since $\tilde{x}(t) \to 0$ as $t \to \infty$, where $\tilde{x}(t)$, $t \ge 0$, satisfies (6), the performance functional (5) reduces to

$$\begin{aligned} J(x(t), x_c(t), x(t-\tau_{\rm d})) &= -\int_0^\infty \dot{V}(\tilde{x})dt \\ &= V(\tilde{x}(0)) - \lim_{t\to\infty} V(\tilde{x}(t)) \\ &= \tilde{x}^T(0)\tilde{P}\tilde{x}(0) + \Phi, \quad (10) \end{aligned}$$

where $\Phi \triangleq \int_{-\tau_d}^{0} \phi^T(s)\phi(s)ds$ is a positive constant. With $L(x(t), x_c(t), x(t-\tau_d))$ given by (9) the performance functional (5) has the same form as the H_2 cost in standard LQG theory. Specifically, $J(\tilde{x}(0)) = \tilde{x}^T(0)\tilde{P}\tilde{x}(0) + \Phi = \operatorname{tr} \tilde{P}\tilde{x}(0)\tilde{x}^T(0) + \Phi$. Hence, we replace $\tilde{x}(0)\tilde{x}^T(0)$ by $\tilde{V} \triangleq \begin{bmatrix} V_1 & 0 \\ 0 & B_c V_2 B_c^T \end{bmatrix}$, where $V_1 \geq 0$ and $V_2 > 0$, and proceed by determining controller gains that minimize $\operatorname{tr} \tilde{P}\tilde{V} + \Phi$. This leads to the following optimization problem.

Auxiliary Minimization Problem. Determine (A_c, B_c, C_c) that minimizes $\mathcal{J}(\tilde{P}, A_c, B_c, C_c) \triangleq \operatorname{tr} \tilde{P}\tilde{V}$ where $\tilde{P} > 0$ satisfies (7) and such that (A_c, B_c, C_c) is minimal.

It follows from Theorem 3.1 that by deriving necessary conditions for the Auxiliary Minimization Problem we obtain sufficient conditions for characterizing dynamic output feedback controllers ensuring stabilization of closed-loop systems with time delay.

4. Fixed-Order Dynamic Compensation for Systems with Time Delay

In this section we present the main theorem characterizing fixed-order dynamic controllers for (1), (2). Note that for design flexibility the compensator order n_c may be less than the plant order n. We shall require for technical reasons that $C_d C_d^T = \sigma^2 V_2$, where the nonnegative scalar σ is a design variable. The following lemma is required for the statement of main theorem.

Lemma 4.1 [24]. Let $\hat{Q}, \hat{P}$ be $n \times n$ nonnegative-definite matrices and suppose that rank $\hat{Q}\hat{P} = n_c$. Then there exist $n_c \times n$ matrices G, Γ and an $n_c \times n_c$ invertible matrix M, unique except for a change of basis in $\mathbb{R}^{n_c}$, such that

$$\hat{Q}\hat{P} = G^T M \Gamma, \qquad \Gamma G^T = I_{n_c}. \tag{11}$$

Furthermore, the $n \times n$ matrices $\tau \triangleq G^T\Gamma$ and $\tau_\perp \triangleq I_n - \tau$ are idempotent and have rank n_c and $n - n_c$, respectively.

For convenience in stating the main result of this section we define the notation $S \triangleq (I + \alpha^{-2}\sigma^2 Q\hat{P})^{-1}$, for arbitrary $n \times n$ nonnegative-definite matrices $Q, \hat{P}$ and

$$\begin{aligned} Q_a &\triangleq Q[C + \alpha^{-2}C_d A_d^T (P + \hat{P})]^T, \\ A_\epsilon &\triangleq A + \frac{1}{2}\epsilon I_n, \\ A_{\hat{P}} &\triangleq A_\epsilon - SQ_a V_2^{-1}(C + \alpha^{-2}C_d A_d^T P) \\ &\quad + \alpha^{-2} A_d A_d^T P, \\ A_Q &\triangleq A_\epsilon + \alpha^{-2}A_d A_d^T (P + \hat{P}) \\ &\quad - \alpha^{-2} A_d C_d^T V_2^{-1} Q_a^T S^T \hat{P}, \\ A_{\hat{Q}} &\triangleq A_\epsilon - BR_2^{-1}B^T P + \alpha^{-2}A_d A_d^T P, \end{aligned}$$

for arbitrary $P, Q, \hat{P} \in \mathbb{R}^{n \times n}$ and $\alpha, \epsilon, \sigma > 0$. Note that since $Q, \hat{P}$ are nonnegative definite and the eigenvalues of $Q\hat{P}$ coincide with the eigenvalues of the nonnegative-definite matrix $Q^{1/2}\hat{P}Q^{1/2}$, it follows that $Q\hat{P}$ has nonnegative eigenvalues. Thus, the eigenvalues of $I + \alpha^{-2}\sigma^2 Q\hat{P}$ are all greater than one so that S exists.

Theorem 4.1. Assume $\alpha, \epsilon, \sigma > 0$ and suppose there exist $n \times n$ nonnegative-definite matrices $P, Q, \hat{P}$, and $\hat{Q}$ satisfying

$$\begin{aligned} 0 &= A_\epsilon^T P + PA_\epsilon + R_1 + \alpha^2 I_n + \alpha^{-2} PA_d A_d^T P \\ &\quad - PBR_2^{-1}B^T P + \tau_\perp^T PBR_2^{-1}B^T P \tau_\perp, && (12) \\ 0 &= A_Q Q + QA_Q^T + V_1 - SQ_a V_2^{-1} Q_a^T S^T \\ &\quad + \tau_\perp SQ_a V_2^{-1} Q_a^T S^T \tau_\perp^T, && (13) \\ 0 &= A_{\hat{P}}^T \hat{P} + \hat{P} A_{\hat{P}} + \alpha^{-2}\hat{P}[\sigma^2 SQ_a V_2^{-1}Q_a^T S^T - A_d C_d^T \\ &\quad \cdot V_2^{-1} Q_a^T S^T - SQ_a V_2^{-1} C_d A_d^T]\hat{P} + \alpha^{-2}\hat{P}A_d A_d^T \hat{P} \\ &\quad + PBR_2^{-1}B^T P - \tau_\perp^T PBR_2^{-1}B^T P\tau_\perp, && (14) \\ 0 &= A_{\hat{Q}}\hat{Q} + \hat{Q}A_{\hat{Q}}^T + SQ_a V_2^{-1} Q_a^T S^T \\ &\quad - \tau_\perp SQ_a V_2^{-1} Q_a^T S^T \tau_\perp^T, && (15) \end{aligned}$$

$$\operatorname{rank} \hat{Q} = \operatorname{rank} \hat{P} = \operatorname{rank} \hat{Q}\hat{P} = n_c, \tag{16}$$

and let A_c, B_c, and C_c be given by

$$\begin{aligned} A_c &= \Gamma[A - SQ_a V_2^{-1}(C + \alpha^{-2}C_d A_d^T P) \\ &\quad + (\alpha^{-2}A_d A_d^T - BR_2^{-1}B^T)P]G^T, && (17) \\ B_c &= \Gamma SQ_a V_2^{-1}, && (18) \\ C_c &= -R_2^{-1}B^T PG^T. && (19) \end{aligned}$$

Then

$$\tilde{P} = \begin{bmatrix} P + \hat{P} & -\hat{P}G^T \\ -G\hat{P} & G\hat{P}G^T \end{bmatrix},$$

satisfies (7) and (A_c, B_c, C_c) is an extremal of $\mathcal{J}(\tilde{P}, A_c, B_c, C_c)$. Furthermore, the feedback interconnection of the delay system (1), (2) and the fixed-order controller (3), (4) is asymptotically stable for all $\tau_d > 0$. Finally, the cost $\mathcal{J}(\tilde{P}, A_c, B_c, C_c)$ is given by

$$\begin{aligned} \mathcal{J}(\tilde{P}, A_c, B_c, C_c) &= \operatorname{tr}[(P + \hat{P})V_1 \\ &\quad + \hat{P}SQ_a V_2^{-1} Q_a^T S^T]. && (20) \end{aligned}$$

Proof. The proof is constructive in nature. Specifically, first we obtain necessary conditions for the Auxiliary Minimization Problem and show by construction that these conditions serve as sufficient conditions for closed-loop stability. For details of a similar proof see [27]. □

In the full-order case set $n_c = n$ so that $\tau = G = \Gamma = I_n$ and $\tau_\perp = 0$. In this case the last term in each of (12)–(15) is zero and (15) is superfluous.

5. Full-State Feedback Control for Nonlinear Systems with Time Delay

In this section we introduce the full-state feedback control problem for nonlinear systems with delay. Specifically, given the dynamical system with nonlinear state delay

$$\begin{aligned}\dot{x}(t) &= Ax(t) + f_{\rm d}(x(t-\tau_{\rm d})) + Bu(t),\ t \in [0,\infty),\\ &\quad \tau_{\rm d} > 0,\ x(t) = \phi(t),\ t\in[-\tau_{\rm d},0],\ \phi(0) = x_0,\\ &\quad f_{\rm d}(0) = 0, \end{aligned} \tag{21}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $f_{\rm d} : \mathbb{R}^+ \to \mathbb{R}^n$, and $\phi : \mathbb{R}^+ \to \mathbb{R}^n$ is a continuous vector valued function specifying the initial state of the system, determine a full-state feedback control law

$$u(t) = Kx(t), \tag{22}$$

such that the closed-loop system (21), (22) is asymptotically stable.

Next, we show that if $f_{\rm d}(\cdot)$ in (21) satisfies $||f_{\rm d}(x)||_2 \le \gamma||x||_2$, where $x \in \mathbb{R}^n$ and $\gamma > 0$, we can construct a full-state feedback control law (22) to stabilize the nonlinear time-delay system (21) independent of the delay amount $\tau_{\rm d}$. This result is an extension of the result in [28] where a stabilizing state feedback controller was obtained for purely linear time-delay systems.

Theorem 5.1 [26]. Let $||f_{\rm d}(x)||_2 \le \gamma||x||_2$, where $x \in \mathbb{R}^n$ and $\gamma > 0$, and suppose there exists an $n \times n$ positive-definite matrix P such that

$$0 = A^TP + PA + \alpha^{-2}P^2 - 2PBR_2^{-1}B^TP + R_1, \tag{23}$$

where $\alpha > 0$, $\lambda_{\min}(R_1) > \alpha^2\gamma^2$, and $R_2 > 0$. Furthermore, let the feedback control gain K in (22) be given by

$$K = -R_2^{-1}B^TP. \tag{24}$$

Then, for all $\tau_{\rm d} > 0$, the closed-loop system (21), (22) is globally asymptotically stable with Lyapunov function

$$V(x) = x^TPx + \alpha^2\int_{t-\tau_{\rm d}}^{t} f_{\rm d}^T(x(s))f_{\rm d}(x(s))ds. \tag{25}$$

6. Illustrative Numerical Examples

In this section we provide two numerical examples to demonstrate the proposed Riccati equation approach for delay systems. For simplicity we consider the design of full-order dynamic compensators and full-state feedback controllers. The design equations (12)–(15) were solved using a homotopy continuation algorithm. For details of a similar algorithm see [27].

Example 6.1. Consider the second-order system

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}0 & 1\\ -0.3 & -1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\\ &+ \begin{bmatrix}0.145 & 0.75\\ 0.275 & -0.2\end{bmatrix}\begin{bmatrix}x_1(t-\tau_{\rm d})\\ x_2(t-\tau_{\rm d})\end{bmatrix} + \begin{bmatrix}0.5\\ 0.2\end{bmatrix}u(t),\\ y(t) &= \begin{bmatrix}2.1 & 4\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\\ &+ \begin{bmatrix}6.0 & 5.0\end{bmatrix}\begin{bmatrix}x_1(t-\tau_{\rm d})\\ x_2(t-\tau_{\rm d})\end{bmatrix},\end{aligned}$$

with design data $V_1 = 0.01I_2$, $V_2 = 1$, $R_1 = 0.5I_2$, $R_2 = 1$, $\alpha = 25$, and $\sigma = 1$. Using Theorem 4.1 a full-order dynamic compensator was designed. To illustrate the closed-loop behavior of the system let $x(0) = \begin{bmatrix}0.4 & -6\end{bmatrix}^T$ and let $\phi(t) = \begin{bmatrix}-384t + 0.4 & -480t - 6\end{bmatrix}^T$ for $t \in [-0.025, 0]$. Figure 1 provides a comparison of the free response of the controlled system states with an LQG controller and the controller designed using Theorem 4.1.

Example 6.2. To illustrate the design of full state-feedback control for dynamic systems with nonlinear state delay consider

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\\ &+0.7\begin{bmatrix}\frac{x_1(t-\tau_{\rm d})}{\sqrt{1+x_1^2(t-\tau_{\rm d})}}\\ x_2(t-\tau_{\rm d})\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t).\end{aligned}$$

Furthermore, note that $||f_{\rm d}||_2 = 0.7\sqrt{\frac{x_1^2}{1+x_1^2} + x_2^2} \le \gamma\sqrt{x_1^2 + x_2^2}$, for $\gamma > 0.7$. Let $\gamma = 0.75$ and choose the design parameters $R_1 = I_2$, $R_2 = 1$, and $\alpha = 1.3$. Using Theorem 5.1, we obtain,

$$P = \begin{bmatrix}9.1707 & 6.0039\\ 6.0039 & 4.9379\end{bmatrix}, \qquad K = \begin{bmatrix}-6.0039\\ -4.9379\end{bmatrix}^T.$$

To illustrate the closed-loop behavior of the system let $x(0) = \begin{bmatrix}3 & 1\end{bmatrix}^T$ and let $\phi(t) = [100t + 3 \ \ -200t + 1]^T$ for $t \in [-0.01, 0]$. Figure 2 provides a comparison of the free response of the controlled system states with an LQR controller and the controller designed using Theorem 5.1.

7. Conclusion

In this paper we developed fixed-order dynamic output feedback controllers and full-state feedback controllers for linear and nonlinear continuous-time systems with time delays, respectively. Specifically, for linear continuous-time systems with state and measurement delay we presented sufficient conditions via fixed-order dynamic compensation. For nonlinear continuous-time systems with nonlinear state delay a constructive procedure was used to obtain full-state feedback stabilizing controllers. For both cases the controllers obtained were delay-independent. Two numerical examples were presented to illustrate the effectiveness of the proposed design approach.

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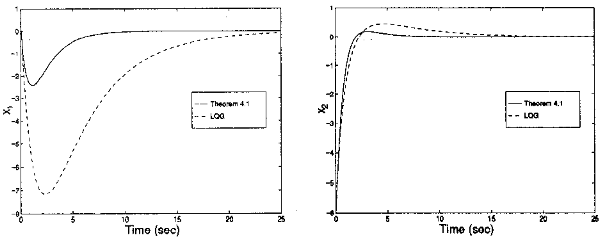

Figure 1: Comparison of LQG and Theorem 4.1 Designs: Example 6.1

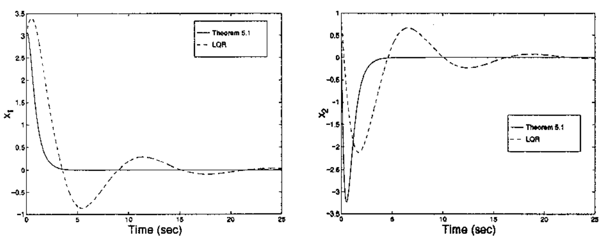

Figure 2: Comparison of LQR and Theorem 5.1 Designs: Example 6.2

References

[1] J.M. Cushing, "Integrodifferential equations and delay models in population dynamics," in *Lecture Notes in Bio-Mathematics*, vol. 20, Springer-Verlag, Berlin, 1977.

[2] P.J. Schwyn and T.A. Bickart, "Stability of networks with distributed and nonlinear elements," *Networks*, vol. 2, pp. 45–96, 1972.

[3] G.F. Franklin, J.D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems.* New York: Addison Wesley, 1994.

[4] A. Thowsen, "Uniform ultimate boundedness of the solutions of uncertain dynamic delay systems with state-dependent and memoryless feedback control," *Int. Journal of Control*, vol. 37, no. 5, pp. 1135–1143, 1983.

[5] E.W. Kamen, P.P. Khargonekar, and A. Tannenbaum, "Stabilization of time-delay systems using finite-dimensional compensators," *IEEE Trans. Automat. Control*, vol. 30, no. 1, pp. 75–78, 1985.

[6] S.-S. Wang, B.-S. Chen, and T.-P. Lin, "Robust stability of uncertain time-delay systems," *Int. Journal of Control*, vol. 46, no. 3, pp. 963–976, 1987.

[7] J.-C. Shen, B.-S. Chen, and F.-C. Kung, "Memoryless stabilization of uncertain dynamic delay systems: Riccati equation approach," *IEEE Trans. Automat. Control*, vol. 36, no. 5, pp. 638–640, 1991.

[8] E.I. Verriest, M.K.H. Fan, and J. Kullstam, "Frequency domain robust stability criteria for linear delay systems," in *Proc. IEEE Conf. on Dec. and Control*, (San Antonio, TX), 1993.

[9] E.I. Verriest and A.F. Ivanov, "Robust stability of systems with delayed feedback," *Circuits, Systems, and Signal Processing*, vol. 13, pp. 213–222, 1994.

[10] E.I. Verriest and A.F. Ivanov, "Robust stability of delay-difference equations," in *Proc. IEEE Conf. on Dec. and Control*, (New Orleans, LA), pp. 386–391, 1995.

[11] S.O.R. Moheimani and I.R. Petersen, "Optimal quadratic guaranteed cost control of a class of uncertain time-delay systems," in *Proc. IEEE Conf. on Dec. and Control*, (New Orleans, LA), pp. 1513–1518, 1995.

[12] S.I. Niculescu, J.-M. Dion, and L. Dugard, "Robust stabilization for uncertain time-delay systems containing saturating actuators," *IEEE Trans. Automat. Control*, vol. 41, pp. 742–747, 1996.

[13] A. Feliachi and A. Thowsen, "Memoryless stabilization of linear delay-differential systems," *IEEE Trans. Automat. Control*, vol. 26, pp. 586–587, 1981.

[14] T. Mori, E. Noldus, and M. Kuwahara, "A way to stabilize linear systems with delayed state," *Automatica*, vol. 19, pp. 571–573, 1983.

[15] T.J. Su, P.-L. Liu, and J.-T. Tsay, "Stabilization of delay-dependence for saturating actuator systems," in *Proc. IEEE Conf. on Dec. and Control*, (Brighton, U.K.), pp. 2891–2892, 1991.

[16] S.I. Niculescu, C.E. de Souza, J.-M. Dion, and L. Dugard, "Robust stability and stabilization of uncertain linear systems with state delay: Single delay case (I)," in *Proc. IFAC Workshop Robust Contr. Design*, (Rio de Janeiro, Brazil), pp. 469–474, 1994.

[17] S.I. Niculescu, C.E. de Souza, J.-M. Dion, and L. Dugard, "Robust stability and stabilization of uncertain linear systems with state delay: Multiple delays case (II)," in *Proc. IFAC Workshop Robust Contr. Design*, (Rio de Janeiro, Brazil), pp. 475–480, 1994.

[18] J. Hale, *Theory of Functional Differential Equations.* New York: Springer-Verlag, 1977.

[19] B.J. Petterson, R.D. Robinett, and J.C. Werner, "Lag-stabilized force feedback damping," Tech. Rep. SAND91-0194, UC-406, Sandia National Laboratories, Albuquerque, NM, 87185, 1991.

[20] C.T. Abdallah, P. Dorato, J. Benitez-Read, and R. Byrne, "Delayed positive feedback can stabilize oscillatory systems," in *Proc. IEEE American Control Conf.*, (San Fransisco, CA), pp. 3106–3107, 1993.

[21] B.D.O. Anderson, R.R. Bitmead, C.R. Johnson, Jr., P.V. Kokotovic, R.L. Koust, I. Mareels, L. Praly, and B.D. Riedle, *Stability of Adaptive Systems: Passivity and Averaging Analysis.* Cambridge, MA: MIT Press, 1986.

[22] S. Phoojaruenchanachai and K. Furuta, "Memoryless stabilization of uncertain linear systems including time-varying state delays," *IEEE Trans. Automat. Control*, vol. 37, no. 7, pp. 1022–1026, 1992.

[23] M.S. Mahmoud and N.F.Al-Muthairi, "Quadratic stabilization of continuous time systems with state-delay and norm-bounded time-varying uncertainties," *IEEE Trans. Automat. Control*, vol. 39, no. 10, pp. 2135–2139, 1994.

[24] D.S. Bernstein and W.M. Haddad, "Robust stability and performance via fixed-order dynamic compensation with guaranteed cost bounds," *Math. Contr. Sig. Sys.*, vol. 3, pp. 139–163, 1990.

[25] K.P.M. Bhat and H.N. Koivo, "Modal characterizations of controllability and observability in time delay systems," *IEEE Trans. Automat. Control*, vol. 21, pp. 292–293, 1976.

[26] W.M. Haddad, V. Kapila, and C. T. Abdallah, "Stabilization of linear and nonlinear systems with time delay," in *Stability and Control of Time-Delay Systems*, L. Dugard and E. Verriest, Eds., Springer-Verlag, to appear.

[27] W.M. Haddad and V. Kapila, "Fixed-architecture controller synthesis for systems with input-output time-varying nonlinearities," *Int. J. Robust and Nonlinear Contr.*, to appear.

[28] W.H. Kwon and A.E. Pearson, "A note on feedback stabilization of a differential-difference system," *IEEE Trans. Automat. Control*, vol. 22, pp. 468–470, 1977.