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ROBUST ADAPTIVE CONTROL FOR A CLASS OF PARTIALLY KNOWN NONLINEAR SYSTEMS

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ABSTRACT

This paper deals with a class of nonlinear systems described by the equation $M(q)\ddot{q} + F(q,\dot{q})\dot{q} + G(q)q = f(t)$ where f(t) is the control input. An adaptive controller is developed that takes advantage of the structure and any known dynamics of the system in order to increase speed of adaptation and relax the conditions required for convergence.

The control design method has two stages. First, the known dynamics are separated out and used to perform a global linearization on the nonlinear system. Second, a model-reference adaptive control, based on the Lyapunov stability criterion, is designed for the remaining unknown portion of the plant. This control scheme is shown to relax several assumptions usually made in applying adaptive control to a manipulator system. For instance, it relaxes the common assumption that the time-varying plant is close to the desired model.

I. INTRODUCTION

The objective of this paper is to design a robust adaptive controller for nonlinear systems described by

$$M(q)\ddot{q} + F(q,\dot{q})\dot{q} + G(q)q = f \tag{1.1}$$

where M(q) is an nxn inertia matrix (symmetric, positive definite), $F(q,\dot{q})$ is an nxn matrix containing the centrifugal and coriolis terms, G(q) is an nx1 vector containing the gravity terms, q(t) is an nx1 joint variable vector, and f(t) is an nx1 input vector. Equation (1.1) describes robot manipulators in the Lagrange-Euler formulation [6].

This nonlinear dynamic equation includes time-varying and uncertain terms. To control such systems, many model-reference adaptive schemes have been introduced. The convergence of such controllers usually depends (e.g.[3,4]) on assuming a slowly time-varying plant that is "close" to the desired model.

In this paper, we attempt to relax such assumptions by separating the plant dynamics into a known part and an unknown part, and by applying a modified adaptive scheme.

II. PROBLEM FORMULATION

Let a system, described by equation (1.1), have some known and some unknown plant dynamics so that we may write

$$M = M_k + \underline{M}_u = M_k(I + M_k^{-1}\underline{M}_u) \equiv M_k M_u,$$

$$G = G_k + G_u,$$

$$F = F_k + F_u, \tag{2.1}$$

where subscript k stands for the known part and subscript u stands for the unknown part. Assume M_u and M_k are both invertible. Note that we are able to deal with both additive and multiplicative uncertainties in M. By substituting (2.1) into (1.1), one gets

$$M_k M_u \ddot{q} + F_u\dot{q} + G_u q = f - F_k\dot{q} - G_k q,$$

or

$$M_u\ddot{q} + M_k^{-1}F_u\dot{q} + M_k^{-1}G_u q = u, \tag{2.2}$$

where

$$u = M_k^{-1}(f - F_k\dot{q} - G_k q). \tag{2.3}$$

The expression in (2.3) reduces to the global linearization described in [5] when M, F, and G are perfectly known. In our case, it is a one-to-one input transformation in terms of the known parameters so that, given u(t), the input f(t) of (1.1) may be recovered by

$$f = M_k u + F_k\dot{q} + G_k q. \tag{2.4}$$

Define $x^T = [q^T \quad \dot{q}^T]$ to obtain

$$\dot{x} = \begin{bmatrix} 0 & I \\ -M_u^{-1}M_k^{-1}G_u & -M_u^{-1}M_k^{-1}F_u \end{bmatrix} x + \begin{bmatrix} 0 \\ M_u^{-1} \end{bmatrix} u \tag{2.5}$$

Note that when the plant is completely known so that $M=M_k$, $F=F_k$, and $G=G_k$, we have that $M_u=I$, $F_u=0$, and $G_u=0$. Then, system (2.5) reduces to the set of n decoupled double integrators whose robust control was analyzed in [1].

At this point, the problem of determining f(t) in (1.1) has been reduced to determining u(t) in (2.2), or equivalently in (2.5). To accomplish this, we proposed the following adaptive scheme.

Let a reference model be given by

$$\dot{x}_d = A_m x_d + B_m v \tag{2.6}$$

where

$$A_m = \begin{bmatrix} 0 & 0 \\ -K_1 & -K_2 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{2.7}$$

K_1 is an nxn diagonal matrix with terms equal to w_i^2, and K_2 is an nxn diagonal matrix with terms equal to $2\delta_i w_i$ [4]. The natural frequency, w_i, and the damping coefficient, δ_i, are chosen to give desired transient and steady-state behavior.

In order to follow a desired trajectory $q_d(t)$, we may define $x_d^T = [\, q_d^T \ \dot{q}_d^T \,]$. If the error between the desired and actual trajectories is defined as

$$e = x_d - x, \tag{2.8}$$

the error dynamics will be given by

$$\dot{e} = A_m e + (A_m - A)x - Bu + B_m v. \tag{2.9}$$

Note that by using (2.6) and (2.7), the required reference input v(t) for (2.6) can be obtained from the desired trajectory using

$$v = \ddot{q}_d + K_2\dot{q}_d + K_1 q_d. \quad (2.10)$$

The control objective is to make the error e(t) vanish asymptotically. To this end, we propose the adaptive control law

$$u = u_k + u_a, \quad (2.11)$$

where

$$u_k = -[K_1 \quad K_2]x + v \quad (2.12)$$

with v(t) given by (2.10), and the adaptive portion of the control is

$$u_a = -[\triangle_1 \quad \triangle_2]x + [\triangle_v]v. \quad (2.13)$$

The gains $\triangle_i$ and $\triangle_v$ are adaptive gains to be chosen using a Lyapunov approach.

III. ADAPTIVE CONTROLLER DESIGN

In this section, we shall see that if some of the dynamics are known, and are removed from system (1.1) by the transformation (2.3), the resulting model-reference adaptive controller (MRAC) is simpler to find and implement. In particular, the known parameters are not required to be slowly-varying, and the frequency content of the control signal can be reduced, since fewer parameters are being identified. This approach is similar in scope to that described in [2], but differs in the use of the linearizing transformation (2.3) and in the structure of the adaptive controller. In particular, we do not attempt to directly estimate the plant's parameters, since our main goal is only to drive the trajectory error to zero. In [4] a similar adaptive controller was presented, but it was designed under the assumption that the slowly time-varying plant is "close" to the desired time-invariant model. Our control scheme relaxes this assumption.

To find the adaptive portion of the control scheme, the direct method of Lyapunov is used. This method permits one to predict sufficient conditions for stability of the system which, as a rule, are more rigid than necessary.

To obtain the error dynamics, the proposed control law (2.11) is substituted into (2.9), yielding

$$\dot{e} = A_m e + B_a x + G_a v \quad (3.1)$$

where A_m is defined in equation (2.7) and

$$G_a = \begin{bmatrix} 0 \\ I - M_u^{-1}(I+\triangle_v) \end{bmatrix}. \quad (3.2)$$

$$B_a = \begin{bmatrix} 0 & 0 \\ M_u^{-1}(M_k^{-1}G_u+K_1+\triangle_1)-K_1 & M_u^{-1}(M_k^{-1}F_u+K_2+\triangle_2)-K_2 \end{bmatrix} \quad (3.3)$$

If the system is completly known (i.e. $M_u=I$, $F_u=0$, $G_u=0$, $\triangle_1=0$, $\triangle_2=0$, $\triangle_v=0$), then (3.1) becomes

$$\dot{e} = A_m e. \quad (3.4)$$

Then this scheme reduces to the computed-torque design [1],[6].

Partition e(t) conformably with equation (3.1) as $e^T = [\, e_1^T \;\; e_2^T]$. The control problem is then to find an adaptation law such that

$$\lim_{t\to\infty} e(t) = 0. \quad (3.5)$$

<u>Theorem 3.1</u>

Define a filtered error as

$$w(t) = P_2 e_1 + P_3 e_2 \quad (3.6)$$

where

$$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix} \quad (3.7)$$

is the positive definite solution to the Lyapunov equation

$$A_m^T P + P A_m = -Q \quad (3.8)$$

with $Q > 0$.

Then, the closed-loop error system (2.9) is asymptotically stable using the control given by equations (2.11)-(2.13) if the adaptive gains are adjusted as

$$\dot{\triangle}_1 = -awq^T$$
$$\dot{\triangle}_2 = -aw\dot{q}^T \quad (3.9)$$
$$\dot{\triangle}_v = bwv^T,$$

where a and b are positive scalars.

<u>Proof</u>:

Select the Lyapunov function candidate

$$L = e^T Pe + tr(B_a^T NF_A B_a) + tr(G_a^T NF_B G_a), \quad (3.10)$$

with $P > 0$, and

$$N = \begin{bmatrix} M_u & 0 \\ 0 & M_u \end{bmatrix}. \quad (3.11)$$

F_A and F_B are weighting matrices to be specified later.

Differentiating both sides of eq.(3.10) with respect to time, we obtain

$$\dot{L} = e^T(A_m^T P + PA_m)e + 2trB_a^T(Pex^T + F_A N\dot{B}_a) + 2trGeT(Pev^T + F_B N\dot{G}_a). \quad (3.12)$$

For the first term of $\dot{L}$ to be negative definite , choose P to satisfy (3.8). The second and the third terms will be identically equal to zero if one chooses the adaptation laws

$$\dot{B}e = -N^{-1}F_A Pex^T \quad (3.13)$$

$$\dot{G}_a = -N^{-1}F_B ev^T, \quad (3.14)$$

where the adaptation gain matrices are chosen as

$$F_A = a\begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \quad F_B = b\begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \quad (3.15)$$

with a>0 and b>0 scalar gains.

By taking the derivative of equations (3.2) and (3.3) with respect to time and assuming the unknown portion of the plant is changing slowly (i.e. $\dot{M}_u=0$, $\dot{G}_u=0$, $\dot{F}_u=0$), we have

$$\dot{B}_e = \begin{bmatrix} 0 & 0 \\ M_u^{-1}\dot{\triangle}_1 & M_u^{-1}\dot{\triangle}_2 \end{bmatrix} \qquad (3.16)$$

$$\dot{G}_e = \begin{bmatrix} 0 \\ -M_u^{-1}\dot{\triangle}_v \end{bmatrix} \qquad (3.17)$$

By using (3.16) and (3.17) in (3.13) and (3.14) respectively, the adaptation laws in equation (3.9) are obtained. ∎

If the fast dynamics are known, they may removed from the plant description using the transformation (2.3). Then the unknown part will change slowly compared to the adaptation mechanism. For example, M_u can include the unknown constant payload (i.e. $\dot{M}_u=0$), while M_k contains the known arm inertia terms, or F_u can include the unknown dynamic friction coefficients, while F_k contains the known coriolis and centripetal terms.

Note that the states of (1.1) are the same as the states of (2.5). Therefore, the trajectory error in (1.1) is the same as in (2.5). Since the poles of the model are specified by K_1 and K_2, our approach yields trajectory following with a desired degree of stability.

IV. ROBUSTNESS ANALYSIS

Although M_k, F_k, and G_k in (2.4) are assumed known, they may have uncertain or inaccurate entries, or it may be desirable to use simplified values for these quantities in the control law. In particular, their calculated, or assumed, values M_c, F_c and G_c could be constants, or else updated only every few samples to save computation time. Then, the calculated control law f_c actually used will be different from the one found when M_k, F_k and G_k are completely known.

The effect of applying f_c, instead of (2.4), to the physical system can be analyzed. Let the calculated control be given by

$$f_c = M_c u + F_c \dot{q} + G_c q. \qquad (4.1)$$

In [1], it is shown, for the case of a completely known system (i.e. $M_u=I$, $F_u=0$, $G_u=0$), how to use information on the structured uncertainties in M, F, and G to carry out a robustness analysis associated with the global linearization (2.3). The approach uses a Lyapunov equation approach in the time domain [7] and the total stability theorem [10] to provide practically meaningful bounds on $|M_c-M_k|$, $|F_c-F_k|$ and $|G_c-G_k|$ for guaranteed closed-loop stability. We plan to extend these results to the case of some unknown dynamics.

On the other hand, one could also carry out a robustness analysis associated with the adaptive portion (2.13) of the proposed control scheme. That is, if $\dot{M}_u$, $\dot{F}_u$, and $\dot{G}_u$ are not exactly zero, the proposed scheme will still work if they are "small enough". Indeed, we should be able to find bounds on the error in (3.1) in terms of the norms of $\dot{M}_u$, $\dot{F}_u$, $\dot{G}_u$, and the desired acceleration $\ddot{q}_d(t)$.

A future publication will provide a complete analysis of these two effects.

V. CONCLUSION

This paper proposes a control scheme which takes advantage of the structure and any known dynamics of a nonlinear system to increase speed of adaptation and relax the conditions required for convergence. The known dynamics are separated out and used to perform a global linearization on the nonlinear system. Then, the adaptive portion of the control scheme uses a modified Lyapunov function to derive adaptation laws which are not dependent on the usual assumption that the time-varying plant is "close" to the desired time-invariant model.

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