Proceedings of the 40th IEEE
Conference on Decision and Control
Orlando, Florida USA, December 2001

Bounded Control of Multiple-Delay Systems with Applications to ATM Networks[1]

Sophie Tarbouriech*, Chaouki T. Abdallah**, Marco Ariola***

* L.A.A.S - C.N.R.S, 7 Avenue du Colonel Roche, 31077 Toulouse cedex 4, FRANCE. E-mail: tarbour@laas.fr
** Electrical and Computer Engineering, 401 Ferris Hall, Knoxville, TN 37996-2100, USA. chaouki@eece.unm.edu
*** Dipartimento di Informatica e Sistemistica, Università degli Studi di Napoli Federico II, Napoli, ITALY. ariola@unina.it.

Abstract

Congestion control in the Available Bit Rate (ABR) class of Asynchronous Transfer Mode (ATM) networks poses interesting challenges due to the presence of multiple-delays, magnitude and rate constraints on the inputs and additive disturbances. We consider a fixed-structure controller for an ATM/ABR network, and solve a robust tracking control problem in which the target is a threshold on the queue level.

1 Introduction

The transmission of multimedia traffic on the broadband integrated service digital networks (B-ISDN) has created the need for new transport technologies such as Asynchronous Transfer Mode (ATM). Briefly, because of the variability of the multimedia traffic, ATM networks seek to guarantee an end-to-end quality of service (QoS) by dividing the varying types of traffic (voice, data, etc.) into short, fixed-size cells (53 bytes each) whose transmission delay may be predicted and controlled. ATM is thus a *Virtual Circuit* (VC) technology which combines advantages of circuit-switching (all intermediate switches are alerted of the transmission requirements, and a connecting circuit is established) and packet-switching (many circuits can share the network resources). In order for the various VC's to share network resources, flow and congestion control algorithms need to be designed and implemented. The congestion control problem is solved by regulating the input traffic rate. In addition, because of its inherent flexibility, ATM traffic may be served under one of the following service classes: 1) The *constant bit rate* (CBR) class: it accommodates traffic that must be received at a guaranteed bit rate, such as telephone conversations, video conferencing, and television. 2) The *variable bit rate* (VBR): it accommodates bursty traffic such as industrial control, multimedia e-mail, and interactive compressed video. 3) The *available bit rate* (ABR): it is a best-effort class for applications such as file transfer or e-mail. Thus, no service guarantees (transfer delay) are required, but the source of data packets controls its data rate, using a feedback signal provided by switches downstream which measure the congestion of the network. Due to the presence of this feedback, many classical and advanced control theory concepts have been suggested to deal with the congestion control problem in the ATM/ABR case [3, 10]. 4) The *unspecified bit rate* (UBR): it uses any leftover capacity to accomodate applications such as e-mail.

Note that the CBR and VBR service categories, a traffic contract is negotiated at the initial stage of the VC setup, and maintained for the duration of the connection. This contract will guarantee the following QoS parameters: 1) Minimum cell rate (MCR), 2) Peak cell rate (PCR), 3) cell delay variation (CDV), 4) maximum cell transfer delay (maxCTD), and 5) cell loss ratio (CLR). This then forces CBR and VBR sources to keep their rate constant regardless of the congestion status of the network. The ABR sources on the other hand, are only required to guarantee an MCR and an PCR, and thus can adjust their rates to accomodate the level available after all CBR and VBR traffic has been accommodated. In order to avoid congestion, the ATM Forum adopted a rate-based ABR control algorithm as opposed to a credit approach whereby the number of incoming cells as opposed to their rate is controlled [6]. This paper will then concentrate on the ABR service category since ABR sources are the ones to adjust their rates using explicit network feedback. In the original ATM forum specification, an ATM/ABR source is required to send one cell called a resource management (RM) cell for every 32 data cells. Switches along the path from the source to the destination then write into the RM cell their required data rate to avoid congestion. The destination switch then has information about the minimum rate required by all switches along the VC which is then relayed back to the ATM/ABR source as a feedback signal which serves to adjust its own data rate.

The earliest control algorithms for ABR consisted of setting a binary digit in the RM cell by any switch along the VC when its queue level exceeds a certain threshold [3]. This was then shown to cause oscillations in the closed-loop system. Other controllers were then suggested by various authors [4, 5], to address this problem. Most of these controllers are either complex or did not guarantee the closed-loop stability (in a sense defined later).

In addition, one of the limiting factors of these earlier proposed controllers was that the ABR bandwidth needed to be known in the implementation of the control algorithm. This however poses a problem in multimedia applications where the ABR bandwidth is bursty and is effectively the remaining available bandwidth after the CBR and VBR traffic have been accommodated. In [10] this particular issue was dealt with using a Smith predictor which then considered the available ABR bandwidth as an unknown dis-

[1] This work has been supported by an international agreement CNRS (France) - NSF (USA).

turbance. While this controller had many desirable properties, it only guaranteed stability in an appropriately defined sense but had no optimality guarantees. In addition, the delays encountered along with the number of ABR sources were assumed known, although the earlier tech report [6] did not require the delays to be exactly known. In [8], robust controllers were designed when both the number of ABR sources and the delays were uncertain.

In the current paper, we present a framework which allows us to deal with the ATM/ABR problem with uncertain delays, and number of sources. Moreover, we shall account for the limitations on the rate of traffic and on the speed of change in such rates. Our formulation will allow us to deal with other performance objectives while maintaining a simple controller structure.

Notation. $\Re_+$ is the set of non-negative real numbers. $A(i,j)$ denotes the element of the ith row and the jth column of matrix A. I_m denotes the $m-$order identity matrix. 1_m denotes in $\Re^m$ the vector $[\ 1\ \ \dots\ \ 1\ ]'$. $\mathcal{C}_\tau = \mathcal{C}([-\tau,0],\Re^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau,0]$ into $\Re^n$ with the topology of uniform convergence. $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. $\|\phi\|_c = \sup_{-\tau\le t\le 0} \|\phi(t)\|$ stands for the norm of a function $\phi \in \mathcal{C}_\tau$. When the delay is finite then "sup" can be replaced by "max". $\mathcal{C}_\tau^v$ is the set defined by $\mathcal{C}_\tau^v = \{\phi \in \mathcal{C}_\tau\ ;\ \|\phi\|_c < v,\ v > 0\}$.

2 The network model and the control problem

2.1 The network model

There are two philosophically distinct approaches to modeling an ATM network. The first assumes a continuous-time flow of the data and thus results in a delay-differential model of the system [10], while the other one assumes a discrete-time flow and results in a difference equation model [8, 4]. In either model however, the eventual controller needs to be implemented in discrete-time. In this paper, we choose the delay-differential model and assume for the time being that the controller is also continuous-time with the understanding that a discrete-time controller may be obtained as discussed for example in [6]. As discussed earlier, the considered ABR class is designed as a best-effort class for applications such as file transfer or e-mail. Thus, no service guarantees are required (beyond meeting the MCR and PCR limits), but the source of data packets controls its data rate, using a feedback signal provided by switches downstream which measure the congestion of the network. Due to the presence of this feedback, many classical and advanced control theory concepts have been suggested to deal with the congestion control problem in the ATM/ABR case [3], [10]. In what follows we present the dynamic model of an ATM queue following [10] and [8]. The data cells enter the network from a source node S_i, and are then stored and forwarded along intermediate links to various intermediate nodes. At each node, the process is repeated until a data cell reaches its destination node D_j. Each node stores its data cells to be transmitted in a queue along each one of its outgoing links. The network is thus modeled as a graph consisting of a set of $N = \{1,\cdots,n\}$ nodes or switches, connected via a set of $L = \{1,\cdots,l\}$ links. Each node $i \in N$ has a set $I(i) \subset L$ of input links and a set $O(i) \subset L$ of output links. Let t_i (sec) be the transmission time of a cell through a link i and the transmission capacity or bandwidth of the corresponding *link* be $c_i = 1/t_i$ (cells/s). Let $t_{di}(sec)$ be the propagation time delay of link i. Let t_{prj} (sec) be the transmission time of a *node* j denoting the time it takes a cell from the time it arrives at node j to the time it goes into one of outgoing links queues. Here, t_{prj} is assumed to be small enough so that any congestion is only due to the transmission capacity and not to any processing delays.

At any particular time, let C be the set of active source/destination pairs $(S,D) \in N \times N$. Let n_c be the cardinality of C, and associate with each pair (S,D) a VC and a path $p(S,D)$ specified by the sequence of links that the VC traverse in going from S to its corresponding D.

In order to provide feedback signals to itself, each source node generates a forward RM cell for every 32 data cells. The destination node or intermediate nodes (switches) then returns this RM cell (which then becomes a backward RM cell) to the source. These RM cells contain a field called the explicit rate (ER) feedback field, a congestion indicator (CI) bit, and a no increase (NI) bit. The RM cells then travel the same path as the data cells and flow through a particular switch (node) which then can take one or more of the following actions: 1) insert feedback control information in the ER field of an RM cell, 2) provide binary feedback information by marking the CI bit or the NI bit, 3) set the explicit forward congestion indicator (EFCI) bit in the data cell header, so that the destination can mark the CI bit in the corresponding RM cell, 4) generate and send its own backward RM cell to the source.

Now, each ABR source has an actual cell rate (ACR) along with its MCR and PCR. The ACR must lie between the lower MCR limit and the upper PCR limit and is adjusted according to the feedback provided though the backwards RM cells. The ER field of a forward RM cell is set by the source at its current ACR, and the source waits until it receives the backward RM cell in order to act according to one of the following scenarios: 1) The CI and NI bits are not set, denoting a no congestion situation. The source node then can increase its ACR by RIF*PCR where RIF is the rate increase factor subject to the new ACR being no greater than the explicit rate specified in the ER field by any of the switches downstream, and of course still less than PCR. 2) The CI bit is set, denoting a congestion situation. The source node will then decrease its ACR by RDF*PCR where RDF is the rate decrease factor subject to the new ACR being no greater than the explicit rate specified in the ER field by any of the switches downstream, and of course still greater than MCR. 3) If the NI bit is set, the source sets its new ACR to be the minimum of the old ACR and the explicit rate specified in the ER field by any of the switches downstream.

This control approach however leads to oscillatory behavior [8]. In what follows, a deterministic fluid model of the cell flow is assumed, so that the source transmission rate is denoted by the continuous variable $u(t) = ACR$

(cells/sec). Then, each ABR source declares its peak cell rate $c_s = 1/t_s = PCR$ and is assumed to always have a cell to send (i.e. be persistent).

The model we consider is that described in [10] and used in [2]. We then assume that each output link of a given node maintains a First-In-First-Out (FIFO) queue shared by all VCs flowing through the link. Hence we suppose that the flow of packets is conserved and therefore the queue level model for each buffer in the ATM network is given as the following continuous-time differential equation:

$$\dot{x}(t) = -d(t) + \sum_{i=1}^{n} u_i(t - T_i^{fw}) \tag{1}$$

with the initial condition, $\forall i = 1, ..., n$:

$$u_i(t_0 + \psi) = \phi_i(\psi), \forall \psi \in [-T_i, 0],\ (t_0, \phi_i) \in \Re_+ \times \mathcal{C}^v_{T_i} \tag{2}$$

where $x(t)$ is the queue level associated with the considered link. n is the number of virtual circuits sharing the queue level associated with the considered link which can be controlled by feedback from the current switch. In other words, there may be many other sources feeding into the current switches but they may be bottlenecked at some other switch (thus can not increase their own rate due to feedback from the current switch) or are already transmitting at their current PCR (and thus can not increase their ACR). u_i is the inflow cell rate (input) from the ith controllable source to the queue. T_i^{fw} is the propagation delay from the ith controlled source to the queue. $d(t) = \mu(t) - r^u(t)$ is the disturbance consisting of the rate of packets leaving the queue $\mu(t)$, minus $r^u(t)$, the rate of all the packets arriving from all uncontrollable sources.

Furthermore we assume that the following assumptions hold with respect to system (1).

Assumption 2.1 *Each input $u_i(t)$ ($ACR = u_i$) is limited in amplitude as follows:*

$$u_i(t) \in \Omega_{0i} = \{u_i, 0 \leq u_i(t) \leq u_{0i}, \forall t \geq 0, i = 1, ..., n\} \tag{3}$$

with $u_{0i} > 0$, $\forall i = 1, ..., n$. Note that in this case, we have chosen $MCR = 0$ and $PCR = u_{0i}$ and that the input constraints are supposed to be satisfied by the initial function $\phi_i(\psi)$, $\forall \psi \in [-T_i, 0]$. This assumption basically states that the inflow rate is bounded above and that each source is persistent.

Assumption 2.2 *The rate of $u_i(t)$, that is, its time-derivative $\dot{u}_i(t)$ is limited in amplitude as follows:*

$$u_i(t) \in \Omega_{1i} = \{u_i, |\dot{u}_i(t)| \leq u_{1i}, \forall t \geq 0, i = 1, ..., n\} \tag{4}$$

with $u_{1i} > 0$, $\forall i = 1, ..., n$. This basically guarantees that no source can change its cell rate instantaneously.

2.2 The control problem

The control objective of this network is to achieve a certain stability property and assure full link utilization as described for example in [10] while simultaneously taking into account the actuator state limitations and external disturbances. Thus, and similarly to [10], let us introduce the fixed-structure controller:

$$u_i(t) = \tfrac{k}{n}\left[r_0 - x(t - T_i^{fb}) - \sum_{i=1}^{n}\int_{t-T_i}^{t} u_i(\tau)d\tau\right] \quad \forall i = 1, ..., n \tag{5}$$

where $r_0 > 0$ represents the queue capacity, T_i^{fb} is the feedback delay of the ith queue, and k is a positive scalar. The author in [10] shows that with the appropriate choice of k, this controller will achieve the desired objectives given the exact knowledge of $T_i = T_i^{fb} + T_i^{fw}$ (otherwise known as the round trip delay RTD) and of the number of sources n.

From paper [10], we note that the VCs are sharing the same link and therefore they all have the same rate $u_i(t)$. Therefore, from (5) it follows that:

$$\sum_{i=1}^{n} u_i(t) = n u_i(t) \tag{6}$$

Hence, from (1) the controller (5) may be equivalently defined as

$$\dot{u}_i(t) = \frac{k}{n}(d(t) - n u_i(t)),\ \forall i = 1, ..., n \tag{7}$$

With this type of controller, the closed-loop system reads:

$$\begin{cases} \dot{x}(t) = -d(t) + \sum_{i=1}^{n} u_i(t - T_i) \\ \dot{u}_i(t) = \frac{k}{n}(d(t) - n u_i(t)),\ \forall i = 1, ..., n \end{cases} \tag{8}$$

Hence, by defining the new vectors of states z as follows:

$$z(t) = \begin{bmatrix} x(t) & u_1(t) & \dots & u_n(t) \end{bmatrix}' \in \Re^{n+1} \tag{9}$$

the initial closed-loop system (8) reads:

$$\dot{z}(t) = Az(t) + \sum_{i=1}^{n} A_{di} z(t - T_i) + Bd(t) \tag{10}$$

with $A = \begin{bmatrix} 0 & 0 \\ 0 & -kI_n \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ \frac{k}{n}1_n \end{bmatrix}$, $A_{di} = \begin{cases} A_{di}(1,1) = 0 \\ A_{di}(1,i) = 1;\ i \geq 2 \\ A_{di}(l,j) = 0 \text{ when } 2 \leq l \leq n+1,\ 1 \leq j \leq n+1 \end{cases}$ and

$$\begin{aligned} z(t_0 + \psi) &= \begin{bmatrix} 0 \\ \phi(\psi) \end{bmatrix}, \forall \psi \in [-\tau, 0],\ t_0, \phi \in \Re_+ \times \mathcal{C}^v_\tau \\ \tau &= \max_i T_i \end{aligned} \tag{11}$$

Furthermore, due to the form of the closed-loop system (10), we can show that the constraints (3) and (4) can be described in the assumption below.

Assumption 2.3 *Let us define*

$$\rho_0 = \begin{bmatrix} u_{01} & \dots & u_{0n} \end{bmatrix}'; \rho_1 = \begin{bmatrix} u_{11} & \dots & u_{1n} \end{bmatrix}'$$

With respect to the closed-loop system (10), the following constraints must be satisfied:

$$z(t) \in \mathcal{Z}_0 = \{z \in \Re^{n+1}; 0 \leq \begin{bmatrix} 0 & I_n \end{bmatrix} z(t) \leq \rho_0, \forall t\} \tag{12}$$

$$z(t) \in \mathcal{Z}_1 = \{z \in \Re^{n+1}; |\begin{bmatrix} 0 & I_n \end{bmatrix} \dot{z}(t)| \leq \rho_1, \forall t\} \tag{13}$$

The control problem can then be re-formulated as follows:

Problem 2.1 *Find a gain k, a set of initial conditions $\mathcal{S}_0 \subseteq \Re^{n+1}$ and a set of admissible disturbances $\mathcal{W}_0 \subset \Re^2$ such that the closed-loop system (10) exhibits the following properties:*
1. **Stability.** $\forall \phi(\psi) \in \mathcal{S}_0$, $\forall \psi \in [-\tau, 0]$, *and* $\forall d \in \mathcal{W}_0$ *one has:*

$$\begin{bmatrix} 1 & 0 \end{bmatrix} z(t) = x(t) \leq r_0, \forall t \geq 0 \tag{14}$$

Since r_0 corresponds to the queue capacity, this condition allows us to guarantee that no cells are lost, but is not an usual stability requirement. It does however guarantee no oscillation, nor overshoot.
2. **Full link utilization.** $\forall \phi(\psi) \in \mathcal{S}_0$, $\forall \psi \in [-\tau, 0]$, *and* $\forall d \in \mathcal{W}_0$ *one has:*

$$\begin{bmatrix} 1 & 0 \end{bmatrix} z(t) = x(t) \geq 0, \forall t \geq 0 \tag{15}$$

3. **Actuator constraints.** *The position and rate constraints of the actuators are linearly satisfied.*

Remark 2.1 *The satisfaction of condition 2) in problem 2.1 may be relaxed to $x(t) \geq 0$, $\forall t \geq T_{tr} \geq 0$ [10], [2]. Moreover, by imposing a linear behavior on the actuator (condition 3)), we avoid the saturation regimes of limited variables.*

In the disturbance free case (that is, $d(t) = 0$, $\forall t \geq 0$), the resulting nonlinear closed-loop system considering the limitations (12) and (13) possesses a basin of attraction of the equilibrium point $z_e = 0$ [11]. Then there exists a subset of this basin of attraction in which the behavior of the closed-loop system remains linear. When $d(t) \neq 0$, it is not possible to strictly define one equilibrium point for the closed-loop system (10) with the time-varying disturbance $d(t)$. At a given time such that $d(t) = d_e$, a corresponding equilibrium point z_e such that $\dot{z}_e = 0$ could be computed, implying that associated with any constant disturbance $d_e \in \mathcal{W}_0$, there exists a set of equilibrium points $\mathcal{Z}_e$. Thus, the closed-loop system due to constraints (12) and (13) exhibits local behaviors around these equilibrium points whose study may be very difficult, if not impossible. Recall that we are interested by a linear behavior of the closed-loop system. Thus, an interesting way to overcome these difficulties is to determine a suitable set of admissible initial conditions, $\mathcal{S}_0$ from which the stability of system (10) with respect to the desired equilibrium points is guaranteed.

Thus the set of equilibrium points under consideration can be defined as follows:

$$\mathcal{Z}_e = \left\{ z_e \in \Re^{n+1}; z_e = \begin{bmatrix} x_e \\ u_e \end{bmatrix}, \dot{z}_e = 0, \forall d_e \in \mathcal{W}_0 \right\} \tag{16}$$

with $u_e = \begin{bmatrix} u_{1e} & \ldots & u_{ne} \end{bmatrix}'$ and $d_e = \text{constant}$.

3 Mathematical Preliminaries

Since we are interested in the linear behavior of the closed-loop system, that is in avoiding the saturation of z and $\dot{z}$, we state the following lemma [13].

Lemma 3.1 *The closed-loop system model (10) subject to constraints (12) and (13) is only valid in the region of linearity $\mathcal{Z}_0 \cap \mathcal{Z}_1$. In other words, the closed-loop system model (10) subject to constraints (12) and (13) is only valid, that is, remains linear if and only if the set of initial conditions $\mathcal{S}_0$ is such that $\forall \phi(\psi) \in \mathcal{S}_0, \forall \psi \in [-\tau, 0]$, $z(t) \in \mathcal{Z}_0 \cap \mathcal{Z}_1, \forall t$.*

When there is a value of $\phi(\psi)$ from which $z(t)$ does not remain in $\mathcal{Z}_0 \cap \mathcal{Z}_1$, $\forall t$, the closed-loop system resulting from (1), (3), (4), and (5) has to be described by using saturation functions. In this case, the occurrence of saturation on the variables u_i and $\dot{u}_i$ has to be investigated and new ways of modeling the resulting closed-loop must be investigated. Such a study will not be considered here, but will be investigated in later research. Note that a solution to such a control problem via statistical learning control was proposed for discrete-time systems with saturation [1].

Lemma 3.2 *Suppose that there exists an equilibrium point $z_e = z(t_e)$ for system (10). Then this equilibrium point satisfies:*

$$\sum_i^n u_i(t_e - T_i) = d_e \tag{17}$$

$$u_i(t_e) = \frac{d_e}{n}, \ \forall i = 1, \ldots, n \tag{18}$$

$$x_e = r_0 - d_e \left(\frac{1}{k} + \frac{1}{n} \sum_{i=1}^n T_i \right) \tag{19}$$

Proof. Relations (17) and (18) are derived by searching $z_e = z(t_e)$ satisfying in (10) $\dot{z}_e = 0$. Relation (19) is derived from (5) by considering that $u_i = \frac{d_e}{n}$ on the interval $[t_e - T_i, t_e]$. □

Remark 3.1 *Condition (18) is consistent with those in [10] and means that the ABR bandwidth d_e is equally shared by the n VC's. Relation (17) is equivalent to*

$$\begin{bmatrix} u_1(t_e - T_1) & \ldots & u_n(t_e - T_n) \end{bmatrix}' = \frac{d_e}{n} 1_n + \zeta_e$$

where ζ_e is any vector of $\Re^n$ such as $1_n' \zeta_e = 0$. Hence, a particular solution consists in choosing $\zeta_e = 0$ leading to $u_i(t_e - T_i) = \frac{d_e}{n}$, $\forall i = 1, \ldots, n$. Condition (19) is consistent with the value exhibited in [10].

4 Main results

A natural way for maintaining the system trajectories in a certain set consists of imposing the positive invariance of such a set with respect to the considered system. Hence, part of our results is based on the use of the extended Farkas lemma applied to delay systems: see [7], [12].

As a first step, we consider that the value of all delays T_i, $i = 1, \ldots, n$, are exactly known.

Proposition 4.1 *If the positive values of r_0, u_{0i}, u_{1i}, k, n, T_i and d_0 satisfy:*

$$0 \le \frac{d_0}{n} \le u_{0i}, \;\; i = 1, ..., n \tag{20}$$

$$r_0 - \frac{d_0}{n}\left(\frac{n}{k} + \sum_{i=1}^{n} T_i\right) \ge 0 \tag{21}$$

$$k u_{0i} + \frac{k}{n} d_0 \le u_{1i}, \;\; i = 1, ..., n \tag{22}$$

$$r_0 - \frac{n}{k} u_{0i} - \sum_{i=1}^{n} u_{0i} T_i \ge 0, i = 1, ..., n \tag{23}$$

then Problem 2.1 is solved for the given values of k and any disturbances satisfying

$$d(t) \in \mathcal{D}_0 = \{d; 0 \le d \le d_0, \; d_0 > 0\} \tag{24}$$

Proof. To solve Problem 2.1, we have to satisfy the three requirements of Problem 2.1.

• We must first verify that $x(t) \le r_0$, $\forall t \ge 0$. From the controller described in (5), one can write the following:

$$r_0 - x(t) = \frac{n}{k} u_i(t) + \sum_{i=1}^{n} \int_{t-T_i}^{t} u_i(\tau) d\tau$$

From Assumption 2.1 one gets:

$$0 \le \sum_{i=1}^{n} \int_{t-T_i}^{t} u_i(\tau) d\tau \le \sum_{i=1}^{n} u_{0i} T_i \tag{25}$$

Therefore, it can be deduced from (3) and (25) that

$$r_0 - x(t) = \frac{n}{k} u_i(t) + \sum_{i=1}^{n} \int_{t-T_i}^{t} u_i(\tau) d\tau \ge 0$$

Furthermore, since from Lemma 3.2, the trajectories of system (10) may attain its equilibrium point $z(t_e) = z_e$ as defined in (17), (18) and (19) we have to verify that $r_0 - x_e \ge 0$. Thus, from (19) it follows that $0 \le r_0 - x_e = d_e \left(\frac{1}{k} + \frac{1}{n}\sum_{i=1}^{n} T_i\right) \le d_0(\frac{1}{k} + \frac{1}{n}\sum_{i=1}^{n} T_i)$. Thus, the first requirement of Problem 2.1 is satisfied for any $u_i(t)$ and $d(t)$ satisfying (3) and (24).

• The second point to verify is the fact that $x(t)$ must be non-negative. Thus, one has to prove that

$$x(t) = r_0 - \frac{n}{k} u_i(t) - \sum_{i=1}^{n} \int_{t-T_i}^{t} u_i(\tau) d\tau \ge 0$$

From (3) it follows that $x(t) \ge r_0 - \frac{n}{k} u_{0i} - \sum_{i=1}^{n} u_{0i} T_i$. Hence, if condition (23) is satisfied one gets $x(t) \ge 0$. This property must also be verified at the equilibrium. Thus, from Lemma 3.2, if relation (21) is satisfied we have $x_e \ge 0$ for any $u_i(t)$ and $d(t)$ satisfying (3) and (24).

• The last point consists in verifying the constraints along the trajectories of the linear closed-loop system (10). Recall that, from Lemma 3.1, system (10) subject to constraints (12) and (13) is only valid in $\mathcal{Z}_0 \cap \mathcal{Z}_1$. Thus, we have to prove that 1) the equilibrium point belongs to this region $\mathcal{Z}_0 \cap \mathcal{Z}_1$ (see Assumption 2.3), and 2) for any u and d such that

$$0 \le u_i \le u_{0i}, i = 1, ..., n$$
$$0 \le d \le d_0$$

it follows

$$0 \le u_i \le u_{0i}, i = 1, ..., n$$
$$-u_{1i} \le \dot{u}_i = \tfrac{k}{n}[d(t) - n u_i(t)] \le u_{1i}, i = 1, ..., n$$

With respect to the point 1, we have shown above that $0 \le x_e \le r_0$ from the satisfaction of (21). Furthermore, from (20) it follows that $u_i(t_e)$ and $u_i(t_e - T_i)$ satisfy (3). With respect to the point 2, from the extended Farkas lemma [7], [12], it follows that if relation (22) is satisfied then there exists a non-negative matrix N_i, $i = 1, ..., n$ such that:

$$\begin{bmatrix} 1 & 0 \\ -k & \frac{k}{n} \\ -1 & 0 \\ k & -\frac{k}{n} \end{bmatrix} = N_i \begin{bmatrix} I_2 \\ -I_2 \end{bmatrix} \text{ and } N_i \begin{bmatrix} u_{0i} \\ d_0 \\ 0 \\ 0 \end{bmatrix} \le \begin{bmatrix} u_{0i} \\ u_{1i} \\ 0 \\ u_{1i} \end{bmatrix} \tag{26}$$

with $N_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ k & \frac{k}{n} & 2k & 0 \\ 0 & 0 & 1 & 0 \\ k & \frac{k}{n} & 0 & \frac{2k}{n} \end{bmatrix}$. □

Remark 4.1 *Relation (22) gives an implicit relation between the bounds u_{0i} and u_{1i}, $i = 1, ..., n$. Indeed, necessarily, we have to satisfy $\frac{n}{k} u_{1i} - n u_{0i} \ge 0$. Moreover, relation (22) comes from the satisfaction of (26) for a certain non-negative matrix N_i. In fact, such a matrix N_i is given by*

$$N_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ N_i(2,1) & N_i(2,2) & N_i(2,1) + k & N_i(2,2) - \frac{k}{n} \\ 0 & 0 & 0 & 1 \\ N_i(4,1) & N_i(4,2) & N_i(4,1) - k & N_i(4,2) + \frac{k}{n} \end{bmatrix}$$

with

$$\begin{bmatrix} u_{0i} & d_0 & 0 & 0 \\ 0 & 0 & u_{0i} & d_0 \end{bmatrix} \begin{bmatrix} N_i(2,1) \\ N_i(2,2) \\ N_i(4,1) \\ N_i(4,2) \end{bmatrix} \le \begin{bmatrix} u_{1i} \\ u_{1i} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} N_i(2,1) \\ N_i(2,2) \\ N_i(4,1) \\ N_i(4,2) \end{bmatrix} \ge \begin{bmatrix} 0 \\ \frac{k}{n} \\ k \\ 0 \end{bmatrix}$$

Note that relation (22) is obtained by choosing $N_i(2,1) = N_i(4,1) = k$ and $N_i(2,2) = N_i(4,2) = \frac{k}{n}$. Another interesting solution, in order to render less conservative conditions of Proposition 4.1, could be to seek $N_i(2,1)$, $N_i(4,1)$, $N_i(2,2)$ and $N_i(4,2)$ satisfying above described conditions instead of relation (22).

In a second stage, we suppose the delays T_i are uncertain but known to lie in a given interval defined as follows:

$$T_i^{min} \le T_i \le T_i^{max}, \forall i = 1, ..., n \tag{27}$$

where T_i^{max} and T_i^{min} are supposed known positive values. It is important to note that we suppose that T_i is not time-varying in its admissible interval. The following proposition can then be stated in the case of uncertain delays.

Proposition 4.2 *If the positive values of* r_0, u_{0i}, u_{1i}, k, n, α_i, T_i^{max}, T_i^{min} *and* d_0 *satisfy relations (20), (22) and the following:*

$$0 \le \alpha_i \le 1 \quad (28)$$

$$r_0 - \frac{d_0}{n}[\frac{n}{k} + \sum_{i=1}^{n}(\alpha_i(T_i^{max} - T_i^{min}) + T_i^{min})] \ge 0 \quad (29)$$

$$r_0 - \frac{n}{k}u_{0i} - \sum_{i=1}^{n}u_{0i}[\alpha_i(T_i^{max} - T_i^{min}) + T_i^{min}] \ge 0 \quad (30)$$

then Problem 2.1 is solved for the given values of k*, any disturbances* $d(t) \in \mathcal{D}_0$ *(described in (24)) and delays* T_i *satisfying (27).*

Proof. The relations of this proposition are directly derived from the relations of Proposition 4.1 by noting that each T_i verifies from (27): $T_i = \alpha_i T_i^{max} + (1-\alpha_i)T_i^{min} = \alpha_i(T_i^{max} - T_i^{min}) + T_i^{min}$ with $0 \le \alpha_i \le 1$. □

In some cases, the value of the number of sources (say n) may be unknown and therefore an interesting problem would be to estimate n. In [8], and considering the case of an ATM switch described in discrete time, a solution for estimating the number of controlled sources is proposed. In order to consider the case where we have also to provide an estimation of n, we suppose the following:

- For simplicity, let $u_{0i} = u_0$ and $u_{1i} = u_1$ for all i.
- All the delays T_i are supposed to satisfy:

$$0 \le T_i \le T_{max}, \ \forall i = 1, ..., n \quad (31)$$

Let us now present a solution to our control problem when the values n and T_{max} are not perfectly known.

Proposition 4.3 *Given* $r_0 > 0$, $u_0 > 0$, $u_1 > 0$ *and* d_0*. If there exist positive values* X, Y *and* Z *satisfying:*

$$0 \le d_0 X \le u_0 \quad (32)$$

$$r_0 - d_0 X - d_0 Z \ge 0 \quad (33)$$

$$u_0 + d_0 X \le u_1 Y \quad (34)$$

$$r_0 X - u_0 Y - u_0 Z \ge 0 \quad (35)$$

$$\frac{r_0}{u_0(Y+Z)} \ge 1 + \max\{\frac{d_0}{u_0}; \frac{d_0}{r_0 - d_0 Z}; \frac{d_0}{u_1 Y - u_0}\} \quad (36)$$

then Problem 2.1 is solved for the values

$$k = \frac{1}{Y} \ and \ n = \mathcal{P}_E(\frac{1}{X})$$

and any disturbances $d(t) \in \mathcal{D}_0$ *(described in (24)) and for all delays verifying (31) with* $T_{max} = Z$.

Proof. Consider relations of Proposition 4.1 with unknown n, k and T_{max}. In order to have linear conditions in the decision variables we choose $X = \frac{1}{n}$, $Y = \frac{1}{k}$ and $Z = T_{max}$. Hence, relations (20) and (22) directly translate into (32) and (34). Relations (33) and (35) comes from relations (21) and (23) by considering (31). Finally, when all conditions are coherent, one obtains: $\frac{u_0(Y+Z)}{r_0} \le X \le \min\{\frac{u_0}{d_0}; \frac{r_0 - d_0 Z}{d_0}; \frac{u_1 Y - u_0}{d_0}\}$. Thus, in order to pick the entire value of $\frac{1}{X}$ to obtain n, there must exist an entire value in the interval $[\max\{\frac{d_0}{u_0}; \frac{d_0}{r_0 - d_0 Z}; \frac{d_0}{u_1 Y - u_0}\}, \frac{r_0}{u_0(Y+Z)}]$. Thus, to ensure this, we have to satisfy condition (36). □

5 Conclusions

We have provided a new approach to deal with the ATM/ABR control problem keeping in mind conditions of simplicity on the controller structure and allowing for various performance objectives to be met. Our approach basically leads to polynomial design inequalities to be satisfied. Such inequalities have been studied by the authors and their collaborators in various papers [9]. The statistical learning control approach discussed by the authors in [1, 2] for example, promises to be effective in this setting. While our controller structure is currently derived in continuous time, it is possible to translate such designs into discrete-time as was done for example in [6]. In addition, and while our controller structure is basically the Smith predictor structure of [10], other controller structures are being investigated.

References

[1] C.T. Abdallah, M. Ariola, R. Byrne. Statistical-Learning Control of an ABR Explicit Rate Algorithm for ATM Switches, *Proc. of the 39th IEEE Conference on Decision and Control*, Sydney (Australia), pp. 53–54, 2000.

[2] C.T. Abdallah, M. Ariola, V. Koltchinskii. Statistical-Learning Control of Multiple-Delay Systems with applications to ATM Networks, *Kybernetica.*

[3] The ATM Forum Traffic Management Working Group. *Traffic Management Specification, Version 4.0*, April 1996.

[4] L. Benmohamed and Y.T. Wang. A Control-Theoretic ABR Explicit Rate Algorithm for ATM Switches with Per-VC Queuing. *Proc. Infocom98*, pp.183–191, San Francisco, CA, 1998.

[5] F. Blanchini, R. Lo Cigno, R. Tempo. Control of ATM Networks: Fragility and Robustness Issues. *Proc. American Control Conference*, pp.2847–2851, Philadelphia, PA, 1998.

[6] D. Cavendish, S. Mascolo, M. Gerla. SP-EPRCA: an ATM rate Based Congestion Control Scheme Based on a Smith Predictor, *1996 UCLA CS Tech Report 960001*, available at: ftp://ftp.cs.ucla.edu/tech-report/.

[7] J-C. Hennet and S. Tarbouriech. Stability and stabilization of delay differential systems, *Automatica*, vol.33, no.3, pp.347-354, 1997.

[8] O.C. Imer, S. Compans, T. Başar, R. Srikant. ABR Control in ATM Networks, *IEEE Control Systems Magazine*, vol.21, no.1, pp.38–56, 2001.

[9] V. Koltchinskii, C.T. Abdallah, M. Ariola, P. Dorato, D. Panchenko. Improved Sample Complexity Estimates for Statistical Learning Control of Uncertain Systems, *IEEE Trans. Autom. Control*, vol.45, no.12, pp.2383–2388, 2000.

[10] S. Mascolo. Smith's Principle for Congestion Control in High-Speed Data Networks, *IEEE Trans. Autom. Control*, vol.45, no.2, pp.358-364, 2000.

[11] A. Saberi, Z. Lin, A.R. Teel. Control of linear systems with saturating actuators, *IEEE Trans. Autom. Control*, vol.41, no.3, pp.368-378, 1996.

[12] G. Seifert. Positively invariant closed-loop systems of delay differential equations, *J. Differential Equations*, vol.22, pp.292-304, 1976.

[13] S. Tarbouriech and J.M. Gomes da Silva Jr. Synthesis of controllers for continuous-time delay systems with saturating controls, *IEEE Trans. Autom. Control*, vol.45, no.1, pp.105-110, 2000.