Haptic Tele-Driving of Wheeled Mobile Robot with Communication Delay

Dongjun Lee, Daye Xu, Oscar Martinez-Palafox, Mark W. Spong, Ivan Lopez, and Chaouki T. Abdallah

Abstract

We propose a novel passivity-enforcing tele-driving control framework, which will enable a human operator to stably tele-control a unicycle-type wheeled mobile robot (WMR) over communication network with constant delays while having some useful haptic force-feedback. We consider both the case of kinematic and dynamic WMRS. Our main objective is to enable human users to tele-drive the WMR as they drive a car, arguably the most efficient human-interface for controlling WMRS: i.e. two degree-of-freedoms of a (nonlinear) master device are respectively used to control the forward velocity and the heading angle of the WMR, much like as the gas-pedal and the steering-wheel of a car. We also enforce passivity of the closed-loop system, so that the interaction of the tele-driving system can be stable for a wide range of human users and slave environments even with communication delays. Experiment is performed to show the efficacy of the proposed control frameworks.

Index Terms

passivity, teleoperation, communication delay, haptic feedback, wheeled mobile robot, relative degree

I. INTRODUCTION

Tele-driving of a wheeled mobile robot (WMR) would enable us to achieve many powerful robotic applications in a remote and possibly unknown/unstructured environment with intelligent human task intervention. Some examples include: planet exploration by astronauts tele-operating a WMR from an orbiting station or on the planet ground (e.g. [1]); remote surveillance, rescue, and payload-transport using an (uninhabited) WMR in/through dangerous areas (e.g. [2]); remote home surveillance with, perhaps,
some simple mechanical operation by the WMR to activate periodic household maintenance (e.g. [1]); and material/part handling WMR in a dynamically-changing factory/warehouse tele-operated by remote or co-existing human workers (e.g. [3], [4]).

Haptic feedback (or force-feedback) may significantly enhance capability/usability of the above applications, especially when other sensory modalities (e.g. vision, auditory) are inaccurate/occluded (e.g. constant-speed cruising or contact-detection of WMR when visibility is very low) and/or the given task heavily relies on mechanical sensations (e.g. pushing an object by the WMR while regulating the contact force). Yet, it is also well-known that achieving such haptic feedback while guaranteeing system stability is challenging when master-slave communication-delay is present ([5], [6]), which, in some cases, is unavoidable either due to the physical master-slave distance (tele-driving of WMR from orbiting station, remote company-headquarter/military-base, or other city) or unreliability of communication medium itself (e.g. Internet [7]).

In this paper, we propose a novel passivity-enforcing tele-driving control framework, which will enable a human operator to tele-control a 3-degree-of-freedom (DOF) unicycle-type WMR via a \( n \)-DOF master-device over delayed communication network while having some useful haptic force-feedback. See Fig. 1. We consider both the second-order dynamic-type WMR (i.e. input is wheels’ torques) and the first-order kinematic-type WMR (i.e. input is wheels’ velocity), where the latter is included here since many commercial WMRs are in that category for implementation simplicity (e.g. Pioneer DX-3, e-puck), although, of course, the former is more complete description of the WMR in Fig. 1. Our control framework also enforces passivity of the closed-loop system, thereby, can ensure interaction stability of the tele-driving system with a wide-range of human users and slave environments (for the dynamic-WMRs) without relying on their (usually complicated/uncertain) mathematical models. Main idea of our control framework may then be summarized as follows.

Compared to conventional teleoperation problems between two non-mobile robots (e.g. serial-links robot with fixed base), one of the unique aspects of the WMR tele-driving is master-slave kinematic dissimilarity, i.e.: 1) workspace of the master joystick is usually bounded, yet that of the slave WMR is not (i.e. \((x_o, y_o)\) in Fig. 1 can go anywhere in \( \mathbb{R}^2 \)); and 2) the slave WMR is a nonholonomic system with its velocity being constrained (e.g. only \( v, w \) possible from the non-slip condition) but its position not (e.g. \((x_o, y_o, \theta)\) can reach admissible any values), while the master-device is usually constraint-free (e.g. serial-link robot) and/or holonomic (e.g. parallel manipulator).

To address this master-slave kinematic dissimilarity, in this paper, we adopt the idea of car-driving metaphor [8]: one-DOF of the \( n \)-DOF master device (e.g. \( q_1 \) in Fig. 1) is used as a gas-pedal to control the WMR’s linear forward velocity \( v \), while another-DOF (e.g. \( q_2 \) in Fig. 1) as a steering-wheel to command the WMR’s heading-angle \( \phi \) (or its rate \( \dot{\phi} \)). In this way, the human operators would tele-drive the slave WMR much as like they drive usual cars, using \( q_1, q_2 \) as the gas-pedal and steering-wheel. Given ubiquitousness of the everyday car driving, we believe this car-driving metaphor would allow human users to tele-drive the slave WMR (probably most) comfortably and efficiently.
This car-driving metaphor would then require \((q_1, v)\)-teleoperation and \((q_2, \phi)\)-teleoperation (or \((q_2, \dot{\phi})\)-teleoperation) over the delayed communication network. For the dynamic-WMR, \((q_2, \phi)\)-teleoperation may then be thought of as the standard position-position teleoperation problem (between two second-order systems), for which many passivity-based techniques are directly applicable (e.g. [9], [10], [11], [12], [13]). Yet, this is not the case for the other \((q_1, v)\) or \((q_2, \dot{\phi})\) teleoperation-loops, since they require position-velocity teleoperation. This notion - it is position-position (or velocity-velocity) teleoperation, thus, standard teleoperation techniques are applicable - becomes even more evasive for the kinematic WMR, since its (kinematic) system dynamics is first-order, yet, the (dynamic) master device is second-order, so, even the \((q_2, \phi)\)-teleoperation cannot be seen as the standard position-position teleoperation problem between two (second-order/dynamic) robotic systems any more.

To clarify this ambiguity, we first develop an argument based on the concept of relative-degree [14] and show that the matching of a certain relative-degree pair between the (dynamic) master-device and (dynamic or kinematic) slave-WMR is a necessary condition to directly apply those standard teleoperation techniques. Based on this observation, we then devise a novel procedure of modified passivity for the \(n\)-DOF (nonlinear Lagrangian) master device, where we design some local state feedback and a new output so that the (locally-modulated) master-device is still passive with this new output (e.g. \(r_1 = \dot{q}_1 + \lambda_1 q_1\) with \(\lambda_1 \geq 0\)), and, moreover, using this new output (e.g. \(r_1\) instead of \(q_1\) for \((q_1, v)\)-teleoperation), we can obtain the matched relative-degree pair.

We then use this procedure of modified passivity for some teleoperation-loops for dynamic/kinematic-WMR tele-driving (i.e. \((q_1, v)\) and \((q_2, \dot{\phi})\) teleoperation-loops for dynamic-WMR; \((q_2, \phi)\) teleoperation-loop for kinematic-WMR) so that those standard passive teleoperation techniques can be directly applied for them. In particular, for this, due to the implementation simplicity, we use the proportional-derivative (PD) control of [13]. On the other hand, for the other teleoperation-loops (i.e. \((q_1, v)\) and \((q_2, \dot{\phi})\) loops...
for kinematic-WMR), we do not need to use this modified passivity procedure, since their algebraic-loop nature (i.e. \( v, \dot{\phi} \) are all inputs for kinematic-WMR) renders their closed-loop dynamics similar to that of a second-order mechanical system with local delayed position feedback, for which we can simply extend the PD-control scheme of [13] without matching the relative-degree pairs. This present paper significantly extends the results of [15] to the case of general nonlinear \( n \)-DOF master devices; kinematic-WMRs; and a different mode of WMR tele-driving (i.e. \((q_2, \dot{\phi})\)-teleoperation), along with some comprehensive experimental results.

Numerous control techniques have been proposed for the standard force-reflecting teleoperation between two non-mobile manipulators with fixed bases (e.g. [9], [10], [11], [12], [13] to name just a few). However, for the WMR haptic tele-driving, only a handful of control frameworks have been proposed, and among them, we believe [8], [16], [17], [18], [19] would adequately cover the state-of-the-art, although this list is certainly not exhaustive.

Some haptic tele-driving schemes were proposed in [8], [16], yet, it is not clear if they can be applicable/extendable for the case of communication delays. This is particularly so, since both works [8], [16] lack (any) stability analysis of the closed-loop system. A passivity-based scheme was proposed in [17], which perhaps may be extendable for delays, although not stated there (e.g. by enforcing passivity of the closed-system’s sub-blocks). Yet, usage of the virtual (i.e. simulated) WMR of [17] may induce, on the top of increasing implementation complexity, delusive human perception arising from discrepancy between the virtual WMR and the real WMR, particularly in fast-speed operations (e.g. real WMR lags behind virtual WMR). The issue of communication delays was considered in [18], [19], yet: 1) in [18], only vision-feedback is considered, with no haptic-feedback; and 2) it is not clear if the event-based scheme proposed in [19] is suitable to address interaction stability issue of the closed-loop tele-driving system, since this stability is a phenomenon of continuous-time, not just of event-indexes (i.e. needs to hold for all (continuous) time, not only at those (discrete) event indexes).

Another limiting aspect common in all of these control schemes ([8], [16]-[19]) is that they only consider kinematic-WMRs. This means that these schemes would not be applicable for “mechanical” tasks, where it is crucial to perceive/control real external forces acting on the WMR (e.g. assembly task), since the kinematic-WMR, due to its first-order nature (i.e. velocity-input/position-output), does not contain any forcing term in its modeling, thus, not allow us to control/perceive that. Of course, either virtual-force (generated by distance sensors) or real-force (measured by force sensors) is (or can be) used in [8], [16]-[19]. Yet, the forcing-information obtained in this way can be possibly inaccurate (e.g. contact with rigid object with no visible deformation), ineffective (e.g. contact outside sensing zones), complex (e.g. increased possibility of malfunctioning), and/or costly (i.e. additional sensors).

In contrast to [8], [16]-[19], our tele-driving control frameworks proposed here: 1) theoretically guarantee haptic tele-driving (continuous-time) stability for a wide-range of human users and slave environments, by enforcing closed-loop passivity against any (constant/asymmetric) communication-delays; 2) are designed both for kinematic and dynamic WMRs, thus, can be applied for those “mechanical” tasks; and
3) achieve haptic-feedback with only basic encoder sensors, without requiring additional force/distance sensors and/or any intervening/artificial virtual dynamics.

The rest of the paper is organized as follows. The general setting of the problem with some preliminary materials are given in Sec. II. Car-driving metaphor and its implication for tele-driving control design (i.e. relative-degree pair mismatch) is discussed in Sec. III, and the procedure of modified passivity to achieve relative-degree pair matching is given in Sec. IV. Tele-driving control laws for both kinematic and dynamics WMRs are then designed and their properties are detailed in Sec. V. Experimental results are presented in Sec. VI, and Sec. VII contains some concluding remarks and comments on future research directions.

Notations: For a vector \( x \in \mathbb{R}^n \), \( ||x|| := \sqrt{x^T x} \) is its Euclidean norm defined. For square matrices \( A, B \in \mathbb{R}^{n \times n} \): \( \bar{\sigma}[A] \) is the largest singular value of \( A \); \( ||A||_{\text{max}} := \max_{ij} |a_{ij}| \) where \( a_{ij} \) is the \( ij \)th element of \( A \); and \( A \succ B \) (or \( A \succeq B \), resp.) implies \( A - B \) is positive-definite (or positive semi-definite, resp.). Also, we will denote by \( I \) the identity matrix with a suitable dimension.

II. THE SETTING

A. System Modeling

Consider a 3-DOF nonholonomically-constrained wheeled mobile robot as shown in Fig. 1 with its configuration \((x, y, \phi) \in \text{SE}(2)\), where \((x, y) := (x_o, y_o) \in \mathbb{E}(2)\) is the position of the geometric center and \(\phi \in \mathbb{S}\) is the heading angle w.r.t. the global frame \((O, X, Y)\). Other choices for the configuration is also possible to be see why. Here, we assume that the WMR has two independently-controlled rear wheels and one front un-actuated castor to avoid tipping over the robot, all of them with negligible inertias.

Then, the nonholonomic constraints (i.e. no slip condition of the wheels) can be written as

\[
\frac{d}{dt} \begin{pmatrix} x \\ y \\ \phi \end{pmatrix} = \begin{bmatrix} \cos \phi & 0 \\ \sin \phi & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} v \\ w \end{pmatrix} \tag{1}
\]

where \(v \in \mathbb{R}\) is the forward velocity of the geometric center and \(w := \dot{\phi} \in \mathbb{R}\) is the heading angle rate w.r.t. \((O, X, Y)\). The two column vectors of the above matrix define the basis of the admissible velocity for the WMR. This constraints is nonholonomic, since the (regular) distribution, obtained by collecting this basis vectors over all the configuration, is not integrable [20].

If the input to the WMR are the torques to the rear wheels, we call it dynamic WMR, whose second-order dynamics equation is then given by:

\[
D(\dot{\phi})\ddot{\phi} + Q(\phi, \dot{\phi})\nu = u + \delta \tag{2}
\]
where $\nu := [v, \dot{\phi}]^T \in \mathbb{R}^2$, $u = [u_1, u_2]^T \in \mathbb{R}^2$, and $\delta = [\delta_1, \delta_2]^T \in \mathbb{R}^2$ are the (projected) velocity, the control, and the external forces respectively, and

$$
D(\phi) := \begin{bmatrix}
  m & 0 \\
  0 & I + ma^2
\end{bmatrix}, \quad Q(\phi, \dot{\phi}) := \begin{bmatrix}
  0 & -ma\dot{\phi} \\
  ma\dot{\phi} & 0
\end{bmatrix}
$$

where $a > 0$ is the distance between the geometric center $(x_o, y_o)$ and the center of mass $(x_c, y_c)$, $m > 0$ is the mass of the WMR, and $I > 0$ is the rotational inertia w.r.t. $(x_c, y_c)$. Here, with the assumption of negligible wheels’ inertia, the control $u$ can be written as

$$
u = u
$$

where $u = [u_1, u_2]^T \in \mathbb{R}^2$ is the control. Here, from the kinematic relation, $u_1, u_2$ can be written by

$$
u_1 = \frac{1}{h}(u_R + u_L), \quad u_2 = \frac{c}{h}(u_R - u_L)
$$

where $(u_R, u_L)$ are the angular torques of the right and left wheels, $c, h > 0$ are the half-width of the cart and the radius of the wheels, respectively. See Fig. 1. By solving this 2-DOF dynamics (2) with the 3-DOF kinematic equation (1), we can then compute the evolution of the 3-DOF configuration $(x, y, \phi)$.

On the other hand, many commercially available WMRs only provide users with capability to control its velocity and not its torque (e.g. Pioneer DX-3 or e-puck). In this case, we call it **kinematic WMR** with its evolution described by the following first-order kinematics equation:

$$
u = u
$$

where $\nu = [v, \dot{\phi}]^T \in \mathbb{R}^2$ is the (projected) velocity, and $u = [u_1, u_2]^T \in \mathbb{R}^2$ is the control. Here, from the kinematic relation, $u_1, u_2$ can be written by

$$u_1 = \frac{1}{h}(u_R + u_L), \quad u_2 = \frac{c}{h}(u_R - u_L)
$$

where $w_R, w_L$ are the angular velocity of the right and left wheels. Again, then, by integrating (1) with this 2-DOF kinematics equation (3), the evolution of the 3-DOF configuration $(x, y, \phi)$ can be computed.

For the master haptic joystick, we assume that it is a $n$-DOF robotic system ($n \geq 2$), with the following standard nonlinear Lagrangian dynamics:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + B^e\dot{q} = \tau + f
$$

where $q, \tau, f \in \mathbb{R}^n$ are the configuration, control torque, and human force, respectively, $M(q) \in \mathbb{R}^{n \times n}$ is the symmetric/positive-definite inertia matrix, $B^e \in \mathbb{R}^{n \times n}$ is the inherent device damping, and $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ is the Coriolis matrix with its $ij$th component given by, from the Lagrangian dynamics structure,

$$c_{ij} = \sum_{k=1}^{n} \frac{1}{2} \left( \frac{\partial m_{ij}}{\partial q_k} + \frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{jk}}{\partial q_i} \right) \dot{q}_k
$$

where $m_{ij}$ and $q_k$ are the $ij$th and $k$th components of $M(q)$ and $q$, respectively.

Here, without loss of generality, we assume that the first two-DOFs of the master device (i.e. $q_1, q_2$) are used to tele-control the WMR (see Fig. 1). By doing so, we can then capture the case of **operation embedding**, which happen frequently in practice: 2-DOF master operation is embedded in a $n$-DOF
general purpose master haptic device with \( n \geq 2 \) (e.g. Sensable Phantom). This also enables us to address the case where the master device is constructed with two separate linear 1-DOF devices (e.g. force-feedback steering-wheel and gas-pedal), with diagonal/constant \( M \) and \( C = 0 \) with \( n = 2 \). Here, we also assume that the eigenvalues of \( M(q) \) are all bounded yet larger than a certain positive constant; and \( \exists a \) bounded \( \gamma \geq 0 \) s.t. \( \gamma \geq |\partial m_{ij}/\partial q_k| \) for all \( q \in \mathbb{R}^n \). These properties are granted for many practical master systems (e.g. linear/revolute-joints robots [21]).

In this paper, we also assume the communication structure as shown in Fig. 1, where the forward (i.e. master to slave) and backward (i.e. slave to master) communication channels are subject to (bounded) constant time-delays, \( d_1 \geq 0 \) and \( d_2 \geq 0 \), respectively.

**B. Passivity and WMR Motion/Power Scaling**

For the tele-driving system we consider here (and also, probably, for any control system), the foremost requirement is the interaction stability (e.g. bounded \( \dot{q}, \nu \)). To address this stability/safety issue, particularly when the communication delays \( d_1, d_2 \) are present, in this paper, we will utilize the open-loop passivity of the kinematic/dynamic WMR (2)-(3) and the master joystick (4). More precisely, 1) for the dynamic WMR (2) and the master joystick (4), using the well-known skew-symmetricity of \( \dot{D} - 2Q \) and \( \dot{M} - 2C \), we can show the open-loop (energetic) passivity with \((u + \delta, \nu)\) and \((\tau + f, \dot{q})\) being their respective input-output pairs, i.e., \( \forall T \geq 0 \),

\[
\int_0^T [u + \delta]^T \nu dt = \kappa_{dWMR}(T) - \kappa_{dWMR}(0) \geq -\kappa_{dWMR}(0) \tag{6}
\]

\[
\int_0^T [\tau + f]^T \dot{q} dt = \kappa_m(T) - \kappa_m(0) + \int_0^T q^T B^c \dot{q} dt \geq -\kappa_m(0) \tag{7}
\]

where \( \kappa_{dWMR}(t) := \frac{1}{2} \nu^T D(\phi) \nu \geq 0 \) and \( \kappa_m(t) := \frac{1}{2} \dot{q}^T M(q) \dot{q} \geq 0 \) are their respective kinetic energies; and 2) for the kinematic WMR (3), we can show its open-loop passivity with \((u, y_{kWMR})\) as the input-output pair with \( y_{kWMR} := [v, \phi]^T \), i.e., \( \forall T \geq 0 \),

\[
\int_0^T u^T y_{kWMR} dt = \int_0^T [v^2 + \dot{\phi} \phi] dt \geq V_{kWMR}(T) - V_{kWMR}(0) \geq -V_{kWMR}(0) \tag{8}
\]

where \( V_{kWMR}(t) := \frac{1}{2} \dot{\phi}^2(t) \geq 0 \). Energetic passivity (6)-(7) has been pivotal to ensure interaction stability/safety with or without communication delays in many teleoperation control schemes (e.g. [9], [10], [11], [22], [12], [13]). Yet, to address the master-slave kinematic dissimilarity, here, we will need to modify the master device passivity (7). See Sec. IV.

In some applications, the motion/power scale of the WMR may be quite different than that of the master system (e.g. micro-WMR, or a large-WMR for remote construction). For this, it may be useful to scale up/down the motion/power of the WMR w.r.t. the master system. We also want this to be done while preserving passivity (6)-(8). For this, similar to [22], [12], we use the following motion scaling
matrix\(^1\)

\[
\eta = \begin{bmatrix}
\eta_1 & 0 \\
0 & \eta_2
\end{bmatrix} \in \mathbb{R}^{2 \times 2}
\]

with \(\eta_1, \eta_2 > 0\), and the power scaling factor \(\rho > 0\) for the WMR, that is, 1) for the dynamic WMR, we rewrite (2) using \(\eta\) s.t.

\[
\rho \left[ \eta^{-T} D(\phi) \eta^{-1} \dot{\nu} + \eta^{-T} Q(\phi, \dot{\phi}) \eta^{-1} \eta \nu \right] = \rho \left[ \eta^{-T} u + \eta^{-T} \delta \right]
\]
or

\[
\dot{D}(\phi) \dot{\nu} + \dot{Q}(\phi, \dot{\phi}) \dot{\nu} = \bar{u} + \bar{\delta}
\]

which possesses passivity similar to (6) with scaled power: for all \(T \geq 0\),

\[
\int_0^T [\bar{u} + \bar{\delta}]^T \dot{\nu} dt = \int_0^T \rho [u + \delta]^T \nu dt = \rho \kappa_{dWMR}(T) - \rho \kappa_{dWMR}(0) \tag{9}
\]

where \(\dot{\nu} = \eta \nu\), \(\bar{u} = \rho \eta \nu\), \(\bar{\delta} = \rho \eta \delta\), \(\bar{D} = \rho \eta^{-T} D \eta^{-1}\), and \(\bar{Q} = \rho \eta^{-T} Q \eta^{-1}\); and 2) for the kinematic WMR, we rewrite (3) using \(\eta\) s.t.

\[
\dot{\nu} = \bar{u}
\]

where \(\dot{\nu} = \eta \nu\) and \(\bar{u} := \eta u\). The relation (9) clearly shows that \(\rho > 0\) is the power scaling (w.r.t. the unscaled master device with (7)).

Here, note that these scaled dynamics/kinematics have exactly the same forms as their counter-parts (2)-(3). This implies that these motion/power scalings will preserve passivity (6)-(8), and we can just consider these scaled kinematics/dynamics instead of (2)-(3) for control design purpose. Thus, from now on, we will just consider (2)-(3) for the WMR, assuming that those scalings are already embedded in them. Of course, the controls \(\bar{u}\), designed for the scaled systems, will need to be decoded before being applied to the real unscaled WMR (e.g. for (2), \(u = \frac{1}{\rho} \eta^T \bar{u}\); and for WMR (3), \(u = \eta^{-1} \bar{u}\)).

### III. Car-Driving Metaphor and Relative Degree Mismatch

Compared to the standard teleoperation problem, the unique challenge of the WMR tele-driving is master-slave kinematic dissimilarity: 1) the workspace of the master joystick (4) is usually bounded, but that of the slave WMR (2)-(3) is not; and 2) the slave WMR is nonholonomically constrained, but the master device is usually unconstrained (or holonomic).

To address this, here, we aim to enable human users to tele-drive the slave WMR as they drive a car, i.e., the linear forward velocity \(v\) of the WMR is controlled by operating the \(q_1\) position of the master joystick (4); and the heading angle \(\phi\) or its rate \(\dot{\phi}\) by the \(q_2\) position. In this way, \(q_1\) and \(q_2\)

\(^1\)This \(\eta\) may also be used to convert between linear motion and angular motion. For instance, \(\eta_2 = 1\text{m/\text{rad}}\) for \((q_2, \phi)\)-teleoperation with linear \(q_2[\text{m}]\) and angular \(\phi[\text{rad}]\); or \(\eta_1 = 1\text{rad/\text{m}}\) for \((q_1, v)\)-teleoperation with angular \(q_1[\text{rad}]\) and linear \(v[\text{m/s}]\).
of the master joystick (4) are respectively used as the gas-pedal and the steering-wheel of a car. This car-driving metaphor has been adopted in [17], [8], and we also believe that, given ubiquitousness of the car driving, it would enable human users to tele-drive the slave WMR (probably most) comfortably and efficiently.

This car-driving metaphor would then require \((q_2, \phi)\)-coordination (or \((q_2, \dot{\phi})\)-coordination) for the tele-steering (i.e. with \(q_2\) as the steering wheel)\(^2\), and \((q_1, v)\)-coordination for the tele-acceleration (i.e. with \(q_1\) as the gas-pedal), over the delayed communication network. Then, for the dynamic WMR (2), the \((q_2, \phi)\)-coordination problem can be thought of as the standard position-position teleoperation problem, for which many standard passivity-based teleoperation techniques are directly applicable (e.g. [9], [10], [11], [12], [13]). Yet, this is not the case 1) for the \((q_1, v)\)-coordination or the \((q_2, \dot{\phi})\)-coordination, since they require position-velocity coordination; and 2) for the kinematic WMR, since its system dynamics (3) is first-order, yet, the (dynamic) master device (4) is of second-order, so, even the \((q_2, \phi)\)-coordination can not be seen as the standard position-position teleoperation problem. Consequently, those standard passivity-based teleoperation schemes, which can provide position-position or velocity-velocity coordination, can not be directly applied here except for the \((q_2, \phi)\)-coordination of the dynamics WMR.

This point can be made a bit clearer by seeing the relative degree pair of each teleoperation task: e.g., for the \((q_2, \phi)\)-coordination of the dynamic WMR, it is defined s.t.

\[
\mathcal{RP}_{(q_2, \phi)} := (\mathcal{R}_{(r_2+f_2)\rightarrow q_2}, \mathcal{R}_{(u_2+\delta_2)\rightarrow \phi}) = (2, 2)
\]

where \(\mathcal{R}_{U\rightarrow Y}\) is the relative degree of a system with the given input \(U\) and output \(Y\)\(^3\). On the other hand, if we consider the \((\dot{q}_2, \dot{\phi})\)-coordination for the dynamic WMR (although not pursued here for the tele-driving), it is the standard velocity-velocity teleoperation problem with

\[
\mathcal{RP}_{(\dot{q}_2, \dot{\phi})} := (\mathcal{R}_{(r_2+f_2)\rightarrow \dot{q}_2}, \mathcal{R}_{(u_2+\delta_2)\rightarrow \dot{\phi}}) = (1, 1)
\]

This shows that a necessary condition to directly apply the standard passivity-based teleoperation techniques is matched relative degree pair (i.e. \(\mathcal{RP}_{(q_2, \phi)} = (2, 2)\) for position-position teleoperation; \(\mathcal{RP}_{(\dot{q}_2, \dot{\phi})}\) for velocity-velocity teleoperation).

Now, let us turn to our tele-driving problem, the relative degree pairs of various teleoperation tasks of which are shown in Table I (in the without matching column). It is then clear from Table I that we have matched relative degree pair only for the \((q_2, \phi)\)-coordination of the dynamic WMR (2), while all the other tasks have mismatched ones: e.g., for the dynamic WMR, \(\mathcal{RP}_{(q_1, v)} = (\mathcal{R}_{(r_1+f_1)\rightarrow q_1}, \mathcal{R}_{(u_1+\delta_1)\rightarrow v}) = (2, 1)\); and, for the kinematic WMR, \(\mathcal{RP}_{(q_2, \phi)} = (\mathcal{R}_{(r_1+f_1)\rightarrow q_1}, \mathcal{R}_{u_2\rightarrow \phi}) = (2, 0)\), as we have an algebraic-loop (or direct feedthrough) from \(u_2\) to \(\dot{\phi}\) because \(\dot{\phi} = u_2\) from (3). This implies that, other than for the

\(^2\)For the tele-steering, we found that the \((q_2, \phi)\)-coordination is preferred, when the human user can see the slave environment via a stationary camera overlooking the slave environment; while the \((q_2, \dot{\phi})\)-coordination is preferred, if the camera is attached to the WMR.

\(^3\)This relative degree \(\mathcal{R}_{U\rightarrow Y}\) is defined as the number of time differentiation of the given output \(Y\) until the given input \(U\) appears from there [14]. If a proper transfer function can be defined from \(U\) to \(Y\), this relative degree is the order difference between its denominator and numerator.
### TABLE I

<table>
<thead>
<tr>
<th>Mode</th>
<th>(q&lt;sub&gt;1&lt;/sub&gt;, v) / (q&lt;sub&gt;2&lt;/sub&gt;, φ)</th>
<th>(2, 1) / (2, 2)</th>
<th>(1, 1) / (2, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dynamic WMR</td>
<td>(q&lt;sub&gt;1&lt;/sub&gt;, v) / (q&lt;sub&gt;2&lt;/sub&gt;, φ)</td>
<td>(2, 1) / (2, 1)</td>
<td>(1, 1) / (1, 1)</td>
</tr>
<tr>
<td>kinematic WMR</td>
<td>(q&lt;sub&gt;1&lt;/sub&gt;, v) / (q&lt;sub&gt;2&lt;/sub&gt;, φ)</td>
<td>(2, 0) / (2, 1)</td>
<td>(2, 0) / (1, 1)</td>
</tr>
</tbody>
</table>

**Relative Degree Pair Matching via Feedback Passivation**

In the next Sec. IV, we will present a way to address this problem. The key idea for this is to define a new output for the master device (4) with a certain (open-loop dynamics modulating) state feedback in such a way that the (modulated) master system with this new output still possesses passivity similar to (7) with a more amicable relative degree for the various modes of telecoordination of Table I, so that those standard passivity-based teleoperation techniques can still be directly applied for achieving the car-driving metaphor.

### IV. Modified Passivity via Feedback Passivation

Let us write the control \( \tau \) for the master device (4) s.t.

\[
\tau := \tau_{\text{local}}(q, \dot{q}) + \tau'
\]

where \( \tau_{\text{local}}(q, \dot{q}) \) is a certain local state-feedback (to be designed), and \( \tau' \) is the additional control (e.g. tele-driving control embedded in it). Suppose that, somehow, we can render this (modulated) master system with the local state-feedback \( \tau_{\text{local}}(q, \dot{q}) \) to be passive in the sense that: for all \( T \geq 0 \),

\[
\int_0^T [\tau' + f]^T r dt \geq -d^2
\]

where \( d \in \mathbb{R} \) is a bounded constant, and \( r \in \mathbb{R}^n \) is the new output as given by

\[
r := \dot{q} + \Lambda q
\]

where \( \Lambda := \text{diag}[\lambda_1, \lambda_2, 0, \ldots, 0] \in \mathbb{R}^{2 \times 2} \) is a diagonal matrix with \( \lambda_1, \lambda_2 \geq 0 \). We call this new passivity from \( \tau' + f \) to \( r \), **modified passivity**.

If we can achieve this modified passivity, we may then use \( r_i \) (with \( \lambda_i > 0 \)) instead of \( q_i \) for the various telecoordination tasks in Table I, since this \( r_i \) contains \( q_i \)-information, thus, particularly if \( \dot{q} \to 0 \) is expected from the tasks, this \( r_i \) will faithfully convey the \( q_i \)-information in the steady-state (with \( \lambda_i > 0 \)). Furthermore, note that, by using this \( r_i \), we actually “shift down” the master relative degree (\( q_2, \phi \))-coordination of the dynamic WMR, the standard passivity-based teleoperation techniques are automatically disqualified.

In the next Sec. IV, we will present a way to address this problem. The key idea for this is to define a new output for the master device (4) with a certain (open-loop dynamics modulating) state feedback in such a way that the (modulated) master system with this new output still possesses passivity similar to (7) with a more amicable relative degree for the various modes of telecoordination of Table I, so that those standard passivity-based teleoperation techniques can still be directly applied for achieving the car-driving metaphor.
from 2 to 1 (since this $r_i$ also has $\dot{q}_i$ in it). This means that we have now matched relative degree pair for the $(r_1, v)$-coordination and the $(r_2, \dot{\phi})$-coordination of the dynamic WMR; as well as for the $(r_2, \phi)$-coordination of the kinematic WMR (see “with matching” column of Table I). Moreover, with the fact that we have this $r_i$ in the above passivity condition (and $\phi$ in (8)), this also implies that we can consider this $r_i$ (and $\phi$ of the kinematic WMR) just as the usual velocities of the Lagrangian systems, and approach the tele-coordination problems of $(r_1, v)$ and $(r_2, \dot{\phi})$ for the dynamic WMR, and that of $(r_2, \phi)$ for the kinematic WMR, just as the standard velocity-velocity teleoperation problem, for which most of the standard passivity-based teleoperation schemes are applicable.

The following Prop. 1 shows that this modified passivity is indeed possible with the nonlinear master system (4) with a simple spring-damper-type local state-feedback. See Fig. 2. This Prop. 1 extend the similar result of [15] to the case of the general nonlinear master-device (4) with its-DOF $n \geq 2$.

**Proposition 1** There exists a 3-tuple of $n \times n$ matrices $(B, K, \Lambda)$ where $B := \text{diag}[b_1, b_2, ..., b_n]$, $K := \text{diag}[k_1, ..., k_p, 0, ..., 0]$, and $\Lambda := \text{diag}[\lambda_1, ..., \lambda_p, 0, ..., 0]$, with $b_i > 0$, $k_i > 0$, and $\lambda_i > 0$, s.t. if $\dot{q}$ is bounded, the modulated dynamics of (4) under the following local state-feedback

$$\tau_{local}(q, \dot{q}) := -B\dot{q} - Kq$$

that is,

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + B\dot{q} + B\dot{q} + Kq = \tau' + f$$

is passive in the sense that: for all $T \geq 0$,

$$\int_{0}^{T} [\tau' + f]^T r dt \geq V_P(T) - V_P(0) \geq -V_P(0)$$

where $r := \dot{q} + \Lambda q$ is the new output (11) and $V_P(t) \geq 0$ is a storage function defined in (16).
Proof: From the modulated master dynamics (13), we can show that

\[
\begin{align*}
[r' + f]^T r &= (\dot{q} + \Lambda q)^T [M\ddot{q} + C\dot{q} + B\dot{q} + B^e\dot{q} + Kq] \\
&= (\dot{q} + \Lambda q)^T [M\dot{r} + Cr - M\Lambda \dot{q} - C\Lambda q + B\dot{q} + B^e\dot{q} + K\dot{q}] \\
&\geq \frac{d}{dt} \left[ \frac{1}{2} r^T Mr - \frac{1}{2} q^T \Lambda M \dot{q} + \frac{1}{2} q^T (\Lambda B + K) q \right] + \left( \begin{array}{c} \dot{q} \\ q_p \end{array} \right)^T Q(q, \dot{q}) \left( \begin{array}{c} \dot{q} \\ q_p \end{array} \right) \\
&= \frac{d}{dt} V_P(t) + s_H(t)
\end{align*}
\]

where

\[
V_P(t) := \frac{1}{2} r^T Mr + q^T [K + \Lambda B - \Lambda M] q
= \frac{1}{2} \left( \begin{array}{c} \dot{q} \\ q_p \end{array} \right)^T \left[ \begin{array}{cc} M & M_{n\times p} \Lambda_p \\ \Lambda_p M_{n\times p}^T & K_p + B_p \Lambda_p \end{array} \right] \left( \begin{array}{c} \dot{q} \\ q_p \end{array} \right)
:= P(q) \in \mathbb{R}^{(n+p)\times (n+p)}
\]

and

\[
\begin{align*}
s_H(t) &:= \left( \begin{array}{c} \dot{q} \\ q_p \end{array} \right)^T \left[ \begin{array}{cc} B + \frac{1}{2} B^e - \frac{1}{2} [M\Lambda + \Lambda M] & \frac{1}{2} C_{n\times p} \Lambda_p \\ \frac{1}{2} \Lambda_p C_{n\times p}^T & \Lambda_p K_p - \Lambda_p B_{p\times p} \Lambda_p \end{array} \right] \left( \begin{array}{c} \dot{q} \\ q_p \end{array} \right) \\
&= Q(q, \dot{q}) \in \mathbb{R}^{(n+p)\times (n+p)}
\end{align*}
\]

where \( q_p = [q_1, q_2, \ldots, q_p]^T \in \mathbb{R}^p, \Lambda_p = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_p] \in \mathbb{R}^{p\times p} \) (similar for \( B_p, K_p \), \( M_{n\times p} \in \mathbb{R}^{n\times p} \) is the “tall” block matrix composed of the first \( p \)-columns of \( M \) (similar for \( C_{n\times p} \)), and \( B_{p\times p}^e \in \mathbb{R}^{p\times p} \) is the square block matrix of \( B^e \) starting from the \((1,1)\) component of \( B^e \). Here, we also use the following facts: 1) for any \( y \in \mathbb{R}^m \),

\[
\frac{d}{dt} \left[ \frac{1}{2} y^T M y \right] = y^T M \dot{y} + \frac{1}{2} y^T [C + C^T] y = y^T M \dot{y} + y^T C y
\]

from the skew-symmetric property of \( \dot{M} - 2C \) (i.e. \( \dot{M} = C + C^T \)) and \( y^T C y = y^T C^T y = \frac{1}{2} y^T [C + C^T] y \); and 2) \( 2q^T \Lambda B^e \dot{q} \leq q^T B^e \dot{q} + q^T \Lambda B^e \Lambda q \).

Thus, if we can achieve \( P(q) > 0 \) and \( Q(q, \dot{q}) \geq 0 \) for all \( q, \dot{q} \), by integrating the inequality (15), we will then have the passivity (14) i.e. \( \forall t \geq 0, \int_0^T [r' + f]^T r dt \geq V_P(t) - V_P(0) + \int_0^T s_H(t) dt \geq V_P(t) - V_P(0) \geq -V_P(0) \)

with \( V_P(t) \geq 0 \) and \( s_H(t) \geq 0 \) for all \( t \geq 0 \). This, in fact, can be attained by choosing small enough \( \lambda_i \) and large enough \( k_i \) with bounded \( \dot{q} \), since 1) following [23, pp.472] with \( M(q) > 0 \) as assumed in Sec. II-A, \( P > 0 \), if (and only if)

\[
K_p + B_p \Lambda_p \succ \Lambda_p M_{n\times p}^T M_{n\times p}^{-1} M_{n\times p} \Lambda_p = \Lambda_p M_{p\times p} \Lambda_p
\]

(18)
which will be granted with large $k_i$, small $\lambda_i$, and bounded $M(q)$ (see the assumption after (5)), where we use the fact that $M_{n \times p} M^{-1} M_{n \times p} = M_{p \times p}$ with $M_{p \times p} \in \mathbb{R}^{p \times p}$ being the square block matrix starting from (1,1)-component of $M \in \mathbb{R}^{n \times n}$; and 2) for the matrix $Q$ in (17), we have

$$Q \succeq \begin{bmatrix}
B - \frac{1}{2} [MA + AM] & \frac{1}{2} A_p C_{n \times p}^T \\
\frac{1}{2} A_p C_{n \times p} & \Lambda_p K_p - \Lambda_p B_{q \times p}^e \Lambda_p
\end{bmatrix} \succeq 0$$

(19)

where the last matrix inequality is because, for the matrix in the middle, with such large enough $k_i$ and small enough $\lambda_i$ and also with bounded $M(q)$ and $C(q, \dot{q})$ (with bounded $\dot{q}$ - see the assumptions after (5)), all the diagonal terms for each row (i.e. disc center) will dominate the sum of the absolute values of their off-diagonal terms (i.e. disc radius), thus, following Gersgorin’s disc theorem [23, pp.344], this middle matrix will be (at least) positive semi-definite, implying $Q \succeq 0$. This completes the proof.

This Prop. and its proof say that the modified passivity (14) will be achieved if we choose small enough $\lambda_i$, and reasonably large $k_i$ and $b_i$. Also, note that, as can be seen in (17), the device damping $B^e$ of (4), which in many cases is not negligible, is not always passivity-enforcing as usual. This is because our modified passivity (14) has different output $r$ than the usual velocity output $\dot{q}$. Yet, if $B^e$ is diagonal as $B$ (or has the same eigen-structure), we can then show that $B^e$ is still passivity-enforcing (e.g. removing $B^e$ from (17) and embedding it into $B$ in (16)-(17)). See Cor. 1. In this work, we will also mainly use $\Lambda = \text{diag}[\lambda_1, \lambda_2, 0, \ldots, 0]$, as only the first two-DOFs of the master device (4) will be used for the tele-driving.

Note that the passivity (14) of this Prop. 1 is state-dependent (i.e. it requires bounded $\dot{q}$). This boundedness of $\dot{q}$ will be enforced by our control design in Sec. V, along with passivity of the human user and the slave environment (w.r.t. the compatible supply rate (14)). This state-dependency of the modified passivity (14) and, consequently, its dependence on the coupled exogenous environments, is in a contrast to the standard notion of passivity (e.g. (6)-(8)), which is usually system’s intrinsic property regardless of how its state evolves or with which exogenous humans/environments it is coupled. Further formalization/investigation of this state-dependent passivity is a topic for future research.

This state-dependency of the modified passivity (14) is, in fact, due to the nonlinearity of the master dynamics, particularly, our way of treating the Coriolis terms. If the master system (4) is linear with $C = 0$, as shown in the following Cor. 1, this state-dependency can be eliminated.

**Corollary 1** Suppose that the master device (4) is linear with $C(q, \dot{q}) = 0$. Then, with the local state-feedback (12), the modulated master system (13) will possess the modified passivity (14), if we choose the 3-tuple $(B, K, \Lambda)$ with small enough $\lambda_i$ s.t.

$$B \succeq \bar{\sigma}[M][\sigma[\Lambda]] I_n, \quad K_p \succeq \bar{\sigma}[B^e][\sigma[\Lambda]] I_p$$

(20)

where $K_p = \text{diag}[k_1, \ldots, k_p] \in \mathbb{R}^{p \times p}$ and $I_n \in \mathbb{R}^{n \times n}$ is the $n \times n$ identify matrix (similar for $I_p \in \mathbb{R}^{p \times p}$). Moreover, if the master device (4) consists of two separate 1-DOF linear systems with $M = \text{diag}[m_1, m_2]$
and $C = 0$, such $(B, K, \Lambda)$ can be chosen s.t.

$$b_i^e + b_i \geq \lambda_i m_i$$

where $b_i^e > 0$ is the device damping, and $b_i > 0$ is the $i^{th}$ components of $B$ in (12), $i = 1, 2$.

**Proof:** The first claim comes directly from (18)-(19) with the Coriolis term $C$ vanishing in (19) and the fact that the first inequality of (20) automatically implies (18), i.e. due to the diagonal structures of $B = \text{diag}[b_1, ..., b_n]$ and $B_p = \text{diag}[b_1, ..., b_p]$, the first inequality of (20) ensures that $b_i \geq \bar{\sigma}[M] \bar{\sigma}[\Lambda]$ for all $i = 1, ..., p$, therefore,

$$B_p \succeq \bar{\sigma}[M] \bar{\sigma}[\Lambda] I_p \succeq \bar{\sigma}[M_{p \times p}] \bar{\sigma}[\Lambda_p] I_p \succeq M_{p \times p} \Lambda_p$$

where we use the relation $\bar{\sigma}[M] \geq \bar{\sigma}[M_{p \times p}]$ [23, pp.419].

The second claim is equivalent to enforce the positive realness (PR) of the following transfer functions (i.e. $\text{Re}[H_i(jw)] \geq 0 \ \forall w \geq 0$ [24]):

$$H_i(s) := \frac{\mathcal{L}[\dot{q}_i + \lambda_i \dot{q}_i]}{\mathcal{L}[\tau_i^r + f_i]} = \frac{s + \lambda_i}{m_i s^2 + b_i^e s + b_i s + k_i}$$

i.e. $k_i + (b_i^e + b_i - \lambda_i m_i)w^2 \geq 0 \ \forall w \geq 0$, where $\mathcal{L}$ and $s$ are respectively the Laplace transform and its corresponding variable, and $i = 1, 2$.

Here, note that, for the second case of Cor. 1, following the structure of $\Lambda$ and $K$ in Prop. 1, $k_i = 0$ if and only if $\lambda_i = 0$. Also, in this case, $B_e$ in Prop. 1 is diagonal (i.e. separate $b_i^e, b_i^d$), thus, we can simply embed $B_e$ into $B$ (i.e. $B_e$ is, just like $B$, always passivity-enforcing) with the second inequality of (20) not necessary any more here (i.e. no condition on $k_i$).

With this Prop. 1 or its Cor. 1, we can now easily achieve the tele-coordination of $(r_1, v)$ and $(r_2, \dot{\phi})$ for the dynamic WMR and that of $(r_2, \phi)$ for the kinematic WMR, by directly applying many standard (velocity-velocity coordinating) passivity-based teleoperation techniques (with matched relative degree pair -see Table I). The WMR motion scaling $\eta$ in Sec. II-B may be useful when this $r$ is used, particularly with small $\lambda_i$. This is because, with such a small $\lambda_i$, a large $q_i$ may be needed to achieve the (originally intended) tele-coordination of $(q_1, v)$ or $(q_2, \dot{\phi})$ for the dynamic WMR and that of $(q_2, \phi)$ for the kinematic WMR, because $r_i$ contains only $\lambda_i q_i$ not $q_i$. Using the (small) motion scaling $\eta$ will address this problem, since it actually scales down the (otherwise possibly big) WMR robot’s motion w.r.t. $\lambda_i q_i$ (e.g. for the $(r_1, \eta_1 v)$ teleoperation with $\dot{q}_1 \rightarrow 0, \lambda_1 q_1 \rightarrow \eta_1 v$).

On the other hand, note that, as shown in Table I, this procedure (of using $r$ with Prop. 1 or Cor. 1) will be used for the $(q_1, v)$ and $(q_2, \dot{\phi})$ tele-driving modes for the dynamic WMR; and the $(q_2, \phi)$ tele-driving mode for the kinematic WMR, but not for the $(q_1, v)$ and $(q_2, \dot{\phi})$ modes for the kinematic WMR, although their relative degree pairs are still mismatched (i.e. $\mathcal{RP} = (2, 0)$). This is because these mismatched modes, in fact, possess algebraic-loops (i.e. $\mathcal{RP} = (2, 0)$ with no integration in the WMR
side), which, even if generating mismatched $R\mathcal{P}$, turn out to be benign and can easily be dealt with for our tele-driving control designs. Thus, we can just leave them as they are here. See Sec. V.

Our idea of defining a new output $r$ (11) with local state-feedback injection (10) may be thought of as feedback passivation [14], yet, used here with a different motivation: feedback passivation usually aims to passify otherwise non-passive systems, while our procedure here aims to modify passivity of the already passive system to address the relative-degree mismatch problem. Our idea to use $r$ with modified passivity (14) is also similar to that in [25], yet, our approach is more robust or simpler, as the procedure of [25] requires either exact knowledge of the inertia/Coriolis matrices of the master device (4) or some adaptation-loop otherwise.

V. TELE-DRIVING CONTROL DESIGN

In this section, we will design the tele-driving control laws for the various teleoperation modes in Table I. The modified passivity (14) with the newly-defined output $r := \dot{q} + \Lambda q$, as detailed in Prop. 1 (or Cor. 1), will be used to address the relative degree mismatch problem discussed in Sec. III, and, then, the control scheme proposed in [13] (along with some simpler control laws) will be used to enforce passivity of the closed-loop tele-driving system with constant communication delay.

A. Control Design 1: Dynamic WMR $(q_1, v)/(q_2, \phi)$ Tele-Driving

Here, we have mismatched relative degree pair for the $(q_1, v)$-coordination with $R\mathcal{P}_{(q_1, v)} = (2, 1)$, while matched one for the $(q_2, \phi)$-coordination with $R\mathcal{P}_{(q_2, \phi)} = (2, 2)$. Thus, we can consider the latter as the standard teleoperation problems, while, for the former, we will use the modified passivity procedure of Sec. IV to obtain the matched relative degree pair. In other words, we will use $r_1 := \dot{q}_1 + \lambda_1 q_1$ with $\lambda_1 > 0$ instead of $q_1$, so that we can have matched relative degree pair $R\mathcal{P}_{(r_1, v)} = (1, 1)$ as shown in Table I. For this, we define the local state-feedback $\tau_{\text{local}}(q, \dot{q})$ with $K = \text{diag}[k_1, 0, ..., 0]$ and the new output $r = \dot{q} + \Lambda q$ with $\Lambda = \text{diag}[\lambda_1, 0, ..., 0]$, with small enough $\lambda_1$ and reasonably large $k_1$ (see Prop. 1).

Once we achieve the matched relative degree pairs by using $r$, we can then simply utilize the control law proposed in [13] for the tele-driving control of the master $\tau' = [\tau'_1, \tau'_2, 0, ..., 0]^T \in \mathbb{R}^n$ in (10) and the dynamic WMR $u = [u_1, u_2]^T \in \mathbb{R}^2$ in (2) s.t.: 1) for the $(r_1, v)$-teleoperation

$$
\tau'_1(t) := -k_v(r_1(t) - v(t - d_2)) \quad (21)
$$

$$
u_1(t) := -k_v(v(t) - r_1(t - d_1)) \quad (22)
$$

where $d_1 \geq 0, d_2 \geq 0$ are the (constant) forward/backward communication-delays, and $k_v > 0$ is the control gain; and 2) for the $(q_2, \phi)$-teleoperation

$$
\tau'_2(t) := -b_\phi(q_2(t) - \dot{\phi}(t - d_2)) - b_d \dot{q}_2(t) - k_\phi(q_2(t) - \phi(t - d_2)) \quad (23)
$$

$$
u_2(t) := -b_\phi(\dot{\phi}(t) - \dot{q}_2(t - d_1)) - b_d \phi(t) - k_\phi(\phi(t) - q_2(t - d_1)) \quad (24)
$$
where \( b_\phi > 0, k_\phi > 0 \) are the PD-control gains, and \( b_d > 0 \) is the damping injection set to be

\[
b_d \geq \frac{d_1 + d_2 k_\phi}{2}.
\]

(25)

according to [13] to passify the delayed P-control actions (i.e. terms with \( k_\phi \)). Here, note that, other than the first two \( \tau'_1, \tau'_2 \), all the other components of the tele-driving control \( \tau' \in \mathbb{R}^n \) are zero, since only the first two-DOFs of the master device (4) are assumed to be used to tele-drive the two-DOF velocity of the WMR (2), \( \nu = [v, \dot{\phi}]^T \in \mathbb{R}^2 \). Also, note that, here, other standard passivity-enforcing teleoperation control schemes would also be applicable (e.g. wave-teleoperation [11]), once we achieve the matched relative degree pairs by using \( r \) and Prop. 1.

The following Th. 1 presents main properties of the closed-loop system under this tele-driving control law (21)-(25). This Th. 1 extends the similar result of [15, Th. 1] to the case of the general \( n \)-DOF nonlinear master device (4).

**Theorem 1** Consider the \( n \)-DOF master device (4) and the dynamic WMR (2), under the PD-type local state-feedback (12) and the tele-driving control (21)-(25), where \( B = \text{diag}[b_1, b_2, \ldots, b_n] \), \( K = \text{diag}[k_1, 0, \ldots, 0] \), and \( \Lambda = \text{diag}[\lambda_1, 0, \ldots, 0] \) with small-enough \( \lambda_1 > 0 \) and large-enough \( k_1 > 0 \).

1) Suppose that the human operator and the slave environment are passive: \( \forall T \geq 0 \)

\[
\int_0^T f^T r dt \leq c_1^2, \quad \int_0^T \delta^T \nu dt \leq c_2^2
\]

(26)

where \( c_1, c_2 \in \mathbb{R} \) are bounded constants. Then, the closed-loop system is two-port passive: \( \forall T \geq 0 \), \( \exists \) a bounded \( c \in \mathbb{R} \) s.t.

\[
\int_0^T [f^T r + \delta^T \nu] dt \geq -c^2
\]

(27)

and its interaction is stable with bounded \( (\dot{q}, q_1, v, \dot{\phi}) \) for all \( t \geq 0 \).

2) Suppose that \( (\ddot{q}, \ddot{q}, \dot{v}, \ddot{\phi}, \dot{\phi}) \to 0 \). Then, for the \((q_2, \phi)\)-teleoperation, we have

\[
f_2 \to -k_\phi (q_2 - \phi) \to -\delta_2
\]

(i.e. heading-angle torque reflection); and, for the \((r_1, v)\)-teleoperation, we have, if \( \delta_1 \to 0 \),

\[
v \to \lambda_1 q_1 \quad \text{with} \quad f_1 \to \frac{k_1}{\lambda_1} v
\]

(i.e. linear velocity haptic feedback), or, if \( v \to 0 \),

\[
f_1 \to -\frac{k_1 + \lambda_1 k_v}{\lambda_1 k_v} \delta_1
\]

(i.e. linear-force reflection).
Proof: 1) Following the procedure in [13] with the facts that \(2r_1(t)v(t - d_2) \leq r_1^2(t) + v^2(t - d_2)\) and \(2v(t)r_1(t - d_1) \leq v^2(t) + r_1^2(t - d_1)\), we can show the (controller) passivity of the \((r_1, v)\) teleoperation-loop: for all \(T \geq 0\),
\[
\int_0^T (\tau'_1 r_1 + u_1 v)dt \leq -V_{LK}^1(T) + V_{LK}^1(0)
\] (28)
where \(V_{LK}^1(t) \geq 0\) is Lyapunov-Krasovskii functional [26] defined by
\[
V_{LK}^1(t) := \frac{k_r}{2} \left[ \int_{-d_1}^0 r_1^2(t + \theta)d\theta + \int_{-d_2}^0 v^2(t + \theta)d\theta \right] \geq 0
\]
i.e. Lyapunov-Krasovskii functional \(V_{LK}^1(t)\) serves as the storage function [27] for the supply rate of the \((r_1, v)\) teleoperation control.

Also, following [13] (or its time-domain translation [28]), under the condition (25), we can show the (controller) passivity of the \((q_2, \phi)\) teleoperation-loop: \(\forall T \geq 0\),
\[
\int_0^T [\tau'_2 q_2 + u_2 \dot{\phi}]dt \leq -V_{LK}^2(T) + V_{LK}^2(0) - V^2(0) + V^2(0)
\] (29)
where \(V_{LK}^2(t)\) and \(V^2(0)\) are Lyapunov-Krasovskii functional and the spring potential energy defined by
\[
V_{LK}^2(t) := \frac{b_d}{2} \left[ \int_{-d_1}^0 \ddot{q}_2^2(t + \theta)d\theta + \int_{-d_2}^0 \ddot{\phi}^2(t + \theta)d\theta \right] \geq 0
\]
\[
V^2(0) := \frac{1}{2} k_\phi (q_2(t) - \phi(t))^2 \geq 0
\]
i.e. \(V_{LK}^2\) and \(V^2\) serve as the storage functions for the delayed D-action of (23)-(24) (i.e. terms with \(b_\phi\)), and the combined damping injection \(b_d\) and delayed P-actions (i.e. terms with \(k_\phi\)), respectively.

Now, suppose that the master’s modified passivity (14) is somehow granted (this assumption will be removed below). Then, combining dynamic WMR’s open-loop passivity (6), modified passivity (14), passivity of human operator and slave environment (26), and the controller passivity (28)-(29), we will have: \(\forall T \geq 0\),
\[
V_p(T) + \kappa_{dWMR}(T) - V_p(0) - \kappa_{dWMR}(0) \leq \int_0^T [\tau' r + u'R]dt + \int_0^T [f'T + \delta' \nu]dt
\]
\[
\leq c_1^2 + c_2^2 + V_{LK}^1(0) + V_{LK}^2(0) + V^2(0)
\] (30)
using the fact that \(V_{LK}^1 \geq 0, V_{LK}^2 \geq 0, V^2 \geq 0\). Here, note that \(r_i = \dot{\phi}_i\) and \(\tau_i' = 0\) for \(i = 2, 3, ..., n\).

Re-arranging this inequality (30) and with \(\kappa_{dWMR} \geq 0\), we then have: for all \(T \geq 0\),
\[
V_p(T) + \kappa_{dWMR}(T) \leq V_p(0) + \kappa_{dWMR}(0) + V_{LK}^1(0) + V_{LK}^2(0) + V^2(0) + c_1^2 + c_2^2
\] (31)
implying that \(\dot{\phi}\) is bounded from the definition of \(V_p\) in (16) (with \(V_p(T) \leq V_p(T) + \kappa_{dWMR}(T)\) for (31)); and moreover, from (16) with small \(\lambda_1\), we can find a uniform bound for \(\dot{\phi}\) from (31) (i.e. bound invariant w.r.t. time).
This means that, if we set $\lambda_1$ small enough (i.e. $\Lambda_p$ in (16)-(17)); and $k_1$ large enough (i.e. $K_p$ in (16)-(17)), particularly, in such a way that $K_p$ in (17) dominates $C_{n\times p}^T(q, \dot{q})$ of the off-diagonal term in (17) for all $\dot{q}$ within the aforementioned uniform-bound, we will then have $P = 0$ and $Q \geq 0$, thereby, the modified-passivity (14) for all time. Here, note that $\dot{q}(0)$ automatically falls within this uniform bound. With such small $\lambda_1$ and large $k_1$, thus, the inequality (31) holds, which then implies boundedness of $\dot{q}, q_1$ (from that of $V_p + q$ and $v, \phi$ (from that of $\kappa_{dWMR}$). Also, similar to (30), with so designed $\lambda_1$ and $k_1$, combining dynamic WMR’s open-loop passivity, (6), modified passivity (14), and the controller passivity (28)-(29), we will have two-port closed-loop system passivity (27): for all $T \geq 0$,

$$\int_0^T [f^T r + \delta^T v] dt \geq V_p(T) + \kappa_{dWMR}(T) - V_p(0) - \kappa_{dWMR}(0) - \int_0^T [\tau^T r + u^T v] dt$$

$$\geq -V_p(0) - \kappa_{dWMR}(0) - V^1_{LK}(0) - V^2_{LK}(0) - V^2_{\varphi}(0) =: -c^2$$

(32)

using the fact that $V^1_{LK} \geq 0, V^2_{LK} \geq 0, V^2_{\varphi} \geq 0$.

2) With $(\dot{q}, \dot{q}, \dot{v}, \delta, \phi, \phi) \to 0$, under the local state-feedback (12) and the tele-driving control (21)-(25), the closed-loop system, consisting of the dynamic WMR (2) and the master device (4), will be reduced to

$$\begin{pmatrix}
 f_1 \\
 \delta_1 \\
 f_2 \\
 \delta_2
\end{pmatrix} \rightarrow
\begin{pmatrix}
 k_1 q_1 + k_v (\lambda_1 q_1 - v) \\
 k_v (v - \lambda_1 q_1) \\
 k_\phi (q_2 - \phi) \\
 -k_\phi (q_2 - \phi)
\end{pmatrix}$$

(33)

showing that: 1) heading-angle torque reflection (i.e. $f_2 \rightarrow -k_\phi (q_2 - \phi) \rightarrow -\delta_2$); and 2) linear velocity haptic feedback, if $\delta_1 \to 0$ (i.e. $v \rightarrow \lambda_1 q_1$ and $f_1 \rightarrow \frac{k_1}{\lambda_1} v$); and 2) linear force reflection, if $v \to 0$ (i.e. $\delta_1 \to -k_\phi \lambda_1 q_1$ and $f_1 \to -\frac{k_1 + \lambda_1 k_v}{\lambda_1 k_v} \delta_1$).

Here, the closed-loop passivity (27) in Th. 1 requires the human and the environment be passive in the sense of (26). This is due to the state-dependency (i.e. boundedness of $\dot{q}$) of the modified passivity (14). This human/environment passivity requirement (26) for the closed-loop passivity (27) can be removed if we a linear master-device is used, since the state-dependency comes from the way we treat the nonlinear Coriolis matrix $C$ in (17) - see Cor. 1 and the statement before it. Or, it can be replaced by any condition enforcing boundedness of $\dot{q}$ (e.g. slow-moving/stablizing human operator). Even so, the passive human/environment assumption (26) will still be needed to enforce boundedness of $\dot{q}, q_1, v, \phi$ (e.g. interaction stability). See [13], [15].

Note that our human passivity assumption in (26) is slightly different than the usual “energetic” passivity assumption (i.e. power $f_1 \dot{q}_1 + f_2 \dot{q}_2$ as supply-rate instead of $f_1 r_1 + f_2 \dot{q}_2$ in (26)). This human $r$-passivity (26) is necessary here to be compatible with the modified passivity (14), which has $r$ as its output. Yet, from our experience and also as shown in experiment in Sec. VI, this human $r$-passivity (26) seems to be a reasonable assumption, at least, for our application, particularly with the local spring of
\( \tau_{local} \) in (12) and the communication-delay. A rigorous experimental validation for this human \( r \)-passivity (26) is beyond the scope of this paper and spared for future work.

B. Control Design 2: Dynamic WMR \((q_1, v)/(q_2, \dot{\phi})\) Tele-Driving

For some applications, human operators may see the slave environment only through the WMR’s (moving) perspective (e.g. camera attached to the WMR), not through some global perspective (e.g. stationary camera fixed to the ceiling and overlooking the slave environment). For the latter case, we found that the \((q_2, \phi)\)-teleoperation of Sec. V-A does make sense to the human operators, as they can relatively easily relate what they see (i.e. \( \phi \)) to what they move (i.e. \( q_2 \)). Yet, we also found that this is not true for the former case, since the human often loses accurate sense of \( \phi \). Even so, they can still perceive \( \dot{\phi} \) well (i.e. perception of local change of \( \phi \) - similar to on-board drivers’ perception during car-driving). This motivates us to consider the teleoperation of \((q_2, \dot{\phi})\) instead of that of \((q_2, \phi)\) of Sec. V-A. In fact, we found that this \((q_2, \dot{\phi})\)-teleoperation is preferred for some cases even with the global/stationary camera information.

If we consider this \((q_2, \dot{\phi})\)-teleoperation, we have the relative degree mismatch problem with \( \mathcal{R}\mathcal{P}(q_2, \dot{\phi}) = (2, 1) \). To address this, similar for the \((q_1, v)\)-teleoperation of Sec. V-A, we will use \( r_2 = \dot{q}_2 + \lambda_2 q_2 \) instead of \( q_2 \) so that we can have the matched relative degree pair (i.e. \( \mathcal{R}\mathcal{P}_{(r_2, \dot{\phi})} = (1, 1) \) - see Table I), while still using the same control (with \( r_1 \)) for the \((r_1, v)\)-teleoperation of Sec. V-A. More precisely, we first define the PD-type local state-feedback \( \tau_{local} \) in (12) with \( B = \text{diag}[b_1, b_2, \ldots, b_n] \) and \( K = \text{diag}[k_1, k_2, 0, \ldots, 0] \) and the new output \( r \) in (11) with \( \Lambda = \text{diag}[\lambda_1, \lambda_2, 0, \ldots, 0] \), where non-zero terms are all strictly positive (i.e. \( p = 2 \) for Prop. 1). We then design the tele-driving control \( \tau' = [\tau_1', \tau_2', 0, \ldots, 0]^T \in \mathbb{R}^n \) and \( u = [u_1, u_2]^T \in \mathbb{R}^2 \) s.t. 1) for the \((r_1, v)\)-teleoperation, we will use the same control (21)-(22) of Sec. V-A; and 2) for the \((r_2, \dot{\phi})\)-teleoperation, similar to (21)-(22), we use

\[
\begin{align*}
\tau'_2(t) & := -k_\phi (r_2(t) - \dot{\phi}(t - d_2)) \\ u_2(t) & := -k_\phi (\dot{\phi}(t) - r_2(t - d_1))
\end{align*}
\]

(34) (35)

where \( d_1 \geq 0, d_2 \geq 0 \) are the communication delays, and \( k_\phi > 0 \) is the control gain.

Theorem 2 Consider the \( n \)-DOF master device (4) and the dynamic WMR (2), under the PD-type local state-feedback (12) and the tele-driving control (21)-(22) and (34)-(35), where \( B = \text{diag}[b_1, b_2, \ldots, b_n] \), \( K = \text{diag}[k_1, k_2, 0, \ldots, 0] \), and \( \Lambda = \text{diag}[\lambda_1, \lambda_2, 0, \ldots, 0] \) with small-enough \( \lambda_1, \lambda_2 > 0 \) and large-enough \( k_1, k_2 > 0 \).

1) Suppose that the human operator and the slave environment are passive in the sense of (26). Then, the closed-loop system is two-port passive (i.e. (27) holds) and its interaction is stable with bounded \((\dot{q}_1, q_2, v, \dot{\phi})\) for all \( t \geq 0 \).
2) Suppose that \((\ddot{q}, \dot{q}, \dot{v}, \ddot{\phi}) \to 0\). Then, for the \((r_1, v)\)-teleoperation, the item 2 of Th. 1 holds. Similarly, for the \((q_2, \phi)\)-teleoperation, we have: if \(\delta_2 \to 0\)

\[ \dot{\phi} \to \lambda_2 q_2 \quad \text{with} \quad f_2 \to \frac{k_2}{\lambda_2} \dot{\phi} \]

(i.e. heading-angle rate haptic feedback), or, if \(\dot{\phi} \to 0\),

\[ f_2 \to -\frac{k_2 + \lambda_2 k_\phi}{\lambda_2 k_\phi} \delta_2 \]

(i.e. heading-angle torque reflection).

**Proof:** 1) Similar to (28), for the \((r_2, \dot{\phi})\)-teleoperation loop, we can show the following controller passivity: for all \(T \geq 0\),

\[ \int_0^T (r'_2 r_2 + u_2 \dot{\phi}) dt \leq -V^2_{LK}(T) + V^2_{LK}(0) \] (36)

where \(V^2_{LK}\) is the Lyapunov-Krasovskii function defined by

\[ V^2_{LK}(t) := \frac{k_\phi}{2} \left[ \int_{-d_1}^0 r_2^2(t + \theta) d\theta + \int_{-d_2}^0 \dot{\phi}^2(t + \theta) d\theta \right] \geq 0 \]

i.e. \(V^2_{LK}(t)\) serves as the storage function for the teleoperation control (34)-(35). Since we still have (28) here, the inequalities (30) and (31) will still hold with \(V^2_{LK}(0) + V^2_{\phi}(0)\) in their respective RHS (right hand side) replaced by \(V^2_{LK}(0)\) with \(V^2_{LK}\) defined by (36).

Therefore, using the same argument in the proof of Th. 1, we can show that, if we choose \(\lambda_1, \lambda_2\) small enough (i.e. small \(\Lambda_p\) for (16)-(17)) and \(k_1, k_2\) large enough (i.e. large \(K_p\) in (16)-(17)), \(\dot{q}(t)\) will be uniformly bounded by (31), \(P \succ 0\) and \(Q \succeq 0\) in (16)-(17), and the modified passivity (14) will hold, for all \(t \geq 0\). This then implies the closed-loop two-port passivity from (30).

2) If \((\ddot{q}, \dot{q}, \dot{v}, \ddot{\phi}) \to 0\), the closed-loop dynamics will reduce to (33) with its third and forth lines replaced by:

\[
\begin{pmatrix}
\dddot{f}_2 \\
\dddot{\delta}_2
\end{pmatrix} \to \begin{pmatrix}
k_2 q_2 + k_\phi (\lambda_2 q_2 - \dot{\phi}) \\
k_\phi (\dot{\phi} - \lambda_2 q_2)
\end{pmatrix}
\]

implying that: heading-angle rate haptic feedback, if \(\dddot{\delta}_2 \to 0\) (i.e. \(\dot{\phi} \to \lambda_2 q_2\) and \(f_2 \to \frac{k_2}{\lambda_2} \dot{\phi}\)); or heading-angle torque reflection, if \(\dddot{\phi} \to 0\) (i.e. \(\dddot{\delta}_2 \to -k_\phi \lambda_2 q_2\) and \(f_2 \to -\frac{k_2 + \lambda_2 k_\phi}{\lambda_2 k_\phi} \delta_2\)).

\[\blacksquare\]

Note that, here, in contrast to the \((q_2, \phi)\)-teleoperation of Sec. V-A, the human operator will control the rate of heading angle \(\dot{\phi}\) by essentially moving \(q_2\) (or, more precisely \(r_2\)). Moreover, once released, due to the spring \(k_2, q_2\) will come back to the origin (i.e. \(q_2 \to 0\)), and the WMR will maintain a constant heading angle, which is yet not necessarily zero as for the case of Sec. V-A (i.e. in general, \(\phi \neq 0\) with \(q_2 \to 0\)). See Sec. VI for the experimental results.
C. Control Design 3: Kinematic WMR \((q_1, v)/(r_2, \phi)\) Tele-Driving

Many commercially-available WMRs (e.g. Pioneer 3-DX, e-puck, etc.) often only provide users with the capability of controlling its velocity, not its (raw) wheel torques. For this case, the WMR’s evolution can be modeled by the first-order kinematic equation (3). Now, let us consider the tele-driving problem of this kinematic WMR (3) via the \(n\)-DOF nonlinear master joystick (4). In particular, let us consider the kinematic-version of the problem in Sec. V-A, that is, tele-driving loop consisting of the \((q_1, v)\)-teleoperation and the \((q_2, \phi)\)-teleoperation (e.g. assuming stationary camera overlooking the slave environment).

We then have the mismatched relative-degree pair problem with \(\mathcal{RP}_{(q_1, v)} = (2, 0)\) and \(\mathcal{RP}_{(q_2, \phi)} = (2, 1)\) - see Table I. For the \((q_2, \phi)\)-teleoperation, as stated in the paragraph after (11) in Sec. IV, we will use the new output \(r_2 = \dot{q}_2 + \lambda_2 q_2\) instead of \(q_2\) to have the matched relative-degree pair (with \(\mathcal{RP}_{(r_2, \phi)} = (1, 1)\) from (3) and (4)), thereby, many many standard velocity-velocity teleoperation techniques can be used for this \((r_2, \phi)\)-teleoperation (relying on the modified passivity (14)). On the other hand, we will not attempt here to match the relative-degree pair for the \((q_1, v)\)-teleoperation using \(r_1\) or by other means, since, as shown below, this mismatched \(\mathcal{RP}_{(q_1, v)} = (2, 0)\) turns out to be benign for our design of the \((q_1, v)\)-teleoperation control due to the algebraic-loop (or direct feed-through) in the WMR side (i.e. \(\mathcal{R}_{u_1-v} = 0\) from (3)).

More specifically, we first define the local state-feedback \(\tau_{\text{local}}(q, \dot{q})\) of (12) with \(B = \text{diag}[b_1, b_2, ..., b_n]\) and \(K = \text{diag}[0, k_2, 0, ..., 0]\), and the new output \(r = \dot{q} + \Lambda q\) with \(\Lambda = \text{diag}[0, \lambda_2, 0, ..., 0]\), with small enough \(\lambda_2 > 0\) and large enough \(k_2 > 0\), so that we can use \(r_2 = \dot{q} + \lambda_2 q_2\) instead of \(q_2\) to circumvent the relative-degree mismatch problem while this \(r_2\) properly showing up in the modified passivity (14) of Prop. I. We then design the tele-driving control \(\tau' = [\tau'_1, \tau'_2, 0, ..., 0]^T \in \mathbb{R}^n\) for (10) and \(u = [u_1, u_2]^T \in \mathbb{R}^2\) for (3) s.t.: 1) for the \((q_1, v)\)-teleoperation,

\[
\tau'_1(t) := -b_d \dot{q}_1(t) - k_d q_1(t) - k_v (q_1(t) - v(t - d_2)) \quad (37)
\]
\[
u_1(t) := q_1(t - d_1) \quad (38)
\]

where \(k_d > 0\) is the local spring to provide the haptic feedback of the WMR linear velocity when \(q_1 \rightarrow v\) and \(b_d > 0\) is the damping injection to passify the delayed control action of \(k_v\) s.t.

\[
b_d \geq (d_1 + d_2) k_v \quad (39)
\]

similar to the condition of [13, Eq.(15)]; and 2) for the \((r_2, \phi)\)-teleoperation,

\[
\tau'_2(t) := -k_\phi (r_2(t) - \phi(t - d_2)) \quad (40)
\]
\[
u_2(t) := -k_\phi (\phi(t) - r_2(t - d_1)) \quad (41)
\]

where \(k_\phi > 0\) is the coupling gain and \(d_1, d_2 \geq 0\) are the (constant) forward/backward communication delays.
Theorem 3 Consider the $n$-DOF master device (4) and the kinematic WMR (2), under the PD-type local state-feedback (12) and the tele-driving control (37)-(41), where $B = \text{diag}[b_1, b_2, \ldots, b_n]$, $K = \text{diag}[0, k_2, 0, \ldots, 0]$, and $\Lambda = \text{diag}[0, \lambda_2, 0, \ldots, 0]$ with small-enough $\lambda_2 > 0$ and large-enough $k_2 > 0$.

1) Suppose that the human operator is passive in the sense of (26). Then, the closed-loop system is one-port passive in the sense that: $\forall T \geq 0$, $\exists$ a bounded $c \in \mathbb{R}$ s.t.

$$\int_{0}^{T} f^T r dt \geq -c^2$$

(42)

and its interaction is stable with bounded $(\dot{\varphi}, q_1, q_2, v, \dot{\phi}, \phi)$ for all $t \geq 0$.
2) Suppose that $(\dot{\varphi}, \dot{\varphi}) \to 0$. Then, we will have the $(q_1, v)$-coordination with linear velocity haptic feedback:

$$q_1 \to v \text{ with } f_1 \to k_d v$$

and the (scaled) $(q_2, \phi)$-coordination with heading-angle haptic feedback:

$$\phi \to \lambda_2 q_2 \text{ with } f_2 \to k_d \phi$$

Proof: 1) We first show that the master linear velocity tele-driving control $\tau'_1$ is (one-port) controller passive, that is: for all $T \geq 0$,

$$\int_{0}^{T} \tau'_1 \dot{q}_1 dt = -b_d \int_{0}^{T} \dot{q}_1^2 dt - k_d \int_{0}^{T} \dot{q}_1 q_1 dt - k_v \int_{0}^{T} \dot{q}_1(t)[q_1(t) - q(t - d_1 - d_2)] dt$$

\[ \leq -(b_d - d k_v) \int_{0}^{T} \dot{q}_1^2 dt - V_\varphi^1(T) + V_\varphi^1(0) + \gamma_1^2 \]  

(43)

where we use the inequality (51) of Appendix, with $v(t - d_2) = q_1(t - d_1 - d_2)$ (from (38) with (3)), $\bar{d} := d_1 + d_2$ (i.e. round-trip delay), $V_\varphi^1(t) := \frac{1}{2} k_d \dot{q}_1^2(t)$ (i.e. spring energy of $k_d$), and $\gamma_1^2 := \frac{k_d d^2}{2} \max_{t \in [-\bar{d}, 0]} |\dot{q}_1(t)|^2$. Then, under the condition (39) and with $V_\varphi^1(T) \geq 0$, we have the last line of this inequality (43) upper-bounded by $V_\varphi^1(0) + \gamma_1^2$. Also, similar to (28), for the $(r_2, \phi)$-teleoperation, we can show controller passivity: for all $T \geq 0$,

$$\int_{0}^{T} \tau'_2 r_2 + u_2 \phi dt \leq -V_{LK}^2(T) + V_{LK}^2(0)$$

(44)

where

$$V_{LK}^2(t) := \frac{k_\phi}{2} \left[ \int_{-d_1}^{0} r_2^2(t + \theta) d\theta + \int_{-d_2}^{0} \phi^2(t + \theta) d\theta \right] \geq 0$$

i.e. the Lyapunov-Krasovskii functional for the $(r_2, \phi)$-teleoperation loop.

Then, similar to (30), if the modified passivity (14) is somehow granted, recognizing that $\dot{q}_1 = r_1$ (with $\lambda_1 = 0$) and combining (14), (8) (with $y_{\text{WMR}} = [v, \phi]^T$ and inequality replaced by $y_{\text{WMR}}' := [0, \phi]$
and equality respectively for (8)), human \( r \)-passivity in (26), and (43)-(44), we can show that: \( \forall T \geq 0, \)
\[
V_p(T) + V_{\text{kWMR}}(T) - V_p(0) - V_{\text{kWMR}}(0) \leq \int_0^T [\tau' + f]^T r dt + \int_0^T u^T y_{\text{kWMR}}' dt
\]
\[
\leq \gamma_1^2 + c_1^2 + V_{\text{LK}}^2(0) - V_1^2(0) - V_1^1(0) + \gamma_1^2 + c_1^2
\]
(45)
where \( u^T y_{\text{kWMR}}' = u_2 \phi \). Similar to (31), we will then also have the following inequality:
\[
V_p(T) + V_{\text{kWMR}}(T) + V_1^1(0) \leq V_p(0) + V_{\text{kWMR}}(0) + V_{\text{LK}}^2(0) + V_1^1(0) + \gamma_1^2 + c_1^2
\]
for all \( T \geq 0 \). Therefore, using the same argument in the proof of Th. 1, we can show that, if we choose \( \lambda_2 \) small enough (i.e. small \( \Lambda_p \) for (16)-(17)) and \( k_2 \) large enough (i.e. large \( K_p \) in (16)-(17)), \( \dot{q}(t) \) will be uniformly bounded, thus, \( P > 0 \) and \( Q \geq 0 \) in (16)-(17), and consequently the modified passivity (14) will hold, for all \( t \geq 0 \). This will then also imply 1) the closed-loop one-port passivity from (45) with \( c^2 := V_p(0) + V_{\text{kWMR}}(0) + V_{\text{LK}}^2(0) + V_1^1(0) + \gamma_1^2; \) and 2) interaction stability with bounded \( (\dot{q}, q_1) \) (from that of \( V_p \)), \( q_1 \) (from that of \( V_1^1 \)), \( v \) (from that of \( q_1 \) with (38) and (3)), \( \phi \) (from that of \( V_{\text{kWMR}} \)), and \( \dot{\phi} \) (from (41) and (3) with bounded \( \phi, r_2 \)).
2) With \( (\ddot{q}, \dot{q}, \dot{\phi}) \to 0 \), the closed-loop dynamics (i.e. open-loop dynamics (3) and (4) under the control (10) and (37)-(41)) will then be reduced to
\[
\begin{pmatrix} f_1 \\ v \\ f_2 \\ \dot{\phi} \end{pmatrix} \to \begin{pmatrix} k_d q_1 + k_v (q_1 - v) \\ q_1 \\ k_2 q_2 + k_\phi (\lambda_2 q_2 - \phi) \\ -k_\phi (\phi - \lambda_2 q_2) \end{pmatrix}
\]
(46)
with \( \dot{\phi} \to v \) with \( f_1 \to k_d v; \) and \( \lambda_2 q_2 \to \phi \) with \( f_2 \to \frac{k_2}{\lambda_2} \phi \).

In contrast to the dynamic WMR teleoperation (i.e. Th. 1 and Th. 2), the human operators here do not perceive the external forces acting on the WMR. This is because the (open-loop) WMR considered here is a first-order kinematic system with no such forces showing up in the modeling (3). This implies that the dynamic WMR (2) would be a preferred and more natural choice for applications where such force information is important (e.g. mechanical manipulation/assembly). Of course, this does not rule out the possibility of other types of control laws, which may still be able to provide the force information even for those kinematic WMRs (e.g. using force-sensors and sensor-fusion as in [29]). Also, here, note that the closed-loop passivity (42) is one-port, relevant only to the interacting human users, not to the WMR. This is again due to the kinematic nature of the WMR (3), which has only one-port connecting to the control action \( u \) (i.e. only \( u \) showing up in (8)).

As shown in Table I, the local PD-type state-feedback \( b_2, k_2 \) for the \( (r_2, \phi) \) teleoperation-loop is used to match the relative-degree pair from \( \mathcal{RP}_{(r_2, \phi)} = (2, 1) \) to \( \mathcal{RP}_{(r_2, \phi)} = (1, 1) \). In contrast, although sharing some similarity with these \( b_2, k_2 \), the local damping and spring \( b_d, k_d \) of (37) are used here, not to achieve the matched relative-degree pair, but to provide haptic feedback (via \( k_d \)) while still enforcing passivity.
(via \( b_d \) with (39)). This is possible in fact here due to the algebraic-loop nature of the \( v \)-dynamics of the kinematic WMR (i.e. \( \mathcal{R}_{u_1 \rightarrow v} = 0 \)), which then allows us to convert the delayed \( k_v \)-control action of (37) to a combination of (passive) position-feedback (i.e. \( k_v q(t) \)) and its delayed one (i.e. \( k_v q(t - d_1 - d_2) \) - see the first line of (43)), so that they can be passified by \( b_d \) under the condition (39). This shows that the mismatched relative-degree pair \( \mathcal{RP}_{(q_1, v)} = (2, 0) \) is benign, in the sense that, because of its algebraic-nature, they can still be addressed without resorting to the new output \( r_1 \) to “shift down” the mismatched relative degree.

Note that, differently from Th. 1 where the haptic feedback will vanish (i.e. \( f_2 \rightarrow 0 \)) if \( q_2 \rightarrow \phi \), here, even if \( \lambda_2 q_2 \rightarrow \phi \), the human will still perceive non-zero haptic feedback \( f_2 \rightarrow -\frac{k_2}{\lambda_2} \phi \). This is a byproduct of using the procedure of the modified passivity of Sec. IV to achieve the matched relative-degree pair \( \mathcal{RP}_{(r_2, \phi)} = (1, 1) \), that is, to enforce \( P > 0 \) and \( Q \geq 0 \) in (16)-(17), we need some \( K_p > 0 \), which is, in this case, \( k_2 \). In many cases, we found that this non-zero haptic feedback is not so detrimental. Also, as done in Sec. VI, by choosing this \( k_2 \) small (yet, still large enough to ensure \( P > 0 \) and \( Q \geq 0 \)), we can make this non-zero haptic feedback not so perceptibly prominent, thereby, making it similar to the vanishing haptic feedback of Th. 1. How to completely/theoretically remove this non-vanishing haptic feedback in \( f_2 \) when \( \phi = \lambda_2 q_2 \), which would probably require us to depart from the procedure of modified passivity in Sec. IV, is a topic for future research.

D. Control Design 4: Kinematic WMR \((q_1, v)/(q_2, \dot{\phi})\) Tele-Driving

This is a dual problem of the control design 2 in Sec. V-B for the case of the kinematic WMR. In this case, we have the mismatched relative-degree pairs with \( \mathcal{RP}_{(q_1, v)} = (2, 0) \) and \( \mathcal{RP}_{(q_2, \phi)} = (2, 0) \) - see Table I. Yet, as shown in Sec. V-C, their algebraic-loop (i.e. \( \mathcal{R}_{u_1 \rightarrow v} = 0 \) and \( \mathcal{R}_{u_2 \rightarrow \phi} = 0 \)) enables us to address these mismatched pairs without invoking the procedure of the modified passivity of Sec. IV and using \( r \) in (11) to obtain the matched relative degree pairs. Moreover, it is also shown in Sec. V-C that, for this case, the teleoperation control laws (37)-(39) can guarantee closed-loop passivity (e.g. (43)). This suggests us to adopt the control laws (37)-(39) here both for the \((q_1, v)\) and \((q_2, \dot{\phi})\) teleoperation-loops.

More specifically, we will still use the PD-type local state-feedback (12), yet, only with the damping \( B = \text{diag}[b_1, b_2, \ldots, b_n] \) and not utilizing \( K = \Lambda = 0 \). This local damping \( B \) is used here rather to achieve some adequate performance (e.g. stabilization), not to obtain the matched relative-degree pairs, as can be seen by \( K = \Lambda = 0 \) (i.e. \( r = \dot{q} \)). We then design the tele-driving controls \( \tau' = [\tau_1', \tau_2', 0, \ldots, 0] \in \mathbb{R}^n \) in (10) and \( u = [u_1, u_2] \in \mathbb{R}^2 \) for the kinematic WMR (3) s.t.: 1) for the \((q_1, v)\)-teleoperation, we will use the same control laws as (37)-(39); and 2) for the \((q_2, \dot{\phi})\)-teleoperation, similar to (37)-(39), we use

\[
\tau_2(t) := -b_d'q_2(t) - k_d'q_2(t) - k_\phi(q_2(t) - \dot{\phi}(t - d_2))
\]

\[
u_2(t) := q_2(t - d_1)
\]

with

\[b_d' \geq (d_1 + d_2) k_\phi\]
i.e. $b_i' > 0$ is set to be large enough to passify delayed $k_\phi$ actions in (47)-(48).

**Theorem 4** Consider the $n$-DOF master device (4) and the kinematic WMR (2) under the PD-type local state-feedback (12) and the tele-driving control (37)-(39) and (47)-(49), with $B = \text{diag}[b_1, b_2, ..., b_n]$ ($b_i \geq 0$), and $K = \Lambda = 0$.

1) The closed-loop system is one-port (energetically) passive: $\forall T \geq 0$,

$$\int_0^T f^T \dot{q} dt \geq -c^2$$

where $c \in \mathbb{R}$ is a bounded constant. Suppose further that the human operator is also (energetically) passive in the sense of (26) with $r$ replaced by $\dot{q}$. Then, the interaction is stable with bounded $(\dot{q}, q_1, q_2, v, \dot{\phi})$ for all $t \geq 0$.

2) Suppose that $(\ddot{q}, \dot{q}) \to 0$. Then, we have $(q_1, v)$-coordination with linear velocity haptic feedback:

$$q_1 \to v \quad \text{with} \quad f_1 \to k_d v$$

and $(q_2, \dot{\phi})$-coordination with heading-rate haptic feedback:

$$q_2 \to \dot{\phi} \quad \text{with} \quad f_2 \to k'_d \dot{\phi}$$

**Proof:** 1) From (43), we have the (controller) passivity for the $(q_1, v)$ teleoperation-control (37)-(39). Also, similar to (43), utilizing (51), we can show the (controller) passivity for the $(q_2, \dot{\phi})$ tele-driving control (47)-(49): for all $T \geq 0$,

$$\int_0^T \tau_2' \dot{q}_2 dt \leq -(b_{d} - \bar{d}k_\phi) \int_0^T \dot{q}_2^2 dt - V_\phi^2(T) + V_\phi^2(0) + \gamma_2^2$$

where $\bar{d} := d_1 + d_2$, $V_\phi^2 := \frac{1}{2} k_d q_2^2$, and $\gamma_2^2 := (k_\phi d^2/2) \max_{t \in [-d, 0]} |\dot{q}_2(t)|^2$. Here, $b_{d}' \geq \bar{d}k_\phi$ from (49).

Now, we have $\tau = -B\ddot{q} - \tau'$ for (10), with $B = \text{diag}[b_1, ..., b_n]$, $b_i \geq 0$ (and $K = \Lambda = 0$). Then, following the derivation of (7), we can show that: $\forall T \geq 0$,

$$\int_0^T [\tau' + f]^T \dot{q} dt = \int_0^T \dot{q}^T [B + B^e] \dot{q} dt + \kappa_m(T) - \kappa_m(0)$$

where $\kappa_m := \frac{1}{2} q^T M(q) \dot{q}$, and $\tau'$ is the tele-driving control. Note that this is similar to (14) of Prop. 1 with $r$ and $V_\rho$ respectively being replaced by $\dot{q}$ and $\kappa_m$ there (with $K = \Lambda = 0$ for (11) and (16)-(17)). Then, combining this inequality with the above controller passivity, we have: $\forall T \geq 0$,

$$\int_0^T f^T \dot{q} dt \geq \kappa_m(T) - \kappa_m(0) + \sum_{i=1}^{2} [V_\phi^i(T) - V_\phi^i(0) - \gamma_i^2]$$

implying closed-loop (one-port) passivity (50) with $c^2 := \kappa_m(0) + \sum_{i=1}^{2} [V_\phi^i(0) + \gamma_i^2]$ (since $\kappa_m(t) \geq 0$, and $V_\phi^i(t) \geq 0$). This inequality also implies interaction stability with the (energetically) passive human assumption (i.e. (26) with $r$ replaced by $\dot{q}$), since, if so, we have: for all $T \geq 0$,

$$c^2 + \kappa_m(0) + \sum_{i=1}^{2} [V_\phi^i(0) + \gamma_i^2] \geq \kappa_m(T) + \sum_{i=1}^{2} V_\phi^i(T)$$
ensuring that $\dot{q}, q_1, q_2$ are bounded (from $\kappa_m(T), V_{\phi}^i(T)$ being bounded), and also so are $v, \dot{\phi}$ (from (3) with (38) and (48) with $q_1, q_2$ being bounded).

2) With $(\ddot{q}, \dot{q}) \to 0$, similar to the case of Th. 3, the closed-loop dynamics reduce to (46) with the third and forth lines replaced by

$$\begin{pmatrix} f_2 \\ \dot{\phi} \end{pmatrix} \to \begin{pmatrix} k'_d q_2 - k_{\phi}(q_2 - \dot{\phi}) \\ q_2 \end{pmatrix}$$

implying that $q_1 \to v$ with $f_1 \to k_d v$; and $q_2 \to \dot{\phi}$ with $f_2 \to k'_d \dot{\phi}$.

Here, note that, in contrast to Theorems 1-3, the closed-loop passivity (50) does not require a priori the passive human assumption (26). This is because, here, exploiting the algebraic-nature of the relative-degree mismatch problem, we do not need to rely on the procedure of the modified passivity of Sec. IV (with the usage of $r$), thereby, such a requirement of passive human/environment (26), which needs to precede the modified passivity (14) to address its state-dependency, can now be completely dropped off here - see Prop. 1 and paragraphs after it. For interaction stability, we still need the passive human assumption though (to limit possible energy accumulation in the system).

Note also that, similar to the case of Sec. V-C, here, the local spring-damper controls, $k_d, k'_d$ and $b_d, b'_d$ in (37) and (47), although sharing the same form as the local PD-type state-feedback (12) (e.g. $b_1, k_1$ in Sec. V-A and $b_2, k_2$ in Sec. V-C), are used here not to achieve the matched relative-degree pairs, but rather to provide some suitable haptic feedback $(k_d, k'_d)$ and passify it $(b_d, b'_d)$. Also, choosing $B = 0$ will not affect closed-loop passivity here (thus, interaction stability either), yet some damping $B$ is desirable for performance (e.g. stabilization). This is in a contrast to the case of Sec. V-C, where we need to choose $B = \text{diag}[b_1, \ldots, b_n]$ with reasonably-large $b_i > 0$, particularly if $B^e$ is small - see (17). Also, similar to Sec. V-C, since the WMR is kinematic (3), haptic feedback of the WMR’s external force is
VI. Experiment

We use a PHANToM Desktop (Sensible Technologies Inc.) as the master device. Among its actuated 3-DOFs (i.e. \( n = 3 \) for (4)), similar to Fig. 1, we use its 2-DOF \( q_1 \) and \( q_2 \) to control the linear and angular motions of the WMR. As the slave robot, we use our own custom-built WMR as shown in Fig. 3, which has two differential wheels, each driven by a dc-motor with 23 : 1 gear-box (Maxon Precision Motors Inc. 310007 and 166936), and one (un-actuated) caster wheel similar to that as depicted in Fig. 1. Wheel-base of the WMR is 29.2cm, and its actuated wheels’ radius is 3.6cm. The update-rates of the master-device and slave-WMR are 1ms and 2ms, respectively. They are connected over WLAN (wireless local area network) communication network with negligible delay/loss. Using some data-buffering, we then set the forward/backward communication delays to be \( d_1 = 0.2 \)sec and \( d_2 = 0.15 \)sec. The DIP-switches of the servo-drivers (Maxon Precision Motors Inc. 145391) allow us to control directly either the torque or the velocity of the dc-motors. The torque-mode then renders the WMR as a dynamic-WMR (i.e. Sections V-A and V-B), while the velocity-mode a kinematic-WMR (i.e. Sections V-C and V-D).

We also use the motion scaling \( \eta \) of Sec. II-B (for both kinematic and dynamic WMR) and the power-scaling \( \rho \) of Sec. II-B (for dynamic-WMR) to match the motion/power between the master and the slave. In particular, during our experiments, we found that the master motion/power are smaller than that of the WMR, as the human operators mainly used their wrist/hand to manipulate the PHANToM, while pivoting the elbow to the ground. Therefore, we used smaller motion scaling and power-scaling so that the slave WMR appears as if shrunk w.r.t. the (unscaled) master PHANToM, or, in other words, the master PHANToM appears as if amplified w.r.t. the (unscaled) WMR (e.g. with (23)-(24), \( q_2 \rightarrow \eta_2 \phi \) or \( \phi \rightarrow \frac{1}{\eta_2} q_2 \); from (27), \( \int_0^T [f^T r + \rho \delta^T \nu] dt \geq -c^2 \) or \( \int_0^T [\frac{1}{\rho} f^T r + \delta^T \nu] dt \geq -c^2/\rho \)). We chose these \( \eta \) and \( \rho \) by trial-and-error until satisfactory performance is achieved, without knowing exact system parameters (e.g. conversion factors of servo-drivers, motor constant, etc). Specific values of \( \eta \) and \( \rho \) we do not provide here, since 1) they are not so informative (yet possibly misleading), as they depend on not-accurate system parameters; and 2) they are completely test-bed dependent, yet also dispensable for others to do the same trial-error process.

We first implement the tele-driving control law of Sec. V-A with the following parameters: 1) for the local state-feedback (12), \( \lambda_1 = 1 \)sec\(^{-1} \), \((b_1, b_3) = (0.5, 0.5)\)Ns/m, \( b_2 = 1\)Nms/rad, \( k_1 = 35\)N/m, and; 2) for the tele-driving control (21)-(24), \( k_v = 10\)Ns/m, \( b_\phi = 3\)Nms/rad with \( b_d = 5\)Nms/rad and \( k_\phi = 25\)Nm/rad, satisfying (25) with \( d_1 + d_2 = 0.35 \)sec. Here, \( \lambda_1 = 1 \) was chosen by trial-and-error, that is, maximizing \( \lambda_1 \) while maintaining stability for a wide-range of scenarios. If the master device is linear, we may directly use the expression of Cor. 1 for \( \lambda_1 \). Yet, this is not the case here, since the PHANToM has complex nonlinear dynamics, although we did not observe any instability problem so far with this empirically-chosen \( \lambda_1 = 1 \) throughout many trials, experiments, and demonstrations.
Experimental result for the case 1 in Sec. V-A is shown in Fig. 4, where the human operator tries to tele-drive the dynamic-WMR with a constant forward-velocity $v$, while making it route around an obstacle (without touching it: around $7 - 15$ sec) and eventually proceed with the constant speed and fixed heading angle $\phi$ (after $15$ sec). As can be clearly seen there (and also expected from Th. 1), 1) tele-driving is stable, although performed over the communication delays with $d_1 + d_2 = 0.35$ sec; 2) human can perceive (constant) forward-velocity $v$ as haptic feedback (via the local spring $k_1$ - see item 2 of Th. 1); and 3) we have heading-angle coordination $q_2 \rightarrow \phi$ with small $f_2$ (after $18$ sec), while the human operator can perceive their coordination error via the haptic feedback (e.g. around $15$ sec).

We also performed an experiment where the WMR’s motion is blocked while human operator tries to move it - see Fig. 5, where for brevity, we only present the results of $(r_1, v)$-teleoperation. As can be seen there, in this case, the human operator can perceive the (constant) blocking force via the haptic feedback, again, as predicted in the item 2 of Th. 1. Of course, solely relying on the haptic feedback (item 2 of Th. 1), the human operator may not be able to discern if the haptic feedback (i.e. $f_1$) is from the (constant) forward-velocity $v$ or from the blocking force $\delta_1$ - see the item 2 of Th. 1. This ambiguity, however, can easily be remedied with some simple situation awareness (e.g. simple camera to see if the WMR stops or moves). Note also the motion-scaling $\eta$ in Figures 4-5 (i.e. instead of $r_1, q_2, \eta_1 q_2, \eta_2$).

In Fig. 4, we have some tracking error between $\lambda_1 q_1$ and $v$, although Th. 1 says otherwise. We found that this is due to the friction inherent to the WMR (e.g. carpet-wheel contact, gear-box, etc), which turn out to be not negligible. By increasing control gain $k_v$, we can reduce this error as can be anticipated in (33) (not shown here). Also, note that this friction effect is much less for the $(q_2, \phi)$-teleoperation. This we believe is because the friction does not always hinder the angular motion, although so it does the linear motion, particularly when these two motions are combined as in the experiment, that is, if $\dot{\phi} > 0$, the WMR’s left wheel would moves slower than the right wheel (i.e. $w_L < w_R$), yet, may still be moving forward (i.e. $w_L > 0$) with its friction still pushing backward, thus, not working against the turning $\dot{\phi} > 0$, although so doing against the forward motion $v > 0$.

The next Fig. 6 shows the experimental results with the control design case 2 of Sec. V-B, for which we use the following control parameters: 1) for the local state-feedback (12), $\lambda_1 = \lambda_2 = 1$ sec$^{-1}$, $(b_1, b_3) = (0.5, 0.5)$ Ns/m, $b_2 = 0.5$ Nms/rad, $k_1 = 30$ N/m, $k_2 = 20$ Nm/rad, and; 2) for the tele-driving control (21)-(22) and (34)-(35), $k_v = 10$ Ns/m, and $k_\phi = 2$ Nms/rad. The result in Fig. 6 is similar to that of Fig. 4. The only noteworthy difference is that, now, due to the change for the heading-angle teleoperation from the $(q_2, \phi)$-mode of Fig. 4 to the $(r_2, \dot{\phi})$-mode, the human can now perceive the WMR’s heading-angle rotation-rate $\dot{\phi}$ as the haptic feedback via the local spring $k_2$ - see the item 2 of Th. 2. Also, when the master device is released, due to the local spring action of $k_2$, $q_2 \rightarrow 0$, inducing $\dot{\phi} \rightarrow 0$, similar to the usual car-driving.

Experimental results for the control design case 3 of Sec. V-C are then presented in Fig. 7, for which we use the following control parameters: 1) for the local state-feedback (12), $\lambda_2 = 1$ sec$^{-1}$, $(b_1, b_3) = (1, 0.5)$ Ns/m, $b_2 = 2$ Nms/rad, $k_2 = 0.5$ Nm/rad, and; 2) for the tele-driving control (37)-(41),
\(b_d = 1\text{Ns/m}, \ k_d = 35\text{N/m}, \ k_v = 2.5\text{N/m}, \ \text{and} \ k_\phi = 2\text{Nm/rad}, \ \text{satisfying (39)}.\)

There, we can see the closed-loop system behaves as predicted in Th. 3, i.e. stable interaction even with communication-delay, coordination between \((q_1,v)\) and \((\lambda_2 q_2, \phi)\) with haptic feedback of the forward velocity (i.e. \(f_1 \rightarrow k_d v\)). Here, we choose \(k_2\) for the \((r_2, \phi)\)-teleoperation to be small to achieve similar behavior as that of the case 1 in Sec. V-A, that is, from (46), \(f_2 \approx -k_\phi(\lambda_2 - \phi)\) with small \(k_2\) and, if \(\phi \rightarrow \lambda_2 q_2, \ f_2 \approx 0\) (i.e. human perceive coordination-error between \(\lambda_2 q_2\) and \(\phi\), and no haptic feedback when \(\lambda_2 q_2 = \phi\): cf. item 2 of Th. 1). In contrast, we choose a large \(k_d\) to provide sharp haptic feedback of the forward velocity - see the item 2 of Th. 3.

The last Fig. 8 then contains the experimental results for the case 4 of Sec. V-D, for which we use the following control parameters: 1) for the local damping \(B\) in (12), \((b_1, b_3) = (0.4, 0.5)\text{Ns/m}, \ b_2 = 0.4\text{Nms/rad}, \ \text{and} \); 2) for the tele-driving control (37)-(38) and (47)-(48), \(b_d = 1.1\text{Ns/m}, \ k_d = 40\text{N/m}, \ k_v = 3\text{N/m}, \ b'_d = 1.1\text{Nms/rad}, \ k'_d = 50\text{Nm/rad}, \ \text{and} \ k'_\phi = 3\text{Nm/rad}, \ \text{satisfying (49)}.\) The closed-loop system behavior is similar to that of Fig. 7, except that, now, for the heading-angle teleoperation, the human can perceive the WMR’s heading-angle rate \(\dot{\phi}\) as the haptic feedback via the local spring \(k_d\), and, when the human releases the master device’s \(q_2\) (i.e. \(f_2 \rightarrow 0\) around 25sec), due to the local spring \(k_d\), \(q_2 \rightarrow 0\), inducing \(\dot{\phi} \rightarrow 0\), emulating the usual car-driving. Note that, here, we do not need \(\lambda_1\) nor \(\lambda_2\), and, also, \(b_1, b_2, b_3\) can be zero, since, for this control design case 4, we do not need to use the procedure of the modified passivity with \(r = \dot{q} + \Lambda q\) (11) of Sec. IV.

VII. Conclusions

In this paper, we propose haptic tele-driving control frameworks between a \(n\)-DOF nonlinear Lagrange master haptic joystick and a kinematic or dynamic WMR over communication channels with constant time-delays. To address master-slave kinematic dissimilarity, the proposed frameworks achieve car-driving metaphor, enabling human users to tele-drive the WMR much like as they drive a car: one-DOF of the master is used as a gas-pedal to control the forward-velocity of the WMR; while another-DOF of the master as a steering-wheel to adjust the heading-angle (or rate) of the WMR. The frameworks also enforce passivity of the closed-loop system, thereby, guaranteeing interaction stability of the tele-driving system with (any passive) human users and slave environments, while providing some useful haptic-feedback (e.g. cruising-speed, contact-force, turning-rate, etc). Experiment is also performed to validate/highlight the properties of the control frameworks.

There are several possible directions for future research. First, we think that the proposed frameworks may also be used/modified for tele-driving of other types of (uninhabited) locomotion systems (e.g. underwater or aerial vehicle [30]), or multiple of them (see [29] for a result in this direction). Another topic, which will be practically important, is how to include obstacle avoidance capability into the proposed frameworks. This can be done for the dynamic-WMR (2) simply by adding some obstacle

\[\text{We may think} \ k_v \ \text{to have dual-units: in (37), unit of [N/m], when multiplied to} \ q_1; \ \text{while [Ns/m] to} \ v(t - d_2). \ \text{In implementation, this can be done just by using value of} \ v \ \text{in (37) as position-signal. For simplicity, here, we provide gains w.r.t. their master variables.}\]
avoidance artificial potential terms in $u$. This will not affect system’s stability, since the potential field is intrinsically passive. For the kinematic-WMR (3), we may think of scaling down $u$ when the WMR enters into “dangerous/collision-prone” zones, although its implication on stability/performance is a topic for future research. The other topics include: experimental verification of $r$-passivity assumption for humans (26) for the WMR delayed tele-driving; and to include/analyze other un-modeled phenomena (e.g. wheel slippage).

**Appendix**

**Derivation of (43):** Among the terms in the first line of (43), only the last term poses some difficulty for obtaining the inequality (43). However, following the procedure for proving [28, Lem.1], we can still show that this last term is bounded as follows: $\forall T \geq 0$,

$$-\int_0^T \dot{q}_1(t)[q_1(t) - q_1(t - \bar{d})] dt = \int_0^T \dot{q}_1(t) \left[ \int_0^\bar{d} \dot{q}_1(t - \theta) d\theta \right] dt$$

$$\leq \int_0^T \dot{q}_1^2 dt \times \int_0^T \left[ \int_0^\bar{d} \dot{q}_1(t - \theta) d\theta \right] \leq \frac{\bar{d}}{2} \int_0^T \dot{q}_1^2 dt + \frac{1}{2} \int_0^T \dot{q}_1(t - \theta) d\theta$$

$$\leq \frac{\bar{d}}{2} \int_0^T \dot{q}_1^2 dt + \frac{\bar{d}^2}{2} \max_{t \in [-\bar{d}, 0]} |\dot{q}_1(t)|^2$$

(51)

where we use: for the second line, Cauchy-Schwartz inequality (in integral form) [31] (i.e. $\left[ \int_0^T x(t) y(t) dt \right]^2 \leq \int_0^T x(t)^2 dt \times \int_0^T y(t)^2 dt$) applied both to the last term of the first line of (51) and $\left[ \int_0^\bar{d} \dot{q}_1(t - \theta) d\theta \right]^2$, and the Young’s inequality (i.e. $2\alpha/\beta \leq \bar{d} \alpha^2 + \frac{1}{2} \beta^2$); and for the third line, we chance the integration order of the last term of the second line of (51) and the facts that $\int_{-\theta}^\theta \dot{q}_1^2(t) dt \geq \int_0^\theta \dot{q}_1^2(t) dt$ and $\int_{-\theta}^\theta \dot{q}_1^2(t) dt \geq \int_{-\theta}^0 \dot{q}_1^2(t) dt$ with $\theta \in [-\bar{d}, 0]$.

This inequality (51) slightly extends [28, Lem.1] to the case of $\dot{q}_1(t) \neq 0$ for $t \leq 0$ (i.e. controller turn-on with non-stationary initial condition). This inequality (51), in fact, can also be derived using the frequency-domain results of [13, e.g. Eq.(45)] with Parseval’s identity, yet, here, for simplicity, we follow their time-domain translations as proposed in [28].

**References**


Fig. 4. Case 1: dynamic WMR \((r_1, v)/(q_2, \phi)\) tele-driving

Fig. 5. Hard Contact - Case 1: dynamic WMR \((r_1, v)/(q_2, \phi)\) tele-driving

Fig. 6. Case 2: dynamic WMR \((r_1, v)/(r_2, \dot{\phi})\) tele-driving
Fig. 7. Case 3: kinematic WMR \((q_1, v)/(r_2, \phi)\) tele-driving

Fig. 8. Case 4: dynamic WMR \((q_1, v)/(q_2, \dot{\phi})\) tele-driving