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EXPERIMENTAL IMPLEMENTATION OF
MRAC FOR A DC SERVO MOTOR

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ABSTRACT

This paper describes a hardware implementation of adaptive controllers using a Digital Signal Processor (TMS320C25). The research explores two model reference adaptive control schemes and some of the practical problems associated with their implementation.

1 Introduction

Our research explores model reference adaptive control (MRAC) schemes in a real system. The motivation being that although the theory of adaptive control is highly developed, its practice has been limited [3]. We successfully implemented two MRAC schemes to control a DC servo motor using a digital signal processor. In addition, we studied the following practical aspects of adaptive control:

1. The existence of "*Limit Cycles*" in the adaptive gains.

2. The influence of the γ modification.

3. The influence of the external input.

Other effects were also studied as described in [1] but are not included in this short paper. Additionally, we derived and implemented a new controller that adjusts the γ in the application of the γ *modification* [3].

2 Hardware Implementation

The overall system - plant, controller, and display block - is shown in Figure 1. The plant is a DC servo motor. The controller includes
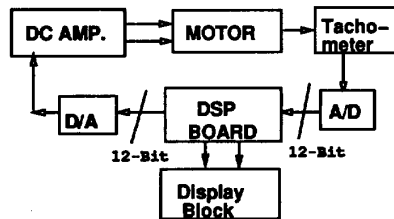

Figure 1: Block Diagram of the Physical System

the hardware (A/D, DSP board D/A, DC power amplifier, etc.) and the software that implements the adaptive controllers. The display block contains the parallel port, D/As, oscilloscopes, etc. and the software necessary to display some signals in real time [1].

3 Software Implementation of the controllers

The TMS320C25 is a fixed-point microprocessor so that in our experiments filters, difference equations, sine wave generations and random sequences are implemented using Q formats [2].

In our experiments, the plant is given by $K_p'/(s + P_p)$, the desired model is $K_m/(s + P_m)$, the output of the plant is y_p, that of the model is y_m, the input to the plant is x_p and that of the model is x_m. The error is defined as $e_1 = y_p - y_m$. All assumptions for the existence of a stable adaptive controller as described in [3] are satisfied. Equation (1) constitute the adaptive law to adjust the proportional gain [3] for case 1, where only the high frequency gain is unknown: Equation (2) is the discrete version of this law, obtained
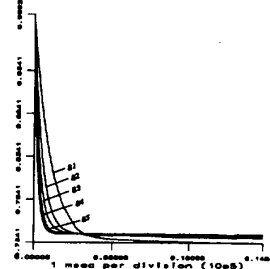

Figure 2: kt vs *time* when $\gamma = 10, 20, 30, 40, 50$

Figure 3: kt vs *time* when $\gamma = 20, 30, 50$

using the backward approximation [4], where h is the sampling time,

$$\dot{kt}(t) = -\gamma \cdot sgn(K_p) \cdot e_1(t) \cdot x_m(t) \tag{1}$$

$$kt(i) = -\gamma \cdot sgn(K_p) \cdot h \cdot e_1(i) \cdot x_m(i) + kt(i-1) \tag{2}$$

Equations (3) and (4) represent the adaptive laws of case 2 where both the high frequency gain and the pole are unknown [3]. Equations (5), and (6) are the discrete time implementations.

$$\dot{kt}(t) = -\gamma sgn(K_p) \cdot e_1(t) \cdot x_m(t) \tag{3}$$

$$\dot{th}(t) = -\gamma sgn(K_p) \cdot e_1(t) \cdot y_m(t) \tag{4}$$

$$kt(i) = -\gamma sgn(K_p) h e_1(i) x_m(i) + kt(i-1) \tag{5}$$

$$th(i) = -\gamma sgn(K_p) h e_1(i) y_p(i) + th(i-1) \tag{6}$$

4 Experimental Results

4.1 K_p Unknown case

The first set of experiments were designed using equation (2) to study the effect of γ. We found that γ has a dual function in our experiments; it makes the convergence possible by eliminating the limit-cycle effect due to truncation, and it increases the convergence speed of the gains. Figure 2 shows the convergence of the gain K_p for different γs. There is an important trade-off in the use of γ; A large γ produces large oscillations of the adaptive parameter(s). Figure 3 shows the oscillations around the final values for 3 different γ's.

The other important study performed using the K_p unknown case examined the influence of the input on the convergence of kt. Figure 4 shows the convergence of kt for 6 different step inputs. A
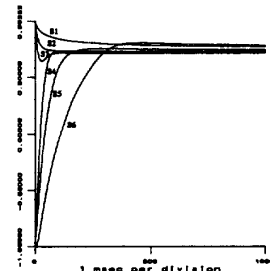

Figure 4: kt vs *time* [$S1$: $kt = 1, x_m = -2.5$ $S2$: $kt = 1, x_m = -5$ $S3$: $kt = 1, x_m = -7.5$ $S4$: $kt = -1, x_m = -2.5$ $S5$: $kt = -1, x_m = -5$ $S6$: $kt = -1, x_m = -7.5$]
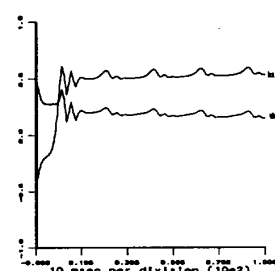
Figure 5: A Close Look at kt and th

larger magnitude step is shown to produce a faster convergence of the adaptive parameter.

4.2 K_p and P_p Unknowns Case

In the second case, two adaptive gains were adjusted using equations (5) and (6). The input to the model used in this study is a sinusoidal wave form. The amplitude of this sinusoid was adjusted experimentally to make the convergence of kt and th possible. The DC value of the sinusoid was set to avoid an input which crossed the level of 0 *Volts* in order to avoid the nonlinear region of the motor [1]. One of the models used to test the controller (*model*1) was :

$$G_m(s) = \frac{100}{s + 100}, \qquad (7)$$

During the first second of operation, the controller adjusted a large portion of the parameters (kt, and th). Figure 5 shows 1 second of the trajectory of kt and th, and Figure 6 shows the first 600 *msecs* of the trajectories of y_m, y_p, and e_1.

An effect similar to the one obtained by updating only K_p was observed in this case. The speed of convergence is related to the magnitude of γ, as is the oscillatory behavior of the parameters. Figure 7 provides a close look at the oscillations in one of the parameters after 20 *secs* of operation of the controller. The curve C2k1 corresponds to a γ of 50, C2k2 corresponds to a γ of 100, and C2k3 corresponds to a γ of 200. Increasing the convergence speed has one drawback; it creates more oscillations in the gains.
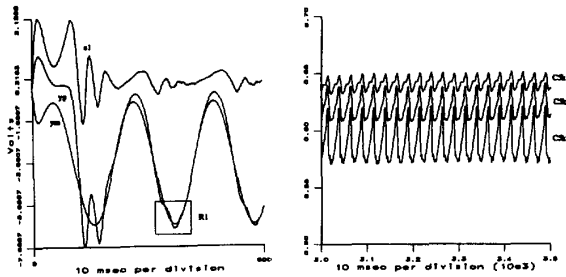

Figure 6: A Close Look at y_m, y_p, and e_1 Figure 7: Close Look of kt oscillations for $\gamma = 50, 100, 200$

4.3 Adjusting γ

The (γ) modification increases the speed of convergence significantly, but it produces an oscillatory behavior around the final value of the adaptive gains. A filter was designed to adjust γ as a function of the output error e_1. Figure 8 shows a block diagram of this filter. The first block represents a low-pass filter followed by a rectifier that finds the absolute value of the filter output. This value is multiplied by a constant term

The new controller runs for 18 *secs* using a fixed γ as the previous controller did. Then the subroutine that implements the γ filter is called every 500 *μsecs* and the process of adjusting the factor γ begins. The results obtained by using this controller can be observed
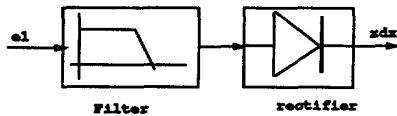

Figure 8: Adjustment of γ esquematic block

in Figure 9 ($kt[0] = -0.5$, $th[0] = 0.5$ and initial $\gamma = 200$). The trajectories that the parameters follow in both plots is similar. The gains kt and th converge to constant values, and the oscillation in the gains disappear.
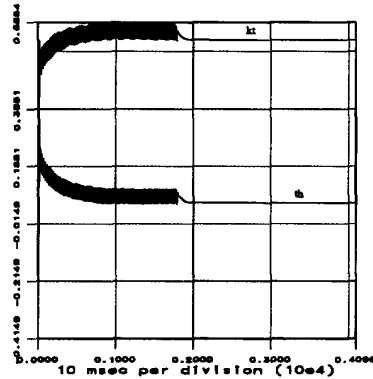

Figure 9: Parameter Trajectories for $kt[0] = -0.5$ and $th[0] = 0.5$

5 Conclusions

The effect of finite register length in digital signal processing is an important factor to consider when a controller is implemented using

a fixed-point microprocessor. In our case, the truncation that occurs when a 32-bit hardware register is stored in a 16-bit memory location, is one of the main factors of the *limit cycle* present in the convergence of the adaptive gains. The solution to this problem was to use the γ-*modification*. The γ *modification* which has commonly been used to increase the speed of convergence in computer simulation, actually has a dual function in our experiments; it makes the convergence possible, and it can be used to increase the convergence speed of the gains. A larger γ produces a faster convergence of the parameters, but unfortunately, the adaptive gains oscillate around the expected final value.

A special controller was designed and implemented to adjust γ as a function of the output error e_1. This controller reduces the oscillations of the adaptive gains around the final value.

Finally, the magnitude of a step input was shown to have an important influence on the time of convergence of the adaptive gain.

References

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