

Robust Mobile Robotic Formation Control Using Internet-Like Protocols

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Abstract—This work presents an Internet-Like Protocol (ILP) to coordinate the formation of n second-order agents in a two dimensional (2D) space. The trajectories are specified through via points and a desired formation at each point. A basis for the proof of convergence is given using Lyapunov second method. Simulink is used to verify the response of the agents for a variety of desired trajectories. The proposed algorithms are robust in the sense that they can accommodate changes in the formation of the agents and more importantly, changes in the number of agents as some of them drop of or join the formation.

I. INTRODUCTION

The coordination and formation of multiple agents is a problem of particular interest to numerous research groups [2], [8], [5], [1]. Applications of such research abound in space (satellite formation), military (remotely-operated clusters of vehicles) and civilian applications (teleautonomy). The problem of distributed coordination and control of such agents has been theoretically studied using various approaches. In [2], a graph-theoretic approach was presented to explain the behavior of n particles in the plane in an attempt to justify the model presented in [11], which had proposed a discrete-time model illustrating the heading alignments of the n particles. Graph theory was also utilized in [8] to define cost functions that govern the movement of the n systems/agents. In [5], virtual potentials were discussed as an analysis tool, while in [1], local sensing and minimal communication was the main focus of the research.

In this paper we present a different approach to the distributed control and coordination problem, inspired by the Internet congestion control protocols [9]. We formulate the coordination and control of various agents as a problem of competing for a common resource. Despite such selfish behavior, it has recently been shown [3] that all users share the resource proportionally and indirectly cooperate to maximize the global utility of all users. The supervisor of such behavior is a master which sets a price to be incurred by a user as a function of the resource usage and resource capacity, then transmits this price to the users. By doing so, all users receive the same feedback price, and the communication overhead

is significantly reduced. The purpose of this paper is to show exactly how such algorithms may be adopted to the coordination and control of physical agents, and in particular to the case of two-dimensional mobile agents. Moreover, we illustrate via simulation that such algorithms are robust to changes in the numbers of agents: if a particular robot (or a group of them) drops out of the formation or if others join the formation, the group continues on a stable trajectory. This is similar to the behavior of a network of computers which remains connected despite the fact that computers are dropping in an out of the network all the time.

II. THE ILP FORMATION COORDINATOR

In this section we discuss how Internet-Like Protocols (ILP) are adapted to our formation and coordination problem. In order to implement the formation coordinator we use the results of [4], [7] which were adopted in [9], to deal with n users sharing a resource of size C . The users update their resource usage according to a non-negative feedback signal called “price” of the resource, where a low price indicates resource availability while a high price reflects resource shortage. An equilibrium point is reached when the users share proportionally in the resource. These results may be applied when a group of users or agents is required to converge to a formation and follow a given trajectory in the plane.

For this particular application, we interpret the sum of positions in the plane as the resource for which the agents are competing. For the sake simplicity, we assumed that the axes in the plane can be decoupled and managed separately, and thus we use the system analyzed in [9] for each axis in the plane. Figure 1 shows a block diagram of the ILP coordinator for one axis. The main controller reads the positions of the agents in the corresponding axis, then computes and broadcasts the feedback price to all agents. In turn, each agent reads the feedback price and computes the next target position in the corresponding axis.

Let us focus our attention on the coordinator for one axis (x axis) of the plane, since the coordinator for the other axis (y axis) is basically the same. The state variable x_i represents the actual position for the i^{th} agent in the x axis, p_x is the feedback price for the x axis, a_{xi} is a constant parameter which defines the proportion of the resource assigned to the

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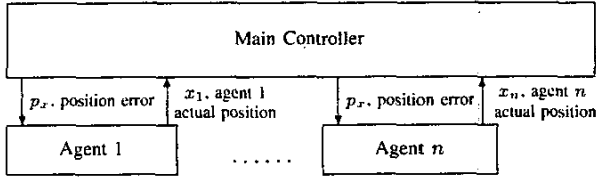


Fig. 1. Block diagram of one axis for the ILP coordinator.

agent. In fact, this proportion is the position on the x axis for this agent in the desired formation. The parameter a_{xi} also defines the speed of convergence to the equilibrium point, as we will show later.

The ILP coordinator then has the following structure (see [9]),

$$\begin{aligned} \dot{x}_i(t) &= -x_i(t)p_x(t) + a_{xi} & 1 \leq i \leq n \\ \dot{p}_x(t) &= \gamma_x \left[\sum_{i=1}^n x_i(t) - C_x \right] \end{aligned} \quad (1)$$

The resource C_x is the sum of the final positions of the agents in the x axis for the desired formation. The parameter γ_x is a positive constant which also defines the speed of convergence to the equilibrium point (formation). In addition, as shown in the linearization section in [9], γ_x specifies the root locus for two of the eigenvalues in the linearized system matrix. The value of γ_x should thus be properly selected to avoid overshoots in the response.

The equilibrium point of (1) is given by

$$x_i^* = \frac{a_{xi}}{\sum_{i=1}^n a_{xi}} \cdot C_x, \quad p_x^* = \frac{\sum_{i=1}^n a_{xi}}{C_x}. \quad (2)$$

As we can see from equation (2), the ratio $a_{xi}/\sum_{i=1}^n a_{xi}$ defines the proportion of resource C_x allocated to the i^{th} agent. We can obtain the same ratio by scaling the values of a_{xi} for all i . However, as shown in equation (12) of [9], the location of $n-1$ eigenvalues of the linearized system is defined by the ratio $\sum_{i=1}^n a_{xi}/C_x$. Thus, a larger value of a_{xi} results in a faster convergence.

In order to show the stability of the equilibrium point, we first translate the equilibrium point of (2) to the origin with the following change of variables

$$\begin{aligned} w_i(t) &= x_i(t) - x_i^*, & \text{for } 1 \leq i \leq n \\ w_m(t) &= p_x(t) - p_x^*, & m = n+1 \end{aligned} \quad (3)$$

Given that $\dot{w}_i(t) = \dot{x}_i(t)$, then

$$\begin{aligned} \dot{w}_i(t) &= -\left(w_i(t) + \frac{a_i C}{S}\right) \left(w_m(t) + \frac{S}{C}\right) + a_i \\ &\text{for } 1 \leq i \leq n \\ \dot{w}_m(t) &= \gamma \left[\sum_{i=1}^n \left(w_i(t) + \frac{a_i C}{S}\right) - C \right] \end{aligned} \quad (4)$$

where $S \triangleq \sum_{i=1}^n a_{xi}$. We have removed the subindex x to avoid confusion, with the understanding that the stability analysis applies for both axes. Simplifying,

$$\begin{aligned} \dot{w}_i(t) &= -\frac{S}{C}w_i(t) - w_i(t)w_m(t) - \frac{a_i C}{S}w_m(t) \\ &\text{for } 1 \leq i \leq n \\ \dot{w}_m(t) &= \gamma \left[\sum_{i=1}^n w_i(t) \right] \end{aligned} \quad (5)$$

Theorem 1: The system (5) is asymptotically stable for $p_x(t) > 0$.

Proof: To analyze the stability of system (5), we use the quadratic Lyapunov function

$$V(w) = \frac{1}{2}w^T P w \quad (6)$$

where $P > 0$ is an $m \times m$ diagonal matrix. Then $V(w) > 0$ for $w \neq 0$, and $V(0) = 0$. Taking the time derivative of $V(w)$ we obtain

$$\dot{V}(w) = \frac{1}{2} [\dot{w}^T P w + w^T P \dot{w}] = \sum_{i=1}^m w_i P_i \dot{w}_i \quad (7)$$

Expanding the terms

$$\begin{aligned} \dot{V}(w) &= \sum_{i=1}^n \left(-\frac{S}{C}P_i w_i^2(t) - P_i \frac{a_i C}{S} w_i(t) w_m(t) \right. \\ &\quad \left. - P_i w_i^2(t) w_m(t) + \gamma P_m w_i(t) w_m(t) \right) \end{aligned} \quad (8)$$

We can cancel out the cross product terms by choosing $P_i = \frac{S\gamma}{a_i C}$, for $1 \leq i \leq n$, and $P_m = 1$, thus simplifying

$$\dot{V}(w) = -\sum_{i=1}^n w_i^2(t) \left(\frac{S\gamma}{a_i C} w_m(t) + \frac{S^2\gamma}{a_i C^2} \right) \quad (9)$$

In order to ensure that $\dot{V}(w) < 0$ we need the term inside the parentheses to be positive, leading to

$$\frac{S\gamma}{a_i C} w_m(t) + \frac{S^2\gamma}{a_i C^2} = \frac{S\gamma}{a_i C} \left(w_m(t) + \frac{S}{C} \right) > 0 \quad (10)$$

By definition, a_i , γ , $S = \sum_{i=1}^n a_i$, and C are positive, and since

$$w_m(t) + \frac{S}{C} = p_x(t) \quad (11)$$

then, the system (5) is asymptotically stable for $p_x(t) > 0$. \square

As we can see from the previous proof, the agents will converge asymptotically to the desired formation. By properly selecting the parameter γ_x , the convergence may be made faster, by just avoiding the undesired overshoot. A more detailed proof of stability in the presence of time delay in the transmission of the signals is presented in [9].

III. AGENT DROPPING/ADDING ROBUSTNESS

A. Dropping one agent

First we present the case when the j^{th} agent suddenly stops moving and therefore is unable to complete the formation. Let (x_d, y_d) the point in the plane where the agent stopped moving. Then the new equilibrium point for the x axis will be at:

$$\dot{x}_i(t) = 0 = -x_i(t)p_x(t) + a_{xi}, \quad x_i^* = \frac{a_{xi}}{p_x^*} \quad \text{for } i \neq j$$

$$\dot{p}_x(t) = 0 = \gamma_x \left[\sum_{i=1}^n x_i(t) - C_x \right], \quad \sum_{i=1}^n x_i^* = C_x \quad (12)$$

Expanding the second equation of (12) with the substitution of the first equation, yields

$$\sum_{i=1}^n x_i^* = C_x = \left[\sum_{i=1}^n a_{xi} - a_{xj} \right] \cdot \frac{1}{p_x^*} + x_d \quad (13)$$

where we have substituted x_d as the equilibrium or final position for the j^{th} agent, then solving for p_x^* we get

$$p_x^* = \frac{\sum_{i=1}^n a_{xi} - a_{xj}}{C_x - x_d} \quad (14)$$

Substituting (14) in the first equation of (12), results

$$x_i^* = \frac{a_{xi}(C_x - x_d)}{\sum_{i=1}^n a_{xi} - a_{xj}} \quad (15)$$

Considering $C_x^- = C_x - x_d > 0$ and $S^- = \sum_{i=1}^n a_{xi} - a_{xj} > 0$, we can rewrite (15) and (14) as

$$x_i^* = \frac{a_{xi}C_x^-}{S^-} \quad \text{for } i \neq j, \quad \text{and} \quad x_j^* = x_d$$

$$p_x^* = \frac{S^-}{C_x^-} \quad (16)$$

We can see from (16) that the equilibrium condition for all the agents but the j^{th} , is affected by the same factor, maintaining the formation shape. However, the formation will suffer a slight scaling, contraction or expansion, according to the initial positions of the agents. Also the equilibrium condition of (16) is very similar to the one in (2), such that the the proof of convergence for the algorithm with one agent dropping is straightforward and presented in detail in [10]. The new equilibrium condition for the y axis can be obtained in a similar fashion.

B. Adding one agent

The other case is when a j^{th} agent joins the formation, where $j = n + 1$. The equilibrium point is now

$$\dot{x}_i(t) = 0 = -x_i(t)p_x(t) + a_{xi}, \quad x_i^* = \frac{a_{xi}}{p_x^*} \quad \text{for } 0 \leq i \leq j$$

$$\dot{p}_x(t) = 0 = \gamma_x \left[\sum_{i=1}^n x_i(t) + x_j(t) - C_x \right], \quad \sum_{i=1}^n x_i^* + x_j^* = C_x \quad (17)$$

Expanding the second equation of (17) with the substitution of the first equation, yields

$$\sum_{i=1}^n x_i^* + x_j^* = C_x = \left[\sum_{i=1}^n a_{xi} + a_{xj} \right] \cdot \frac{1}{p_x^*} \quad (18)$$

solving for p_x^* we get

$$p_x^* = \frac{\sum_{i=1}^n a_{xi} + a_{xj}}{C_x} \quad (19)$$

Substituting (19) in the first equation of (17), results

$$x_i^* = \frac{a_{xi}C_x}{\sum_{i=1}^n a_{xi} + a_{xj}} \quad (20)$$

Considering $S^+ = \sum_{i=1}^n a_{xi} + a_{xj}$, we can rewrite (20) and (19) as

$$x_i^* = \frac{a_{xi}C_x}{S^+} \quad \text{for } 0 \leq i \leq j$$

$$p_x^* = \frac{S^+}{C_x} \quad (21)$$

Comparing equation (21) with equation (2), we can see that the equilibrium positions of all the agents are scaled by the same factor, and the formation will then keep the shape but will suffer a slight contraction. The convergence proof for the algorithm with the addition of one agent is also presented in [10]. The extension for the y axis is also straightforward. The next section shows simulation results of both situations analyzed in this section.

IV. SIMULATION RESULTS

A. Dropping one agent

In this subsection we will show first the Simulink simulation of one agent dropping the formation. The commanded formation consists of six agents completing a triangle-like shape. Figure 2 shows the results when the agent in the right vertex of the triangle removes itself from the formation, and the agents were commanded to complete the formation in 20 seconds. Five seconds after the start of the trajectory, the agent is forced to stay in its current position. Despite the loss of one agent, the remaining agents keep the formation and complete the trajectory in the commanded time.

B. Adding one agent

Figure 3 shows the simulation when a new agent joins an original wedge formation with five agents. The new agent takes the center position in the wedge, changing the formation to a triangle formation, and the agents are commanded to reach the final formation in 20 seconds. The joining takes place ten seconds after the start of the trajectory. Despite the addition of a new agent, the other agents keep the formation and complete the trajectory in the commanded time.

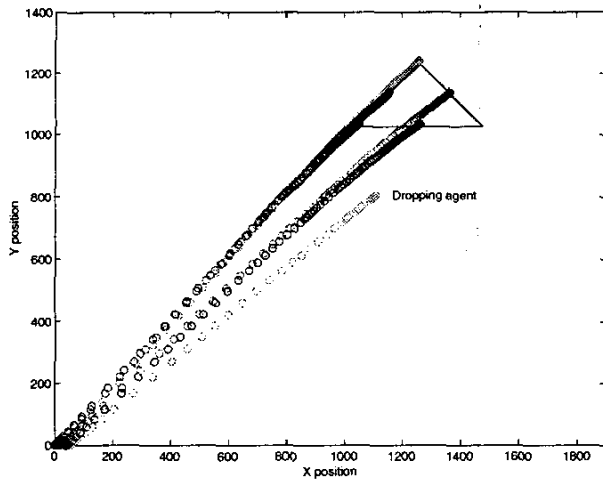


Fig. 2. Six agents completing a triangle formation, with the removal of one agent in the middle of the trajectory.

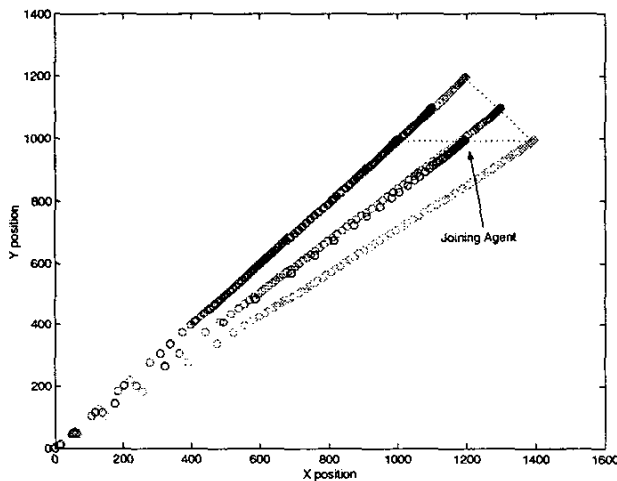


Fig. 3. Five agents completing a wedge formation, with the addition of a new agent the middle of the trajectory.

V. CONCLUSIONS

The proposed algorithm shows stability of convergence when all the agents are able to complete the commanded formation. The algorithm is robust in the sense of maintaining the shape of the formation when one agent is dropping the formation. Also the algorithm shows robustness when one agent is joining a previously commanded formation. The proof of convergence can be easily extended to the case of multiple drops or additions. The algorithm then presents an interesting alternative to the problem of coordination of multiple agents, especially when the communication channel is limited, given that all the agents receive the same feedback

signal, and need not communicate amongst each other.

Our current work focuses on implementing our coordination algorithm on a number of indoor mobile robots specifically modified for this purpose.

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