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Bounded controller design of an ABR explicit rate algorithm for ATM switches

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Abstract— Congestion control in the Available Bit Rate (ABR) class of Asynchronous Transfer Mode (ATM) networks poses interesting challenges due to the presence of delays, magnitude and rate constraints, and additive disturbances. In this paper, we consider a discrete-time fixed-structure controller for an ATM/ABR switch, and solve a robust tracking control problem in which the target is a threshold on the queue level.

I. INTRODUCTION

The transmission of multimedia traffic on the broadband integrated service digital networks (B-ISDN) has created the need for new transport technologies such as Asynchronous Transfer Mode (ATM). Briefly, because of the variability of the multimedia traffic, ATM networks seek to guarantee an end-to-end quality of service (QoS) by dividing the varying types of traffic (voice, data, etc.) into short, fixed-size cells (53 bytes each) whose transmission delay may be predicted and controlled. ATM is thus a *Virtual Circuit* (VC) technology which combines advantages of circuit-switching (all intermediate switches are alerted of the transmission requirements, and a connecting circuit is established) and packet-switching (many circuits can share the network resources). In order for the various VC's to share network resources, flow and congestion control algorithms need to be designed and implemented. The congestion control problem is solved by regulating the input traffic rate. In addition, because of its inherent flexibility, ATM traffic may be served under one of the following service classes: 1) The *constant bit rate* (CBR) class: it accommodates traffic that must be received at a guaranteed bit rate, such as telephone conversations, video conferencing, and television. 2) The *variable bit rate* (VBR): it accommodates bursty traffic such as industrial control, multimedia e-mail, and interactive compressed video. 3) The *available bit rate* (ABR): it is a best-effort class for applications such as file transfer or e-mail. Thus, no service guarantees (transfer delay) are required, but the source of data packets controls its data rate, using a feedback signal provided by switches downstream which measure the congestion of the network. Due to the presence of this feedback, many classical and advanced control theory concepts have been suggested to deal with the congestion control problem in the ATM/ABR case [2], [10]. 4) The *unspecified bit rate* (UBR): it uses any leftover capacity to accommodate applications such as e-mail.

Note that the CBR and VBR service categories, a traffic contract is negotiated at the initial stage of the VC setup, and maintained for the duration of the connection. This contract will guarantee the following QoS parameters: 1) Minimum cell rate (MCR), 2) Peak cell rate (PCR), 3) cell delay variation (CDV), 4) maximum cell transfer delay (maxCTD), and 5) cell loss ratio (CLR). This then forces CBR and VBR sources to keep their rate constant regardless of the congestion status of the network. The ABR sources on the other hand, are only required to guarantee an MCR and an PCR, and thus can adjust their rates to accommodate the level available after all CBR and VBR traffic has been accommodated. In order to avoid congestion, the ATM Forum adopted a rate-based ABR control algorithm as opposed to a credit approach whereby the number of incoming cells as opposed to their rate is controlled [7]. This paper will then concentrate on the ABR service category since ABR sources are the ones to adjust their rates using explicit network feedback. In the original ATM forum specification, an ATM/ABR source is required to send one cell called a resource management (RM) cell for every 32 data cells. Switches along the path from the source to the destination then write into the RM cell their required data rate to avoid congestion. The destination switch then has information about the minimum rate required by all switches along the VC which is then relayed back to the ATM/ABR source as a feedback signal which serves to adjust its own data rate.

The earliest control algorithms for ABR consisted of setting a binary digit in the RM cell by any switch along the VC when its queue level exceeds a certain threshold [2]. This was then shown to cause oscillations in the closed-loop system. Other controllers were then suggested by various authors [5], [6], to address this problem. Most of these controllers are either complex or did not guarantee the closed-loop stability (in a sense defined later).

In addition, one of the limiting factors of these earlier proposed controllers was that the ABR bandwidth needed to be known in the implementation of the control algorithm. This however poses a problem in multimedia applications where the ABR bandwidth is bursty and is effectively the remaining available bandwidth after the CBR and VBR traffic have been accommodated. In [10] this particular issue was dealt with using a Smith predictor which then considered the available ABR bandwidth as an unknown disturbance. While this controller had many desirable properties, it only

guaranteed stability in an appropriately defined sense but had no optimality guarantees. In addition, the delays encountered along with the number of ABR sources were assumed known, although the earlier tech report [7] did not require the delays to be exactly known. In [9], robust controllers were designed when both the number of ABR sources and the delays were uncertain.

In this paper, we consider a discrete-time model for an ATM/ABR switch and source which was presented in [5] and attempt to control the resulting nonlinear system. The technique developed is based on the use of Finsler's lemma and a generalized sector nonlinearity description. Considering quadratic Lyapunov functions, the use of Finsler's lemma allows us to express stability and invariance conditions without explicitly substituting the dynamic system equations into the Lyapunov function. Such an approach corresponds to weighing the dynamic system equations through multipliers which represent additional variables [12], [11]. The introduction of these new variables increases the degrees of freedom in the problem, and relaxes the conditions of applicability.

Notations. For two vectors x, y of $\Re^n$, the notation $x \succeq y$ means that $x_{(i)} - y_{(i)} \geq 0$, $\forall i = 1, \ldots, n$. $A_{(i)}$ denotes the ith row of matrix A. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. A' denotes the transpose of A. $sat_{\{A_1, A_2\}}(\nu)$ denotes the scalar saturation function:

$$sat_{\{A_1, A_2\}}(\nu) = \begin{cases} A_1 & \text{if } \nu < A_1 \\ \nu & \text{if } A_1 \leq \nu \leq A_2 \\ A_2 & \text{if } \nu > A_2 \end{cases}$$

II. PROBLEM FORMULATION

As in [5] and [1], we consider the closed-loop discrete-time system:

$$Q(n+1) = sat_{\{0,B\}}(Q(n) + \lambda(n - d^f) - \mu(n)) \quad (1)$$

$$R(n+1) = sat_{\{0,C\}}(R(n) - \sum_{j=0}^{J} \alpha_j(Q(n-j) - Q_0)$$

$$- \sum_{k=0}^{K} \beta_k R(n-k)) \quad (2)$$

where R denotes the explicit rate (ER) computed by a switch for a given VC and Q denotes the buffer occupancy of this VC at the switch. Furthermore, $\lambda(n)$ and $\mu(n)$ are respectively, the rate at the ABR source and the service rate at the switch during the interval $[n, n+1]$, d^f is the forward delay from the source to the switch and the saturation level B represents the buffer size. The saturation level C is the maximum ER and Q_0 is the desired buffer occupancy. The ABR source is greedy if the source's rate $\lambda(n - d^f)$ is equal to $R(n+1-d)$, where $d = d^f + d^b$ is the round trip delay (d^b is the feedback delay from the switch back to the source).

Note that equation (1) describes the plant dynamics, i.e. the state of the buffer, and equation (2) defines a particular

controller structure. In (1)-(2), the numbers J, K and the parameters α_j, β_k have to be found such that closed-loop stability and some performance levels are attained [4]. In [4], the author showed that when considering the linearized model of system (1)-(2) it is sufficient to consider $J = 1$ and $K = d$ in order to completely place the closed-loop poles. Hence, we consider (1)-(2) with $J = 1, K = d$ but without removing the saturation functions as was done in [5]. Moreover we assume that $\lambda(n - d^f)$ is equal to $R(n+1-d)$.

Consider both the extended state and disturbance vectors

$$X(n) = \begin{bmatrix} Q(n) \\ Q(n-1) \\ R(n) \\ R(n-1) \\ \vdots \\ R(n+1-d) \\ R(n-d) \end{bmatrix} \in \Re^{d+3} \; ; \; W(n) = \begin{bmatrix} \mu(n) \\ Q_0 \end{bmatrix} \in \Re^2$$

and define the following matrices

$$\mathbb{A} = \begin{bmatrix} 0 & 0 & \ldots & \ldots & 0 & 0 & 0 \\ 1 & 0 & \ldots & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & \ldots & 0 \\ 0 & 0 & 1 & 0 & \ldots & \ldots & 0 \\ \vdots & \ldots & \ldots & \ddots & \ldots & \ldots & \vdots \\ \vdots & \ldots & \ldots & \ldots & \ddots & \ldots & \vdots \\ 0 & \ldots & \ldots & \ldots & \ldots & 1 & 0 \end{bmatrix} \; ; \; \mathbb{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}$$

$$\mathbb{B}_2 = \begin{bmatrix} -1 & 0 \\ 0 & \alpha_0 + \alpha_1 \end{bmatrix}$$

$$\mathbb{G} = \begin{bmatrix} 1 & 0 & \ldots & \ldots & 0 & 1 & 0 \\ -\alpha_0 & -\alpha_1 & 1 - \beta_0 & -\beta_1 & \ldots & \ldots & -\beta_d \end{bmatrix}$$

with $\mathbb{B}_1 = \begin{bmatrix} \mathbb{B}_{11} & \mathbb{B}_{12} \end{bmatrix}$ and $\mathbb{G} = \begin{bmatrix} \mathbb{R} \\ \mathbb{K} \end{bmatrix}$. Hence, the system (1)-(2) reads:

$$X(n+1) = (\mathbb{A} + \mathbb{B}_1\mathbb{G})X(n) + \mathbb{B}_1\mathbb{B}_2W(n) + \mathbb{B}_1\Psi(n) \quad (3)$$

with

$$\Psi(n) = sat_{\{0,B\},\{0,C\}}(\mathbb{G}X(n)+\mathbb{B}_2W(n))-(\mathbb{G}X(n)+\mathbb{B}_2W(n)) \quad (4)$$

where $sat_{\{0,B\},\{0,C\}}(\nu) = \begin{bmatrix} sat_{\{0,B\}}(\nu_{(1)}) \\ sat_{\{0,C\}}(\nu_{(2)}) \end{bmatrix}$. By definition, $\Psi(n)$ is a decentralized dead-zone nonlinearity and satisfies the following sector condition [8]:

$$\Psi(n)'[\Psi(n) + \Lambda(\mathbb{G}X(n) + \mathbb{B}_2W(n))] \leq 0, \; \forall n \quad (5)$$

for some X and W belonging to the set $S(\mathbb{G}, B^\lambda, C^\lambda)$:

$$S(\mathbb{G}, B^\lambda, C^\lambda) = \{X \in \Re^{d+3}, W \in \Re^2; 0 \preceq \mathbb{G}X$$
$$+ \mathbb{B}_2 W \preceq \begin{bmatrix} \frac{B}{1-\lambda_{(1)}} \\ \frac{C}{1-\lambda_{(2)}} \end{bmatrix} \} \quad (6)$$

where Λ is a diagonal matrix whose diagonal elements $\Lambda_{(i,i)}$, denoted by $\lambda_{(i)}$, are such that $0 \leq \lambda_{(i)} < 1$.

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The problem we aim to solve is summarized as follows.

Problem 1: Determine a matrix $\mathbb{K}$, a set of admissible initial conditions S_0, and a set of admissible disturbances D_0 such that:

1) The closed-loop matrix $\mathbb{A} + \mathbb{B}_1\mathbb{G}$ is asymptotically stable.
2) The closed-loop trajectories remain bounded for any $X(0) \in S_0$ and any admissible disturbance $W(n) \in D_0$, $\forall n$.
3) The steady state buffer occupancy is equal to the desired threshold Q_0.

We consider that the saturation functions are effectively taken into account, that is, the nonlinear behavior of the closed-loop system is studied. The implicit objectives in Problem 1 consist of optimizing the size of the region of stability and/or the size of the region of admissible disturbances.

III. PRELIMINARY RESULTS

Satisfying point 3 in Problem 1 allows us to study the existence of possible equilibrium points corresponding to the case $W(n) = W_e$ where W_e is a constant value.

Lemma 1: Suppose that there exists an equilibrium point X_e for system (3). Then this equilibrium point satisfies:

$$R_e(n-k) = R_e(n-k+1) = R_e = \mu_e, k = 0,...,d \quad (7)$$

$$Q_e(n) = Q_e(n-1) = Q_e = Q_0 \quad (8)$$

$$\sum_{k=0}^{d}\beta_k R_e = \sum_{k=0}^{d}\beta_k\mu_e = 0 \quad (9)$$

provided that the following two conditions hold

$$0 \le Q_0 \le B \quad (10)$$

$$0 \le \mu_e \le C \quad (11)$$

Proof. Relations (7) to (11) are obtained by letting $X_e(n+1) = X_e(n) = X_e$, $W(n) = W_e(n) = W_e = \begin{bmatrix} \mu_e \\ Q_0 \end{bmatrix}$ in (3), and by assuming that this equilibrium point is in the region of linearity of the closed-loop system (3). Indeed, it is sufficient to solve $X_e = (\mathbb{A} + \mathbb{B}_1\mathbb{G})X_e + \mathbb{B}_1\mathbb{B}_2 W_e$ with $\Psi_e = 0$. $\square$

Remark 1: The conditions of Lemma 1 are consistent with those given in [5]. Moreover, in general μ_e is not equal to 0, therefore condition (9) implies that $\sum_{k=0}^{d}\beta_k = 0$. This equality implies that β_d is computed from the last d entries as:

$$\beta_d = -\sum_{k=0}^{d-1}\beta_k \quad (12)$$

Remark 2: If we do not require that β_d verifies (12) then the equilibrium point Q_e is such that $Q_e = Q_0 - \dfrac{\sum_{k=0}^{d}\beta_k\mu_e}{\alpha_0+\alpha_1} \ne$

Q_0. Hence, the steady-state buffer occupancy will be equal to Q_e (i.e. different from Q_0). Furthermore, in this case, condition (10) reads:

$$0 \le Q_0 - \dfrac{\sum_{k=0}^{d}\beta_k\mu_e}{\alpha_0+\alpha_1} \le B \quad (13)$$

Thus, from Lemma 1 and Remark 1, we can consider another representation of model (1)-(2) and therefore of model (3). Towards this aim, consider the following vectors

$$Y(n) = \begin{bmatrix} Q(n) - Q_0 \\ Q(n-1) - Q_0 \\ R(n) - \mu_e \\ R(n-1) - \mu_e \\ \vdots \\ R(n-d) - \mu_e \end{bmatrix} \in \Re^{d+3} , \nu(n) = \mu(n)-\mu_e \in \Re$$

which correspond to a change of variables around the equilibrium point X_e, and define the matrix

$$\mathbb{B}_3 = \begin{bmatrix} -1 & 0 & 0 & 0 & ... & 0 \end{bmatrix}' \in \Re^{d+3}$$

Thus, the closed-loop system under consideration reads:

$$Y(n+1) = (\mathbb{A}_0 + \mathbb{B}_{12}\mathbb{K})Y(n) + \mathbb{B}_3\nu(n) + \mathbb{B}_1\Psi(n) \quad (14)$$

where $\mathbb{A}_0 = \mathbb{A} + \mathbb{B}_{11}\mathbb{R}$, $\mathbb{B}_0 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \in \Re^2$, and $\Psi(n) = sat_{\{-Q_0,B-Q_0\},\{-\mu_e,C-\mu_e\}}(\mathbb{G}Y(n) + \mathbb{B}_0\nu(n)) - (\mathbb{G}Y(n) + \mathbb{B}_0\nu(n))$. Given a diagonal matrix Λ, whose elements $\lambda_{(i)}$ are such that $0 \le \lambda_{(i)} < 1$, the nonlinearity $\Psi(n)$, satisfies the following sector condition

$$\Psi(n)' [\Psi(n) + \Lambda(\mathbb{G}Y(n) + \mathbb{B}_0\nu(n))] \le 0, \forall n \quad (15)$$

for some Y and ν belonging to the set $S(\mathbb{G},\Lambda)$:

$$S(\mathbb{G},\Lambda) = \left\{ Y \in \Re^{d+3}, \nu \in \Re; \begin{bmatrix} \frac{-Q_0}{1-\lambda_{(1)}} \\ \frac{-\mu_e}{1-\lambda_{(2)}} \end{bmatrix} \preceq \mathbb{G}Y \right.$$
$$\left. +\mathbb{B}_0\nu \preceq \begin{bmatrix} \frac{B-Q_0}{1-\lambda_{(1)}} \\ \frac{C-\mu_e}{1-\lambda_{(2)}} \end{bmatrix} \right\} \quad (16)$$

Remark 3: By considering that the closed-loop system is linear (i.e., $\Psi(n) = 0$) and by considering that the service rate $\mu(n)$ is constant (i.e., $\nu(n) = 0$), one obtains the same system studied in [5].

Remark 4: If we work in the context of Remark 2, the closed-loop system defined in (14) is obtained by replacing Q_0 by Q_e in the definitions of $Y(n)$ and of $\Psi(n)$ and (16).

IV. MAIN RESULTS

Some conditions in terms of matrix inequalities are now presented in order to guarantee that the closed-loop system (3) satisfies conditions 1 and 2 of Problem 1 statement. Moreover, and in order to simplify the statement of the results, we assume that the constraints in (16) are symmetrical, that is, we let $B = 2Q_0$ and $C = 2\mu_e$.

A. Results without structural conditions on $\mathbb{K}$

We are working in the context of Remarks 2 and 4.

Proposition 1: If there exist matrices of appropriate dimensions $P = P' > 0$, F, G, H, L, $\mathbb{K}$, diagonal positive matrices T and Λ, positive scalars γ, ν_0, δ and ω verifying[1]

$$\begin{bmatrix} M_1 & \star & \star & \star \\ M_2 & M_3 & \star & \star \\ M_4 & M_5 & M_6 & \star \\ M_7 & M_8 & M_9 & M_{10} \end{bmatrix} < 0 \tag{17}$$

$$-1 + \delta + \omega\gamma\nu_0^2 \leq 0 \tag{18}$$

$$\begin{bmatrix} P & \star \\ (1 - \lambda_{(1)})\mathbb{R} & \gamma\nu_1^2 \end{bmatrix} \geq 0 \tag{19}$$

$$\begin{bmatrix} P & \star \\ (1 - \lambda_{(2)})\mathbb{K} & \gamma\mu_e^2 \end{bmatrix} \geq 0 \tag{20}$$

$$\nu_1 = Q_e - (1 - \lambda_{(1)})\nu_0 \geq 0 \tag{21}$$

$$0 \leq \Lambda < I_2 \tag{22}$$

with $M_1 = -\delta P + F(\mathbb{A}_0 + \mathbb{B}_{12}\mathbb{K}) + (\mathbb{A}_0 + \mathbb{B}_{12}\mathbb{K})'F'$, $M_2 = G(\mathbb{A}_0 + \mathbb{B}_{12}\mathbb{K}) - F'$, $M_3 = P - G - G'$, $M_4 = H(\mathbb{A}_0 + \mathbb{B}_{12}\mathbb{K}) + \mathbb{B}_1'F' - T\Lambda G$, $M_5 = -H + \mathbb{B}_1'G'$, $M_6 = -2T + H\mathbb{B}_1 + \mathbb{B}_1'H'$, $M_7 = \mathbb{B}_3'F' + L(\mathbb{A}_0 + \mathbb{B}_{12}\mathbb{K})$, $M_8 = -L + \mathbb{B}_3'G'$, $M_9 = L\mathbb{B}_1 + \mathbb{B}_3'H' - \mathbb{B}_0'\Lambda T$, $M_{10} = L\mathbb{B}_3 + \mathbb{B}_3'L' - \omega$, then

(a) the gain $\mathbb{K}$ is such that $\mathbb{A} + \mathbb{B}_1 G$ is asymptotically stable,
(b) the closed-loop trajectories remain bounded in the set $\mathcal{E}(P, \gamma) = \{Y \in \Re^{d+3}; \; Y'PY \leq \gamma^{-1}\}$ for any admissible disturbance satisfying $-\nu_0 \leq \nu(n) \leq \nu_0$,
(c) the steady-state occupancy is equal to $Q_0 - \frac{(\mathbb{K}e-1)\mu_e}{\mathbb{K}(e-1_{d+3})}$ with $e = \begin{bmatrix} 0 & 0 & 1 & \cdots & 1 \end{bmatrix}'$.

Proof. Consider the quadratic function $V(n) = Y(n)'PY(n)$ with $P = P' > 0$ and the following definitions:

$$\xi \triangleq \begin{bmatrix} Y(n) \\ Y(n+1) \\ \Psi(n) \\ \nu(n) \end{bmatrix} ; \mathcal{Q} \triangleq \begin{bmatrix} -\delta P & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega \end{bmatrix} ; \mathcal{R} \triangleq \begin{bmatrix} F \\ G \\ H \\ L \end{bmatrix}$$

$$\mathcal{B} \triangleq \begin{bmatrix} \mathbb{A} + \mathbb{B}_1 G & -I_{d+3} & \mathbb{B}_1 & \mathbb{B}_3 \end{bmatrix}$$

If

$$\begin{cases} V(n+1) - \delta V(n) - \omega\nu(n)'\nu(n) \leq 0 \\ -1 + \delta + \omega\gamma\nu_0^2 \leq 0 \end{cases} \tag{23}$$

are satisfied, then all the closed-loop trajectories initiated in $\mathcal{E}(P, \gamma) = \{Y \in \Re^{d+3}; \; Y'PY \leq \gamma^{-1}\}$ remain in it for any admissible disturbance satisfying $\nu(n)'\nu(n) = \nu(n)^2 \leq \nu_0^2$. Indeed, letting $n = 0$ in (23) we have that if $V(0) = Y'(0)PY(0) \leq \gamma^{-1}$ then also $V(1) = Y'(1)PY(1) \leq \gamma^{-1}$. Then we can go on and show that $V(n) = Y'(n)PY(n) \leq \gamma^{-1}$ for all $n > 0$.

Then the first inequality in (23) is equivalent to:

$$\xi'\mathcal{Q}\xi \leq 0 \text{ such that } \mathcal{B}\xi = 0, \xi \neq 0 \tag{24}$$

[1]The symbol $\star$ stands for a symmetric block in the matrix.

where T is a positive definite, diagonal matrix. Now, applying Finsler's lemma [12], it follows by using inequality (15) that if there exist a matrix $\mathcal{R}$ such that

$$\xi'(\mathcal{Q} + \mathcal{R}\mathcal{B} + \mathcal{B}'\mathcal{R}')\xi \leq \xi' \begin{bmatrix} 0 & \star & \star & \star \\ 0 & 0 & \star & \star \\ T\Lambda G & 0 & 2T & \star \\ 0 & 0 & \mathbb{B}_0'\Lambda T & 0 \end{bmatrix} \xi \tag{25}$$

where T is a positive definite, diagonal matrix, then for all $Y, \nu \in S(\mathbb{G}, \Lambda)$ the first inequality in (23) is verified. Thus, provided that relations (17) and (18) are verified and $Y, \nu \in S(\mathbb{G}, \Lambda)$, inequality (25), and as a consequence, (23) is verified.

Furthermore, if relation (21) is satisfied, the bounds of the set $\begin{bmatrix} \frac{-Q_e}{1-\lambda_{(1)}} \\ \frac{-\mu_e}{1-\lambda_{(2)}} \end{bmatrix} - \mathbb{B}_0\nu(n) \preceq \mathbb{G}Y(n) \preceq \begin{bmatrix} \frac{Q_e}{1-\lambda_{(1)}} \\ \frac{\mu_e}{1-\lambda_{(2)}} \end{bmatrix} - \mathbb{B}_0\nu(n)$ under the constraints $-\nu_0 \leq \nu(n) \leq \nu_0$ are well defined. Finally, from the definition of $\mathbb{G}$, if relations (19), (20) and (21) are verified, then $\mathcal{E}(P, \gamma) \subset S(\mathbb{G}, \Lambda)$ provided that ν is such that $-\nu_0 \leq \nu(n) \leq \nu_0$.

Hence if all the inequalities of Proposition 1 are verified, we can conclude that:

• when $\nu \neq 0$, $\mathcal{E}(P, \gamma) \subset S(\mathbb{G}, \Lambda)$ is a positively invariant set with respect to the trajectories of the closed-loop system for $Y(0) \in \mathcal{E}(P, \gamma) \subset S(\mathbb{G}, \Lambda)$ and any admissible $\nu(n)$.
• when $\nu = 0$ (constant service rate), we have that for any $Y(0) \in \mathcal{E}(P, \gamma) \subset S(\mathbb{G}, \Lambda)$, $V(n+1) < V(n)$ and therefore $Y(n) \rightarrow 0$ as $n \rightarrow \infty$. Hence, $X(n) \rightarrow Q_e$ which, from Remark 4, implies that point (c) is satisfied. □

In the matrix inequalities of Proposition 1, some nonlinearities appear due to the product between the multipliers (F, G, H, L) and the gain $\mathbb{K}$, to the product between T, Λ and $\mathbb{K}$, and due to the product involving ν_0. However, it is important to note that from (17), that $G + G' > P > 0$ and therefore matrix G must be nonsingular. Hence, we can investigate a suitable choice of multipliers with an adequate change of variables in order to simplify a major part of these nonlinearities. The following corollary, which is a particular application of Proposition 1, is provided in this case.

Corollary 1: If there exist matrices of appropriate dimensions $V = V' > 0$, S, Z, diagonal positive matrices D and Λ, positive scalars γ, ν_0, δ and ω verifying

$$\begin{bmatrix} N_1 & \star & \star & \star \\ N_2 & N_3 & \star & \star \\ N_4 & N_5 & N_6 & \star \\ N_7 & N_8 & N_9 & N_{10} \end{bmatrix} < 0 \tag{26}$$

$$-1 + \delta + \omega\gamma\nu_0^2 \leq 0 \tag{27}$$

$$\begin{bmatrix} V & \star \\ (1 - \lambda_{(1)})\mathbb{R}S' & \gamma\nu_1^2 \end{bmatrix} \geq 0 \tag{28}$$

$$\begin{bmatrix} V & \star \\ (1 - \lambda_{(2)})Z & \gamma\mu_e^2 \end{bmatrix} \geq 0 \tag{29}$$

$$\nu_1 = Q_e - (1 - \lambda_{(1)})\nu_0 \geq 0 \qquad (30)$$

$$0 \leq \Lambda < I_2 \qquad (31)$$

with $N_1 = -\delta V + \mathbb{A}_0 S' + S \mathbb{A}_0' + \mathbb{B}_{12} Z + Z' \mathbb{B}_{12}'$, $N_2 = \mathbb{A}_0 S' + \mathbb{B}_{12} Z - S$, $N_3 = V - S - S'$, $N_4 = D \mathbb{B}_1' - \Lambda \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbb{R} S' - \Lambda \begin{bmatrix} 0 \\ 1 \end{bmatrix} Z$, $N_5 = D \mathbb{B}_1'$, $N_6 = -2D$, $N_7 = \mathbb{B}_3'$, $N_8 = \mathbb{B}_3'$, $N_9 = -\mathbb{B}_0' \Lambda$, $N_{10} = -\omega$, then

(a) the gain $\mathbb{K} = Z(S')^{-1}$ is such that $\mathbb{A} + \mathbb{B}_1 \mathbb{G}$ is asymptotically stable,

(b) the closed-loop trajectories remain bounded in set $\mathcal{E}(S^{-1} V (S')^{-1}, \gamma) = \{Y \in \Re^{d+3}; \ Y' S^{-1} V (S')^{-1} Y \leq \gamma^{-1}\}$ for any admissible disturbance satisfying $-\nu_0 \leq \nu(n) \leq \nu_0$,

(c) the steady state occupancy is equal to $Q_e = Q_0 - \frac{(\mathbb{K}e-1)\mu_e}{\mathbb{K}(e-1_{d+3})}$ with $e = \begin{bmatrix} 0 & 0 & 1 & \dots & 1 \end{bmatrix}'$.

The following comments apply to the previous results:

• As opposed to the classical approach [15] where $\mathbb{K} = ZW^{-1}$ with $W^{-1} = P$, here the state feedback gain $\mathbb{K}$ is computed from matrix S ($\mathbb{K} = Z(S')^{-1}$) which needs not be positive definite.

• The matrix V which allowed us to define the Lyapunov matrix P ($P = S^{-1} V (S')^{-1}$) is a decision variable. Such a fact allows us to consider different matrices V (and therefore implicitly different matrices P) in other LMI constraints, for example, to deal with the regional assignment of the poles of the closed-loop system [14].

B. Results with structural conditions on $\mathbb{K}$

According to Remark 1, the gain $\mathbb{K}$ can be written as:

$$\mathbb{K} = \mathbb{K}_0 + \mathbb{K}_1 \mathbb{C} \qquad (32)$$

with $\mathbb{K}_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & \dots & \dots & 0 \end{bmatrix} \in \Re^{1 \times (d+3)}$, $\mathbb{K}_1 = \begin{bmatrix} -\alpha_0 & -\alpha_1 & -\beta_0 & -\beta_1 & \dots & -\beta_{d-1} \end{bmatrix} \in \Re^{1 \times (d+2)}$, $\mathbb{C} = $

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & \dots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & 1 & 0 & -1 \\ 0 & 0 & \dots & \dots & \dots & 0 & 1 & -1 \end{bmatrix} \in \Re^{(d+2) \times (d+3)}.$$

Thus from the definition (32) of $\mathbb{K}$, the problem of designing the gain $\mathbb{K}$ consists of designing the gain $\mathbb{K}_1$, since $\mathbb{K}_0$ is fixed. A very important fact is that the synthesis problem is now a static output design problem and no more a state feedback design problem due to the presence of matrix $\mathbb{C}$. Note that in this case the closed-loop system reads:

$$Y(n+1) = (\mathbb{A}_1 + \mathbb{B}_{12} \mathbb{K}_1 \mathbb{C}) Y(n) + \mathbb{B}_3 \nu(n) + \mathbb{B}_1 \Psi(n) \qquad (33)$$

with $\mathbb{A}_1 = \mathbb{A}_0 + \mathbb{B}_{12} \mathbb{K}_0$.

A result equivalent to Proposition 1 in the current case can be stated by replacing $\mathbb{K}$ by $\mathbb{K}_0 + \mathbb{K}_1 \mathbb{C}$. The presence of matrix $\mathbb{C}$ implies that an equivalent result to Corollary 1

cannot be directly stated. In order to overcome this difficulty, we consider the following modifications. First, since matrix $\mathbb{C} \in \Re^{(d+2) \times (d+3)}$ is full row rank ($rank(\mathbb{C}) = d + 2$) there always exists a matrix $N \in \Re^{1 \times (d+3)}$ such that matrix $M = \begin{bmatrix} \mathbb{C}' & N' \end{bmatrix}'$ is non-singular. For example a trivial solution is $N = \begin{bmatrix} 0 & 0 & \dots & \dots & \dots & 0 & 0 & 1 \end{bmatrix} \in \Re^{1 \times (d+3)}$. From this non-singular matrix M we can use the following change of variables: $\tilde{Y}(n) = \begin{bmatrix} \mathbb{C} \\ N \end{bmatrix} Y(n) = M Y(n)$, which gives the augmented closed-loop system:

$$\tilde{Y}(n+1) = (\tilde{\mathbb{A}}_1 + \tilde{\mathbb{B}}_{12} \begin{bmatrix} \mathbb{K}_1 & 0 \end{bmatrix}) \tilde{Y}(n) + \tilde{\mathbb{B}}_3 \nu(n) + \tilde{\mathbb{B}}_1 \Psi(n) \qquad (34)$$

Hence, relative to this system our objective consists in being able to compute a state feedback $\tilde{\mathbb{K}}_1$ with a structural constraint since one wants $\tilde{\mathbb{K}}_1 = \begin{bmatrix} \mathbb{K}_1 & 0 \end{bmatrix}$. Such a constraint can be linearly treated. At this stage, we can consider some changes of variables as in Corollary 1.

Corollary 2: If there exist matrices of appropriate dimensions $V = V' > 0$, $S = \begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix}$, $Z = \begin{bmatrix} Z_1 & 0 \end{bmatrix}$, diagonal positive matrices D and Λ, positive scalars γ, ν_0, δ and ω verifying relations of Corollary 1, in which Q_e has been replaced by Q_0, then

(a) the gain $\mathbb{K} = Z(S')^{-1}$ is such that $\mathbb{A} + \mathbb{B}_1 \mathbb{G}$ is asymptotically stable,

(b) the closed-loop trajectories remain bounded in set $\mathcal{E}(S^{-1} V (S')^{-1}, \gamma) = \{Y \in \Re^{d+3}; \ Y' S^{-1} V (S')^{-1} Y \leq \gamma^{-1}\}$ for any admissible disturbance satisfying $-\nu_0 \leq \nu(n) \leq \nu_0$,

(c) the steady state occupancy is equal to Q_0.

The structural constraints imposed on the matrices Z and S induce more conservatism than in the state feedback case. Indeed the problem arises from the fact that the output feedback gain is dependent on a state transformation whose choice is still an open problem. In fact, there is no particular difficulty in choosing a matrix N such that M is non-singular, but in no way guarantees that a feasible solution to the set of matrix inequalities (of Corollary 2) will be obtained [3]. Other approaches could be investigated to deal with this output feedback problem in a less conservative way [13].

C. Robustness issues

Suppose that d is uncertain (constant but unknown integer) but known to lie in a given interval defined as follows: $d_{min} \leq d \leq d_{max}$, where d_{min} and d_{max} are known positive integers. Hence, the delay d can take $d_{max} - d_{min} + 1$ integer values (the first value being d_{min} and the last one d_{max}). For each of these $d_{max} - d_{min} + 1$ values, we can associate $d_{max} - d_{min} + 1$ closed-loop systems. In this case the problem consists in being able to simultaneously stabilize the $d_{max} - d_{min} + 1$ systems and therefore to solve Problem 1 with respect to these $d_{max} - d_{min} + 1$ systems.

V. EXAMPLE AND COMPUTATIONAL ISSUES

Some conditions of Corollaries 1 or 2 are bilinear in decision variables δ, V, Λ, S, Z, ν_0 due to products between them. An interesting way to overcome the difficulty of directly solving BMI conditions consists in using relaxation schemes: that is, to fix some of the variables and to search the other ones. Notice that in this case the relations become linear. Moreover, the implicit objectives are to maximize the region of stability of the closed-loop system and/or the region of admissible disturbances. From Proposition 1, Corollaries 1 or 2, the region of stability associated to the closed-loop system (14) is the ellipsoid $\mathcal{E}(P,\gamma) = \{Y \in \Re^{d+3}; Y'PY \leq \gamma^{-1}\}$, whereas the set of admissible disturbances $\nu(n)$ is given through the positive scalar ν_0. By noting that the volume of the ellipsoid is proportional to $\sqrt{\det(\frac{P^{-1}}{\gamma})}$, it is then possible to maximize its size by minimizing the function $\log(\det(\gamma P))$. Hence, depending on the weight that we want to give to the set $\mathcal{E}(P,\gamma)$ or to the set of admissible disturbances $\nu(n)$, one can consider the following optimization problem: $\min\{\beta_1((d+3)\log(\gamma) + \log(\det(V)) - 2\log(\det(S))) + \beta_2\nu_0\}$ for which relations of Corollaries 1 or 2 are the constraints and the β_i's are tuning parameters.

Example 1: Consider an ATM network with a bandwidth of 100 Mbps and apply our controller to a switch located at a distance of 500 Km from the source; with these values of the parameters the round trip delay $d = d^f + d^b$ amounts to 5ms [4]. We assume that the state of the switch is updated every 200 cells (the length of each cell in the ATM networks is equal to 53 bytes), which corresponds to 0.848ms; therefore the delay in terms of computation cycles is $d = 6$. We fixed $Q_0 = 200$, $\mu_e = 10$. The controller guarantees a steady-state buffer occupancy $Q_e = 86$ and a region of admissible disturbances with $\nu_0 = 0.9$. Figure 1 shows the controller performance in the presence of a constant $\mu = \mu_e$ in terms of queue length (QL) and explicit rate (ER).
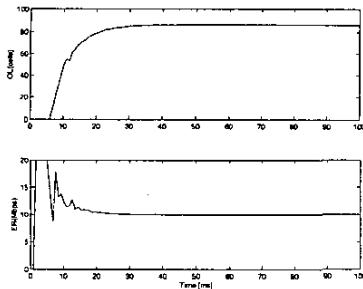

Fig. 1. Controller performance

VI. CONCLUSIONS

This paper presented a nonlinear, discrete-time model of an ABR/ATM switch and showed how queue saturation may be incorporated directly into the control design step. The resulting fixed-structure controller is simple, implementable, and robust.

VII. ACKNOWLEDGMENTS

VIII. REFERENCES

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