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Nonlinear Observer Design Using Dynamic Recurrent Neural Networks

*Young H. Kim, *Frank L. Lewis, and **Chaouki T. Abdallah

*Automation and Robotics Research Institute
The University of Texas at Arlington
7300 Jack Newell Blvd. South
Fort Worth, TX 76118-7115
e-mail. ykim@arri.uta.edu

**Department of Electrical and Computer Engineering
The University of New Mexico
Albuquerque, NM 87131
e-mail. chaouki @rachana.eece.unm.edu

Abstract

A nonlinear observer for a general class of single-output nonlinear systems is proposed based on a generalized Dynamic Recurrent Neural Network (DRNN). The Neural Network (NN) weights in the observer are tuned on-line, with no off-line learning phase required. The observer stability and boundness of the state estimates and NN weights are proven. No exact knowledge of the nonlinear function in the observed system is required. Furthermore, no linearity with respect to the unknown system parameters is assumed. The proposed DRNN observer can be considered as a universal and reusable nonlinear observer because the same observer can be applied to any system in the class of nonlinear systems.

1 Introduction

Ever since the introduction of the Luenberger observer [10], there have been many papers devoted to the subject of nonlinear observers. Most of the early attempts were based on extending the linear methodology *through various kinds of linearization techniques*. A survey of these results can be found in [14] and [20].

The first nonlinear adaptive observer was proposed in [2] based on certain coordinate transformations and an auxiliary filter. Marino [11] presented a simple but restricted observer based on the satisfaction of strict positive real (SPR) conditions. A global adaptive observer for a class of single-output nonlinear systems which are linear with respect to an unknown constant parameter vector was presented in [12]. Recently, adaptive observers with arbitrary exponential rate of convergence were considered by Marino and Tomei [13]. However their assumption of linearity with respect to any unknown system parameters and their conditions on transforming the original system into special canonical form are not often met for many physical systems.

Neural networks (NN) have been used for approximation of nonlinear systems, for classification of signals, and for the associative memory. For control engineers, the function approximation capability of NN is usually used for system identification or identification-based ("indirect") control. In [19], a state estimator has been designed for use with Radial Basis Function Neural Networks. Recently, Levin and Narendra [7] has addressed the problem of estimating unknown system states for certain discrete-time nonlinear systems. An "off- line" trained feedforward NN is employed to generate the estimated states. However, very little is known about the use of NN for designing an on-line NN observer with stability proven for a prescribed class of nonlinear systems.

In this paper a generalized DRNN is used for designing a nonlinear observer. A feedforward NN is inserted in the feedback path to capture the nonlinear characteristics of the observer system. We will show that the state estimation errors are suitably small and the NN weight parameter errors are bounded.

Compared with other NN techniques, the NN weights are tuned on-line, with no off-line learning phase required. Comparedwith other adaptive observer methods, a nonlinear state-space transformation of the nonlinear system is not required, and a general class of nonlinear systems is considered. The "output matching" condition [6] is not required. Of course, no exact knowledge of the *function or functional in the system is required.*

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2 Preliminaries

We define the norm of a vector $x \in R^n$ and a matrix $A \in R^{m\times n}$

$$\|x\| = \sqrt{x^T x}\,,\ \ \|A\|_s = \sqrt{\lambda_{max}[A^T A]} = \sigma_{max}[A] \quad (2.1)$$

where $\lambda_{max}[\cdot]$ and $\lambda_{min}[\cdot]$ are respectively the maximum and minimum eigenvalue.

Given $A=[a_{ij}]$ and $B \in R^{m\times n}$, the Frobenius norm is defined by

$$\|A\|_F^2 = tr(A^T A) = \sum a_{ij}^2 \quad (2.2)$$

with $tr(\cdot)$ the trace. The associated inner product is $< A,B >_F = tr(A^T B)$.

The norm $\|x\|_2^\alpha$ with $x(t) \in R^n$ is defined as [5]

$$\|x\|_2^\alpha = (\int_0^t e^{-\alpha(t-\tau)} x^T(\tau) x(\tau) d\tau)^{1/2}\,. \quad (2.3)$$

2.1 Stability of Systems

Consider the nonlinear system

$$\dot{x} = f(x,t)\,,\ y = h(x,t) \quad (2.4)$$

with state $x(t) \in R^n$. We say the solution is *uniformly ultimately bounded (UUB)* if there exists a compact set $U \subset R^n$ such that for all $x(t_0) = x_0 \in U$, there exists an $\varepsilon > 0$ and a number $T(\varepsilon, x_0)$ such that $\|x(t)\| < \varepsilon$ for all $t \geq t_0 + T$ [8].

2.2 Nonlinear Plant, Observer, and Error Dynamics

Definition 2.1: The linear system

$$\begin{aligned} \dot{z} &= Az, & z &\in R^n, \\ y &= C^T z, & y &\in R \end{aligned} \quad (2.5)$$

is said to be in observer canonical form if A and C are given by

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$$A=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ & & \vdots & & \\ 0 & 0 & \cdots & 1 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},\quad C=\begin{bmatrix}1\\0\\ \vdots\\0\\0\end{bmatrix}. \tag{2.6}$$

Consider the class of single-input single-output (SISO) nonlinear plants with (A, C) in observer canonical form given by

$$\dot{x}=Ax+b[f(x)+g(x)u+d(t)] \qquad y=C^T x \tag{2.7}$$

with $x\in R^n$, $y\in R$, $u\in R$, $b\in R^n$, $d(t)$ the unknown disturbances with a known upper bound b_d, and $f, g: R^n\to R$ unknown smooth functions. Vector b is general and not in a restricted form; if $b=[0\cdots 1]^T$, then the system (2.7) is said to be in *Brunovsky canonical form* [8]. We assume $f(x)$ and $g(x)$ contain parameter uncertainties which are not necessarily linear parameterizable. Only output y is assumed to be measurable. Note that nonlinearities of functions $f(x)$ and $g(x)$ depend on the system state x and output y.

A nonlinear observer for the states in (2.7) is

$$\begin{aligned}\dot{\hat{x}}&=A\hat{x}+b[\hat{f}(\hat{x})+\hat{g}(\hat{x})u-v(t)]+K(y-C^T\hat{x})\\ \hat{y}&=C^T\hat{x}\end{aligned} \tag{2.8}$$

where $\hat{x}$ denotes the estimates of the state x and $K=[k_1\ k_2\ \cdots\ k_n]^T$ is the observer gain matrix chosen so that the characteristic polynomial of $A+KC^T$ is strictly Hurwitz. The functions $\hat{f}(\hat{x})$ and $\hat{g}(\hat{x})$ are estimates of $f(x)$ and $g(x)$ respectively. The robustifying term $v(t)$ is a function that provides robustness in the face of bounded disturbances.

Defining the state estimation error as $\tilde{x}=x-\hat{x}$, we obtain the estimation error dynamics

$$\begin{aligned}\dot{\tilde{x}}&=(A+KC^T)\tilde{x}+b[\tilde{f}+\tilde{g}u+d(t)+v(t)]\\ \tilde{y}&=C^T\tilde{x}\end{aligned} \tag{2.9}$$

where the functional estimate errors $\tilde{f}$, $\tilde{g}$ are given by

$$\tilde{f}=f(x)-\hat{f}(\hat{x}),\qquad \tilde{g}=g(x)-\hat{g}(\hat{x}). \tag{2.10}$$

The output estimation error $\tilde{y}=y-\hat{y}$ is given by

$$\tilde{y}=W(s)[\tilde{f}+\tilde{g}u+d(t)+v(t)] \tag{2.11}$$

where s denotes the differential operator d/dt. The linear transfer function $W(s)$ is realized using standard techniques by the triple ($A+KC^T$, b, C). It has the form

$$W(s)=\frac{b_1 s^{n-1}+\cdots+b_{n-1}s+b_n}{s^n+k_1 s^{n-1}+\cdots+k_{n-1}+k_n}. \tag{2.12}$$

If the entries of the vector b are positive constants $[b_1\ b_2\ \cdots\ b_n]$, then $W(s)$ may be strictly positive real (SPR).

Lemma 2.1: If a strictly proper rational function $H(s)=C^T(sI-A)^{-1}b$ with A a Hurwitz matrix is SPR, then there exists a positive definite symmetric matrix P such that

$$A^T P+PA=-Q,\qquad Pb=C \tag{2.13}$$

with Q a positive definite symmetric matrix. ■

Lemma 2.2-Boundness of systems with exponentially stable strictly proper transfer function: Consider the linear time invariant system in state-space representation

$$\dot{x}(t)=Ax(t)+Bu(t)\qquad x(0)=x_0 \tag{2.14}$$

with $x(t)\in R^n$, $u(t)\in R^m$, the matrices $A\in R^{n\times n}$, $B\in R^{n\times m}$, and let transition matrix $\Phi(t)$ be bounded by

$$\|\Phi(t)\|_s=\left\|e^{At}\right\|_s\le m_0 e^{-\alpha t} \tag{2.15}$$

where the number $\alpha=-\max_i \operatorname{Re}\lambda_i[A]$ if all the eigenvalues of A are distinct, and m_0 is a positive constant [21]. Then every solution $x(t)$ of (2.14) is such that

$$\|x(t)\|\le k_1+k_2\|u\|_2^{\alpha},\qquad \forall t\ge 0 \tag{2.16}$$

with k_1 is exponentially decaying term due to x_0 and k_2 is a positive constant which depends on eigenvalues of A.

Proof: The solution $x(t)$ of (2.14) can be expressed as

$$x(t)=\Phi(t,0)x_0+\int_0^t\Phi(t,\tau)Bu(\tau)d\tau. \tag{2.17}$$

Therefore,

$$\|x(t)\|\le\|\Phi(t,0)\|_s\|x_0\|+B_M\int_0^t\|\Phi(t,\tau)\|_s\|u(\tau)\|d\tau \tag{2.18}$$

Taking the condition (2.15) into account and applying the Cauchy-Schwartz inequality, we obtain

$$\begin{aligned}\|x(t)\|&\le m_0\|x_0\|e^{-\alpha t}\\ &+m_1\left(\int_0^t e^{-\alpha(t-\tau)}d\tau\right)^{1/2}\left(\int_0^t e^{-\alpha(t-\tau)}\|u(\tau)\|^2 d\tau\right)^{1/2}\end{aligned} \tag{2.19}$$

with $m_1=B_M m_0$. Using the fact $\int_0^t e^{-\alpha(t-\tau)}d\tau\le 1/\alpha$,

$$\|x(t)\|\le m_0\|x_0\|e^{-\alpha t}+\frac{m_1}{\sqrt{\alpha}}\left(\int_0^t e^{-\alpha(t-\tau)}\|u(\tau)\|^2 d\tau\right)^{1/2} \tag{2.20}$$

which completes the proof. ■

2.3 Dynamic Recurrent Neural Networks

A generalized DRNN can be constructed using an arbitrary linear transfer function and a nonlinear mapping from a feedforward NN as shown in Fig. 2 [16]. The linear transfer function and the NN constitute a closed feedback loop since a subset of the NN's inputs are a function, through the evolution of the linear system, of the outputs from previous operations. Given the system state vector $\chi\in R^n$, the systematic description of a generalized DRNN can be given by

$$\begin{aligned}\dot{\chi}&=A\chi+B\left[\sum_{j=1}^{N_h}\left\{w_{ij}\sigma\left(\sum_{k=1}^{n}v_{jk}\chi_k+\theta_{vj}\right)+\theta_{wi}\right\}+u\right];\ i=1,\ldots,q\\ \zeta&=C^T\chi\end{aligned} \tag{2.21}$$

with $A\in R^{n\times n}$, $B\in R^{n\times n}$, $C\in R^{n\times m}$, $u\in R^n$ and $\zeta\in R^m$. It is plain that the Hopfield net is a special case of this equation, which is also true of many other dynamical NN in the literature. Typical examples of the function $\sigma(\cdot)$ are sigmoid, hyperbolic tangent, gaussian.

The DRNN equation is in vectors as

$$\dot{\chi}=A\chi+B[W^T\sigma(V^T\chi)+u] \tag{2.22}$$

with the vector of activation functions defined by $\sigma(z)=[\sigma(z_1)\ \ldots\ \sigma(z_n)]^T$ for a vector $z\in R^n$. The thresholds are included as the first columns of the weight matrices; any tuning of W and V then includes tuning of the thresholds as well.

The main property of a feedforward NN inserted in the feedback path is the *function approximation property* [4]. Let $f(\chi)$ be a smooth function from R^n to R^q. Then, it can be shown that, as long as χ is restricted to a compact set S of $\chi \in R^n$, for some sufficiently large number of hidden layer neurons N_h there exist weights and thresholds such that any continuous function on a compact set can be represented as

$$f(\chi) = W^T \sigma(V^T \chi) + \varepsilon. \tag{2.23}$$

The value of ε is called the *NN functional approximation error*. For any choice of a positive number ε_N one can find a NN such that $\varepsilon \le \varepsilon_N \quad \forall \chi \in S$.

For estimation and control purposes, the *ideal* approximating NN weights exist for a specified value of ε_N. Then, an estimate $\hat{f}(\chi)$ of $f(\chi)$ can be given by

$$\hat{f}(\chi) = \hat{W}^T \sigma(\hat{V}^T \chi) \tag{2.24}$$

where $\hat{W}$, $\hat{V}$ are estimates of the ideal NN weights.

The NN in this paper is considered with the first layer weight fixed. This makes the NN linear in the parameters. Therefore, select $V =' I'$ so that the static NN output y is given by

$$y = W^T \sigma(\chi). \tag{2.25}$$

Then, for suitable NN approximation properties, some conditions (e.g., [18]) must be satisfied by $\sigma(\chi)$. In fact, it must be a basis.

Therefore, there exist constant weights W so that the nonlinear function can be represented as

$$f(\chi) = W^T \sigma(\chi) + \varepsilon(\chi) \tag{2.26}$$

where $\|\varepsilon(\chi)\| \le \varepsilon_N(\chi)$ with the bounding function $\varepsilon_N(\chi) \in C^1(S)$ known. It has been shown in [1] that the *NN functional approximation error* $\varepsilon(\chi)$ for 1-layer NN is fundamentally bounded below by a term of order $(1/n)^{2/d}$, where n is the number of fixed basis functions and d is the dimension of the input to the NN. However, as seen in our main result of Theorem 3.1, the NN observer estimation error may still be made small through the judicious choice of certain observer gains.

3 DRNN Observer Design

3.1 Observer Error Dynamics and Structure

The continuous nonlinear functions in the system (2.7) can be represented by NNs with constant "ideal" weights W and sufficient number of basis functions $\sigma(.)$,

$$f(x) = W_f^T \sigma_f(x) + \varepsilon_f, \qquad \varepsilon_f \le \varepsilon_{f,N} = \text{constant}$$

$$g(x) = W_g^T \sigma_g(x) + \varepsilon_g, \qquad \varepsilon_g \le \varepsilon_{g,N} = \text{constant} \tag{3.1}$$

where subscripts "f" and "g" denote the function $f(x)$ and $g(x)$, respectively. We assume that the ideal weights, W_f and W_g are bounded by known positive values [9] so that

$$\|W\|_{i,F} \le W_{i,M}, \qquad i = f, g \tag{3.2}$$

where $W_{i,M}$ are known values.

Let the NN functional estimates for the nonlinear functions of $f(x)$ and $g(x)$ be given by

$$\hat{f}(\hat{x}) = \hat{W}_f^T \sigma_f(\hat{x}), \qquad \hat{g}(\hat{x}) = \hat{W}_g^T \sigma_g(\hat{x}) \tag{3.3}$$

where the current weights $\hat{W}_f$ and $\hat{W}_g$ are provided by the weight tuning algorithms. The expression for the functional estimate error $\tilde{f}$ defined in (2.10) is given by

$$\tilde{f} = W_f^T \sigma_f(x) - \hat{W}_f^T \sigma_f(\hat{x}) + \varepsilon_f. \tag{3.4}$$

The input layer output error with sigmoid activation for a given x is defined as

$$\tilde{\sigma}_f = \sigma_f(x) - \sigma_f(\hat{x}). \tag{3.5}$$

Adding and subtracting $W_f^T \sigma_f(\hat{x})$ from (3.4) yield

$$\tilde{f} = \tilde{W}_f^T \sigma_f(\hat{x}) + w_f(t) + \varepsilon_f \tag{3.6}$$

where the weight estimation error is defined as

$$\tilde{W}_f = W_f - \hat{W}_f \tag{3.7}$$

and the disturbance terms $w_f(t)$ is given by

$$w_f(t) = W_f^T \tilde{\sigma}_f. \tag{3.8}$$

Following the same arguments for $\tilde{f}$, we have an expression for $\tilde{g}$

$$\tilde{g} = \tilde{W}_g^T \sigma(\hat{x}) + w_g(t) + \varepsilon_g \tag{3.9}$$

where the disturbance terms $w_g(t)$ is given by

$$w_g(t) = W_g^T \tilde{\sigma}_g. \tag{3.10}$$

Fact 1: The disturbance function $w(t)$ is bounded according to

$$\|w_i(t)\| \le a_i \qquad i = f, g \tag{3.11}$$

with a_f, $a_g > 0$. It is obvious from (3.2) and the property of the neural activation functions.

Then the proposed observer system (2.8) and the observation error dynamics (2.9) become

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + b[\hat{W}_f^T \hat{\sigma}_f + \hat{W}_g^T \hat{\sigma}_g u - v_f(t) - v_g(t)] + K(y - C\hat{x}) \\ \hat{y} &= C^T \hat{x} \end{aligned} \tag{3.12}$$

and

$$\begin{aligned} \dot{\tilde{x}} &= (A + KC^T)\tilde{x} + b[\tilde{W}_f^T \hat{\sigma}_f + w_f(t) + \{\tilde{W}_g^T \hat{\sigma}_g \\ &\quad + w_g(t) + \varepsilon_g\}u + \varepsilon_f + d(t) + v_f(t) + v_g(t)] \\ \tilde{y} &= C^T \tilde{x} \end{aligned} \tag{3.13}$$

with $\hat{\sigma} = \sigma(\hat{x})$.

It is required to find the output estimation error $\tilde{y}$

$$\begin{aligned} \tilde{y} &= W(s)[\tilde{W}_f^T \hat{\sigma}_f + w_f(t) + \{\tilde{W}_g^T \hat{\sigma}_g + w_g(t) + \varepsilon_g\}u \\ &\quad + \varepsilon_f + d(t) + v_f(t) + v_g(t)] \end{aligned} \tag{3.14}$$

where $W(s)$ is a known proper transfer function with stable poles, and is realized by the pair ($A + KC^T$, b, C). The block diagram representation of the proposed observer and the plant is shown in Fig. 1.

3.2 Stability Analysis-SPR Lyapunov Approach

The output estimation error (3.14) can be written in the form of

$$\begin{aligned} \tilde{y} &= W(s)L(s)[\tilde{W}_f^T \bar{\hat{\sigma}}_f + \tilde{W}_g^T \bar{\hat{\sigma}}_g u + \bar{w}_f(t) + \bar{w}_g(t)u \\ &\quad \delta_f + \bar{\varepsilon}_f + \delta_g + \bar{\varepsilon}_g u + \bar{d}(t) + \bar{v}_f(t) + \bar{v}_g(t)] \end{aligned} \tag{3.15}$$

where $L^{-1}(s)$ is proper transfer functions with stable poles, $\bar{\hat{\sigma}} = L^{-1}(s)\hat{\sigma}$ and $L(s)$ is chosen so that $W(s)L(s)$ is an SPR transfer function. Therefore the "-" notation indicates the signal filtered by $L^{-1}(s)$.

The mismatch terms δ_f and δ_g is defined as

$$\delta_f = L^{-1}(s)[\widetilde{W}_f^T\hat{\sigma}_f] - \widetilde{W}_f^T L^{-1}(s)[\hat{\sigma}_f] \quad (3.16)$$

$$\delta_g = L^{-1}(s)[\widetilde{W}_g^T\hat{\sigma}_g u] - \widetilde{W}_g^T L^{-1}(s)[\hat{\sigma}_g]u \quad (3.17)$$

and we have the following fact for the subsequent proof.

Fact 2: The mismatch values of δ_f and δ_g are bounded according to

$$\|\delta_f\| \le c_f \|\widetilde{W}_f\|_F \quad (3.18)$$

$$\|\delta_g\| \le c_g \|\widetilde{W}_g\|_F \quad (3.19)$$

where c_f and c_g are computable positive constants. It can be easily seen from the standard norm inequality and the property of proper transfer function.

For some cases $W(s)$ it is possible that no $L(s)$ can be found such that $W(s)L(s)$ is an SPR transfer function. In such cases the output estimation error still can be manipulated and put in the form of (3.15). For example, see [5]. Note that each term (3.14) need to be filtered by $L^{-1}(s)$. But this is not important because we can realize the filtering on the basis function only for implementation as shown later. Therefore this error dynamics can be used only for analysis purposes to show that the state estimation error $\tilde{x}$ and the weight estimation error $\widetilde{W}$ are bounded.

The state-space realization of (3.15) is given by

$$\dot{\tilde{z}} = A_c\tilde{z} + b_c[\widetilde{W}_f^T\hat{\sigma}_f + \widetilde{W}_g^T\hat{\sigma}_g u + \overline{w}_f(t) + \overline{w}_g(t)u + \delta_f + \bar{\varepsilon}_f + \delta_g + \bar{\varepsilon}_g u + \bar{d}(t) + \bar{v}_f(t) + \bar{v}_g(t)] \quad (3.20)$$

$$\tilde{y} = C^T\tilde{z}$$

where ($A_c \in R^{n\times n}, b_c \in R^n, C_c \in R^n$) is a minimal state representation of $W(s)L(s) = C_c^T(sI - A_c)^{-1}b_c$ with $C_c = [1\ 0\ \cdots\ 0]^T$.

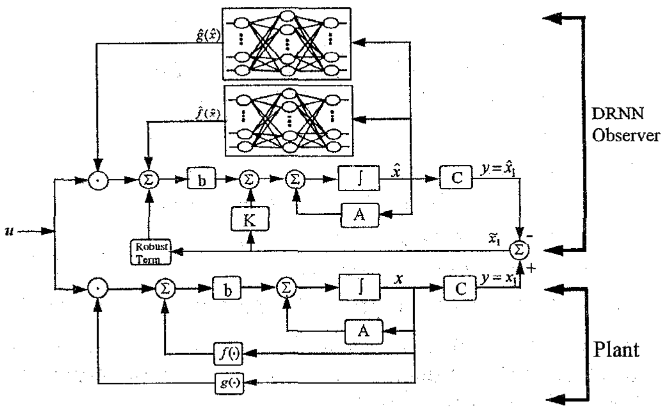

Fig. 1. Nonlinear observer using dynamic recurrent neural network.

Theorem 3.1: Assume the control input u is bounded by positive constant u_d. Consider the observer system (3.12) and the robustifying terms are given by

$$v_i(t) = -D_i\tilde{y}/|\tilde{y}| \qquad i = f, g \quad (3.21)$$

with $D_f \ge a_f\sigma_M$, $D_g \ge a_g\sigma_M u_d$ and $\sigma_M = \sigma_{\max}[L^{-1}(s)]$. Let the NN weight tunings be provided by

$$\dot{\hat{W}}_f = F_f\hat{\bar{\sigma}}_f\tilde{y} - \kappa_f F_f|\tilde{y}|\hat{W}_f$$
$$\dot{\hat{W}}_g = F_g\hat{\bar{\sigma}}_g\tilde{y}u - \kappa_g F_g|\tilde{y}|\hat{W}_g \quad (3.22)$$

where $F_i = F_i^T > 0$ is any constant design matrix governing the speed of convergence and $\kappa_i > 0$ is a design parameter with $i = f, g$. Then the state estimates $\hat{x}(t)$ and the NN weight estimates $\hat{W}(t)$ are *UUB*.

Proof: Consider the Lyapunov function candidate

$$L = \tfrac{1}{2}\tilde{z}^T P\tilde{z} + \tfrac{1}{2}tr(\widetilde{W}_f^T F_f^{-1}\widetilde{W}_f) + \tfrac{1}{2}tr(\widetilde{W}_g^T F_g^{-1}\widetilde{W}_g) \quad (3.23)$$

with $P = P^T > 0$. By manipulation $W(s)L(s)$ is SPR, according to the Lemma 2.1, the time derivative $\dot{L}$ is

$$\dot{L} = -\tfrac{1}{2}\tilde{z}^T Q\tilde{z} + \tilde{y}\{\widetilde{W}_f^T\hat{\bar{\sigma}}_f + \widetilde{W}_g^T\hat{\bar{\sigma}}_g u + \overline{w}_f + \overline{w}_g u + \bar{\varepsilon}_f + \bar{\varepsilon}_g u + \bar{d} + \delta_f + \bar{v}_f + \delta_g + \bar{v}_g\} + tr(\widetilde{W}_f^T F_f^{-1}\dot{\widetilde{W}}_f^T) + tr(\widetilde{W}_g^T F_g^{-1}\dot{\widetilde{W}}_g^T) \quad (3.24)$$

Evaluating (3.24) along the trajectories of (3.21) and (3.22), we have

$$\dot{L} \le -\tfrac{1}{2}\lambda_{min}(Q)\|\tilde{z}\|^2 + |\tilde{y}|\{\bar{d} + \bar{\varepsilon}_f + \bar{\varepsilon}_g u + c_f\|\widetilde{W}_f\|_F + c_g\|\widetilde{W}_g\|_F\} + k_f F_f|\tilde{y}|\, tr\{\widetilde{W}_f^T(W_f - \widetilde{W}_f)\} + k_g F_g|\tilde{y}| tr\{\widetilde{W}_g^T(W_g - \widetilde{W}_g)\} \quad (3.25)$$

Using fact 2, $tr\{\widetilde{W}^T(W - \widetilde{W})\} \le W_M\|\widetilde{W}\|_F - \|\widetilde{W}\|_F^2$, and $-\lambda_{min}(Q)\|\tilde{z}\|^2 \le -\lambda_{\min}(Q)|\tilde{y}|^2$, we have

$$\dot{L} \le -|\tilde{y}|[\tfrac{1}{2}\lambda_{\min}(Q)|\tilde{y}| - \sigma_M\{b_d + \varepsilon_{f,N} + \varepsilon_{g,N}u_d\} - \beta_f(\alpha_f\|\widetilde{W}_f\|_F - \|\widetilde{W}_g\|_F^2) - \beta_g(\alpha_g\|\widetilde{W}_f\|_F - \|\widetilde{W}_g\|_F^2)] \quad (3.26)$$

with $\beta_i = \kappa_i F_i$ and $\alpha_i = W_{i,M} + c_i/\beta_i$, $i = f, g$. Furthermore, squaring the terms $\|\widetilde{W}_f\|_F$ and $\|\widetilde{W}_g\|_F$, we obtain the following conditions for the time derivative $\dot{L}$ of L to be negative

$$|\tilde{y}| \ge \min\left\{\begin{array}{l}\{4\sigma_M(b_d + \varepsilon_{f,N}) + \beta_f\alpha_f^2\}/\lambda_{\min}(Q),\\ \{4\sigma_M\varepsilon_{g,N}u_d + \beta_g\alpha_g^2\}/\lambda_{\min}(Q)\end{array}\right\} \quad (3.27)$$

or

$$\|\widetilde{W}_f\|_F \ge \alpha_f/2 + \sqrt{\sigma_M(b_d + \varepsilon_{f,N})/\beta_f + \alpha_f^2/4},$$
$$\|\widetilde{W}_g\|_F \ge \alpha_g/2 + \sqrt{\sigma_M\varepsilon_{g,N}u_d/\beta_f + \alpha_g^2/4}. \quad (3.28)$$

According to a standard Lyapunov theorem [8][15], this demonstrates the *UUB* of $\|\tilde{z}\|$, $\|\widetilde{W}_f\|_F$, and $\|\widetilde{W}_g\|_F$. In order to show the boundness of the state estimation error $\tilde{x}$, consider the observation error dynamics (3.13).

The state trajectory $\tilde{x}(t)$ of the system (3.13) can be expressed as

$$\tilde{x}(t) = \Phi(t,0)\tilde{x}(0) + \int_0^t\Phi(t,\tau)b\tilde{u}(\tau)d\tau \quad (3.29)$$

where $\Phi(t,\tau)$ is bounded by $m_0e^{-\alpha(t-\tau)}$ with m_0, $\alpha > 0$, $\tilde{u}(\cdot) = \widetilde{W}_f^T\hat{\sigma}_f + w_f(t) + \varepsilon_f + \{\widetilde{W}_g^T\hat{\sigma}_g + w_g(t) + \varepsilon_g\}u + d(t) + v_f(t) + v_g(t)$.
After straightforward manipulation of the state trajectory

(3.29) using the standard norm inequality and Lemma 2.2, we obtain

$$\|\tilde{x}(t)\| \le (c_1 + \frac{c_2}{\sqrt{\alpha}}) + \frac{c_3}{\sqrt{\alpha}}\|\tilde{W}_f\|_F^{\alpha} + \frac{c_4}{\sqrt{\alpha}}\|\tilde{W}_g\|_F^{\alpha} \qquad \forall t \ge 0 \quad (3.30)$$

where c_1 is an exponentially decaying term due to initial condition and c_2, c_3, and c_4 are positive and computable constants. And arbitrarily small estimation errors can be achieved by the appropriate choice of observer gain K. From the expression (3.30) the state trajectory $\|\tilde{x}(t)\|$ of the system (3.13) is bounded by the weight estimation errors $\|\tilde{W}_f\|_F$ and $\|\tilde{W}_g\|_F$, which are shown to be bounded by the Lyapunov stability proof. So we can conclude that every signal $\tilde{x}$, $\tilde{z}$, and $\tilde{W}$ is *Uniformly Ultimately Bounded*. This completes our stability proof. ■

Corollary 3.2: Assume the control input u is bounded by positive constant u_d and that for the given vector b, for the transfer function $W(s)$ realized by the triple $(A+KC^T$, b, $C)$ is an SPR transfer function. Let the robustifying terms be given by

$$v_i(t) = -D_i\tilde{y}/|\tilde{y}| \qquad i = f, g \qquad (3.31)$$

with $D_f \ge a_f$, $D_g \ge a_g$, weight tunings be provided by

$$\begin{aligned} \dot{\hat{W}}_f &= F_f\hat{\sigma}_f\tilde{y} - \kappa_f F_f|\tilde{y}|\hat{W}_f \\ \dot{\hat{W}}_g &= F_g\hat{\sigma}_g\tilde{y}u - \kappa_g F_g|\tilde{y}|\hat{W}_g \end{aligned} \qquad (3.32)$$

where $F_i = F_i^T > 0$ is any constant design matrix governing the speed of convergence and $\kappa_i > 0$ is a design parameter with $i = f, g$. Then the state estimates $\hat{x}(t)$ and the NN weight estimates $\hat{W}(t)$ are *UUB*.

Proof: The proof is similar to the one of Theorem 3.1 and hence it is omitted. ■

Remarks: Although the stability analysis of the proposed observer is similar to that of standard adaptive techniques, there are fundamental differences between the two approaches which render the proposed DRNN observer universal and reusable.

1) Most parameter identification techniques require the unknown system parameters to be linearly parameterizable. The NN technique reported here does not assume the linearity in the unknown system parameters for the unknown functions. Hence the NN technique can be applied to systems with nonlinear functions which may not be linearly parameterizable. our NN technique does not require any preliminary analysis to determine the regression matrix.
2) The proposed DRNN observer can be applied to a broad class of nonlinear systems. The nonlinearities of $f(x)$ and $g(x)$ are not restricted to depend on the output *y* only, but may depend on the unknown system state *x*.
3) When the relative degree of the system is greater than unity, $W(s)$ cannot be SPR [15]. In order to overcome this difficulty, the augmented error method and the normalized error method are used in [15] and [5], respectively. However, notice that these two approaches cannot be applied to the analysis of the parameter identification of the proposed observer since the nonlinear functions $f(x)$ and $g(x)$ are in terms of the unknown system state x. As pointed out in [5], there is no possible solution for this kind of problem from the standard adaptive scheme unless some *a priori* information about the unknown modeling error is available. In our NN approach, the chosen sigmoid basis function $\sigma(x)$ is a bounded function for which $\sigma(x) \to 1$ as $x \to \infty$ and $\sigma(x) \to 0$ as $x \to -\infty$. Using this property of neural activation functions we can circumvent the problem of the unknown modeling error term without resorting to any special method, e.g. normalized signal or augmented error. This can be considered as a step in extending the adaptive control theory to nonlinear NN techniques.
4) Finally, it is emphasized that the NN weights may be initialized at zero, and stability will be maintained by the observer gain K until the NN learns. This means there is no off-line learning phase, which is one of the main features of our tuning algorithms.

4 Simulation Results

The DRNN observer strategies have been simulated on single link robot and Van der Pol oscillator. In each of the examples, the number of sigmoid functions was chosen to be 10. The linear filter $L(s) = s + 3$ was used to compensate the transfer function $W(s)$ which is not SPR. All initial conditions were taken so that a non-zero error affected the performance of the DRNN observer.

Example 4.1: Consider a single-link robot rotating in a vertical plane whose equations of motion [13] are

$$M\ddot{q} + \frac{1}{2}mgl\sin q = u, \qquad y = q \qquad (4.1)$$

in which *q* is the angle, *u* the input torque, *M* the moment of inertia, *g* the gravity constant, *m* and *l* are the mass and the length of the link. The robot parameters are (in SI units): *m*=1, *l*=1, *M*=0.5, and *g*=9.8. Letting $x_1 = q$ and $x_2 = \dot{q}$, the state-space description of the system (4.1) is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}(u - \frac{1}{2}mgl\sin x_1)/M \qquad (4.2)$$
$$y = x_1$$

Then the observer for the above system (4.2) is given by (3.12). with the theorem 3.1. The simulation parameters: $K = [400 \;\; 800]$, $\kappa_f = \kappa_g = 0.001$, $F_f = diag[5\times10^4 \;\; 5\times10^4]$, $F_g = diag[5\times10^3 \;\; 5\times10^3]$, initial conditions $x = [0 \;\; 0.5]^T$ and $\hat{x} = [0.1 \;\; 0]^T$, control input $u(t) = \sin 2t + \cos 20t$.

Fig. 2 shows the trajectories of the estimated states are bounded.

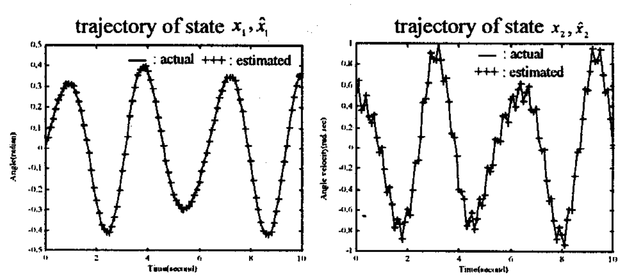

Fig.2. State estimation.

Example 4.2: Consider a Van der Pol oscillator

$$\ddot{x} + (x^2 - 1)\dot{x} + x = (1 + x^2 + \dot{x}^2)u, \qquad y = x. \quad (4.3)$$

The state-space representation by letting $x_1 = x$ and $x_2 = \dot{x}$ is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \{(1 - x_1^2)x_2 - x_1 + (1 + x_1^2 + x_2^2)u\} \quad (4.4)$$
$$y = x_1$$

Then the observer for the above system is given by (3.12). Two separate simulations were conducted; 'zero control input' and 'non-zero control input'. The simulation parameters: for 'zero input': $K = [600 \;\; 600]$, $F_f = diag[500 \;\; 500]$, $\kappa_f = 0.001$, initial conditions $x = [0 \;\; 0.25]^T$ and $\hat{x} = [0.5 \;\; 0.5]^T$. The results are shown in Fig. 3, clearly demonstrating the limit cycle in the Van der Pol oscillator. For 'non-zero control input', simulation parameters: $\kappa_f = \kappa_g = 0.001$, $F_f = F_g = diag[500 \;\; 500]$, $K = [600 \;\; 600]$, $x(0) = [0 \;\; 0.25]^T$ and $\hat{x}(0) = [0.5 \;\; 0.5]^T$. Fig.4 shows boundness of the estimation errors.

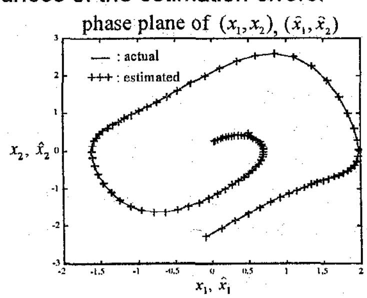

Fig.3. State estimation for 'zero control input'.

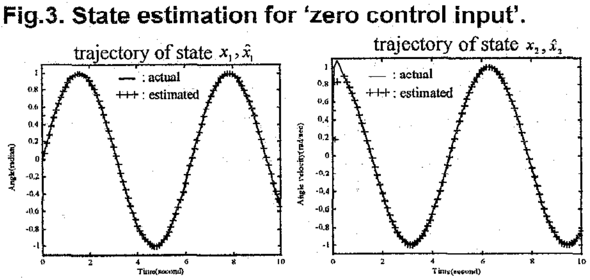

Fig.4. State estimation for 'non-zero input'

5 Conclusions

We have presented a nonlinear observer based on the DRNN. The proposed observer does not require the nonlinear state-space transformation under some severe conditions and linearity with respect to unknown system parameters which is hard to be satisfied in the physical systems. Furthermore, the NN observer technique reported in this paper does not require the exact form of function or functional in the system considered. Even the nonlinearity of the unknown function is not required to depend on the system output only. Compared with other NN techniques, we do not require any preliminary off-line "training or learning phase."

The main contribution of this paper is to provide a universal and reusable observer for the class of highly nonlinear system. A key point in developing an intelligent system is the reusability of the proposed system, i.e. the same proposed system works even if the behavior of the system has changed. This is the case of the observer reported in this paper.

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