Application of Game Theory to Power and Rate Control for Wireless Data

by

Mohammad Suleiman Hayajneh

B.S., Electrical Engineering, Jordan University of Science and Technology, 1995

M.S., Electrical Engineering, Jordan University of Science and Technology, 1998

DISSERTATION

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

Engineering

The University of New Mexico

Albuquerque, New Mexico

July, 2004

Dedication

To my dear teacher, Professor Chaouki T. Abdallah
and to my parents, sisters and brothers for their continuous inspiration

إِلَى أُستَاذي العَزيز، د. شَوقِي ط. عَبدَاللّٰه
وَ إِلَى والدَيَّ وَ أَخَوَاتِي وَ إِخوَا ني لإِلهَامِهِم المُستَمِر

Acknowledgments

I would like to thank my advisor, Professor Chaouki Abdallah, for his support, continuous guidance and inspiration and for introducing me to the field of this interesting research. I would also like to thank the dissertation committee: Professor Majid Hayat, Professor Christos Christodoulou and Professor Vladimir Koltchinskii for their insightful comments to make this research a better one. Deep thanks go to my friends: Majid Khodier, Moad Mowafi and Ousseini Lankoande for their support and encouragement.

Application of Game Theory to Power and Rate Control for Wireless Data

by

Mohammad Suleiman Hayajneh

ABSTRACT OF DISSERTATION

Submitted in Partial Fulfillment of the
Requirements for the Degree of

Doctor of Philosophy
Engineering

The University of New Mexico

Albuquerque, New Mexico

July, 2004

Application of Game Theory to Power and Rate Control for Wireless Data

by

Mohammad Suleiman Hayajneh

B.S., Electrical Engineering, Jordan University of Science and Technology, 1995
M.S., Electrical Engineering, Jordan University of Science and Technology, 1998

Ph.D., Engineering, University of New Mexico, 2004

Abstract

The new generations of CDMA communications systems (3G, B3G and 4G) and *adhoc* networks are expected to support multirate services (multimedia applications, email, Internet, etc.) in addition to telephone service (fixed-rate service), which was the only service offered by 1G and 2G. Each user in these new generations of communications systems has different quality of services (QoSs) (e.g., signal-to-interference ratio (SIR), frame error rate (FER) and data rate) that he/she is willing to fulfill by accessing the common radio interface. These new services establish a strong need for new algorithms that enable the efficient spectral use of the common radio interface.

Due to the strong relation between the SIR and the data rate at which a user can send information as was shown by Shannon, it was natural to propose joint power

and rate control algorithms for wireless data. Prior work that has emerged to address this problem has used a centralized algorithm.

Our approach to solve the problem of jointly optimizing rate and power for wireless data in game-theoretic framework relies on two layered games: Game $G1$ is a non-cooperative rate control game with pricing (NRGP). It sets the rules for the users to enable them to reach a unique rate Nash equilibrium (NE) operating point that is the most socially desired operating point (Pareto efficient)in a distributed fashion. Game $G2$, on the other hand, is a non-cooperative power control game with pricing (NPGP). $G2$ admits a unique power Nash equilibrium operating point that supports the resulting rate Nash equilibrium point of game $G1$ with lowest possible transmit power level (Pareto efficient).

In this dissertation we also propose two new distributed games: New NPGP game to optimize the transmit power for wireless data in CDMA uplink. With the rules of this game, mobile users were able to achieve higher than their minimum required SIRs (signal-to-interference ratio) with a reasonable small transmit power levels as compared to other existing NPGP games. New NPGpr game to minimize the fading induced outage probability in an interference limited wireless channels by maximizing certainty-equivalent-margin(CEM) under Nakagami and Rayleigh channels. Analysis of this NPG game shows that under Rayleigh and Nakagami (with fading figure $m = 2$) the best policy for all users is to set their transmit power to the minimum level.

Moreover, we studied the performance of NPG and NPGP games proposed by Saraydar *et. al.* in realistic fading wireless channels and we showed how the strategy spaces of the mobile users should be modified to guarantee the existence and uniqueness of NE operating point.

Contents

List of Figures

List of Tables

Glossary

p_i	transmit power level of user i in Watts
d_i	distance between the BS and user i in meters
h_i	path gain between user i and the BS
P_i	power strategy space of the ith user
p_{i-max}	maximum transmit power level in P_i
p_{i-min}	minimum transmit power level in P_i
p_{-i}	transmit power vector of all users except for the ith user
R_i	transmission rate (bits/s) of the ith user
u_i	utility factor of the ith user
λ; c	pricing factor chosen by the BS
γ_i	attained SIR of the ith user at the BS
W	spread spectrum bandwidth or the chip rate in Hz
G_i	spreading gain of user i

N	number of users currently served in the cell
σ^2	variance of the background AWGN at the receiver in the BS
L_i	target function of the ith user to be optimized
NPG	Non-cooperative power control game
NPGP	Non-cooperative power control game with pricing
NRGP	Non-cooperative rate control game with pricing
$\mathcal{N}$	indexing set of the users in the cell
p_i^o	optimizing transmit power level (watts) of user i
I_{-i}	sum of received powers from all users except the ith user
I	sum of received powers from all users and AWGN power
γ_i^*	minimum required SIR of user i
γ^*	vector of all minimum required SIRs of all users
p	Nash equilibrium operating point
$\varphi_i(p_i, \mathrm{p}_{-i})$	arbitrary scalar function
$\mathcal{S}$	equilibria set of an S-modular game

p_s, p_l	smallest and largest elements in the set $\mathcal{S}$, respectively
$\vec{\phi}(\mathrm{p})$	arbitrary vector function
δ	scalar larger than one
$J_i(p_i, \mathrm{p}_{-i})$	cost function in [26]
p_i^*	minimizer transmit power
$\vec{u}$	vector of all utility factors of users in the cell
G_2^{λ}	NPGP game with pricing factor λ
$\alpha_{i,j}$	fading coefficient between users j and transmitter i
M	number of bits in a packet (frame)
L	number of information bits in a frame
Δt_c	Coherence time of the fading channel
Δf_c	Coherence bandwidth of the fading channel
T_m	Multipath delay spread
B_d	Doppler spread of multipath channel
TX	Transmitter
RX	Receiver
BS	Base station
QoS	Quality of service

Glossary

CDMA	Code division multiple access
DS	Direct sequence
SIR	Signal-to-interference ratio
MS	Mobile station

Chapter 1

Introduction

The demand for high data rates in modern wireless CDMA communications systems, which support multirate services, increases the need to efficiently use the available radio channel bandwidth as the shared resource by the network users. The relationship between the signal-to-interference ratio (SIR) and channel capacity, as shown by Shannon [25], inspired many centralized schemes of balancing a target SIR for all users using the communication system [11]-[14].

Because of the difficulty in implementing centralized power control algorithms, and to avoid the extensive number of control signals that cause delays in the system operation, a need for distributed algorithms arose. In earlier distributed algorithms that does not use game-theoretic framework, each user is expected to allocate his own power iteratively based on local measurements to meet SIR constraints. In general, these algorithms result in large transmitter power requirements. Such algorithms may be found in [1]-[4].

To find distributed algorithms that efficiently use the transmit power, game theory was proposed. In a game-theoretic distributed power control algorithm, each user efficiently chooses his transmit power level in an attempt to optimize a target function. This target function maps the preferences and desires of the user (e.g., SIR, FER, data rate, etc.) into the real line. Such distributed algorithms are found in [5]-[10].

The mathematical theory of games was introduced by John Von Neumann and Oskar Morgenstern in 1944 [17]. In the late 1970's game theory became an important tool in the analyst's hand whenever he or she faces a situation in which a player's decision depends on what the other players did. Game theory has been used by economists for long time to study how rational individuals interact to reach their goals.

Our focus in this research will be on non-cooperative games which is a subclass of game theory. A player in a non-cooperative game, responds individually to the actions of other players by choosing a strategy from his strategy space in an attempt to optimize a target function that quantifies its QoS.

The power control problem for wireless data CDMA systems was first addressed in the game theoretic framework in [5], then in a more detailed manner in [6] and [8]. The reason that game theory attracts researchers in the power control field is that it offers a good insight into the strategic interactions between rational agents (cellular users), and generates efficient outcomes according to the players' preferences [19], [29].

In the remainder of this Chapter we will introduce some of the vocabulary of game theory that appear in the dissertation.

1.1 What is a Game ?

A game is a situation where a rational agent's decision or choice that is selected from his action profile to maximize his pay off, depends on the actions and decisions of the other rational agents. And all these rational agents have potential conflicting objectives. Therefore, the elements of a game are :

- A group of rational players
- Strategy spaces from which players choose their actions
- A functional description of player's preferences
- Strategic interdependence, that is a player's decision depends on others decisions

In this research, we shall concentrate on one type of games, that is the non-cooperative game, since it represents the best game theoretic framework for the distributed power and rate control algorithms.

1.2 Utility Function

A player in a game has objectives and preferences which are combined into what is called a utility function by the economists. A utility function quantifies the level of satisfaction a player obtains by adopting a strategy from his action (strategy) profile. Mathematically, a utility function maps the preferences and goals of a player into the real numbers. One should note, that the utility function is not unique, since any function that puts all the elements of the game in a desired order is a candidate function. For this reason, we may use different utility functions in the various Chapters of this dissertation.

1.3 Non-Cooperative Game

A non-cooperative game is a game in which each player decides, based on local information, his strategy in response to the other players' decisions. The functional description of the player's preferences (utility function) in this kind of game should be designed such that the best responses of all players converge to a fixed operating point. This operating point is known as a Nash equilibrium point as introduced by John Nash in 1951 [43].

A Nash equilibrium point is interpreted as the operating point where no player can unilaterally improve his utility by changing his strategy (on the individual's level it is the best point). Unfortunately, Nash equilibrium point may not be the most desired social point, that is the point seen by all users. The most desired social point is called a *Pareto optimal* point in game theory [29]. The interpretation of Pareto optimal point is that no player can improve his utility by adopting a different strategy without harming at least one other player. In other words, a Pareto optimal point maximizes the aggregate utility of all players in the game.

To improve the performance of a Nash equilibrium point in non-cooperative games, a modified version of non-cooperative games was suggested by economists. In this modified version, a pricing technique on the game resources was introduced. Such modified non-cooperative game is called non-cooperative game with pricing. Any improvement in the Nash equilibrium point performance of a non-cooperative game with pricing, is called *Pareto dominance* with respect to the performance of Nash equilibrium point of pure non-cooperative game.

1.4 Motivation

In wireless data systems that are expected to support multirate services (multimedia applications, Internet, etc.), users may desire to have a different SIRs at their receivers coupled with the lowest possible transmit powers. The importance of having a high SIR in such systems results from the need of a low error rate, a more reliable system, and high channel capacity, which allow users to transmit at higher bit rates [11],[15]. It is also important to decrease the transmit power because low-power levels help alleviate the ever present near-far problem in CDMA systems [16] and increase the lifetime of the battery in a mobile unit or in a node of *adhoc* networks.

To simultaneously achieve these two goals many papers have emerged within the game-theoretic framework [6]-[8], or outside such framework [1]-[4]. Our focus in this research will be on power-control algorithms and joint rate and power control algorithms within a game-theoretic framework.

In [8] the authors proposed asynchronous distributed algorithms for uplink CDMA wireless data in a single cell. The authors did not, however, consider the statistical variation of the power in a realistic wireless channel, as they only considered an additive-white-Gaussian noise (AWGN) channel. Wireless channels are known to exhibit multipath fading, and multipath wireless channels experience two kinds of fading: Large-scale and small-scale fading. Large-scale fading results if the distance between the transmitter and the receiver is relatively large and the main contribution of the received signal comes from the reflections of the transmitted signal. The large-scale fading parameter is modeled as a log normal random variable, and it is sometimes called log-normal shadowing. Small-scale fading, on the other hand, occurs in heavily populated urban areas with a short distance between the transmitter and the receiver, and the main contribution of the received signal comes from the scattering of the transmitted signal. The small-scale fading parameter is usually modeled as Rayleigh, Rician, or Nakagami random variables.

In this research we examine the algorithms of [8] in a flat-fading channel (see Appendix B for the description of flat-fading channels) and modify the algorithms and the strategy spaces to fit the flat-fading channel model.

In [26], the authors proposed a cost (utility) function, which is the difference between a utility function and a pricing function. The proposed utility function is proportional to the capacity of the channel, while the pricing function is linear in the user's transmit power in market-based pricing scheme. The existence and uniqueness of a Nash equilibrium point was established in [26], and moreover, the authors offered different schemes of pricing and two methods of updating the user's transmit power. One limitation, however, of the cost function in [26] is that under *market-based pricing*, and if the users desire SIRs such that their utility factors are the same, the transmit power level will increase as the mobile comes closer to the base station (BS). Another limitation of the proposed algorithms in [26], is that they result in an unnecessarily high power at equilibrium.

In light of the two limitations mentioned above, we propose a new target (utility or cost) function that helps the users of CDMA systems, or *adhoc* networks, to operate on equilibrium points that support the lowest possible power with guaranteed different QoSs for the different users.

Each user in the new CDMA wireless generations and *adhoc* wireless networks has unique QoS requirements. Therefore, there must be realistic algorithms that take care of supporting the needs of each user in the network. To adopt more realistic (multi-objective) algorithms we study a joint power and rate control algorithm for wireless data in a game-theoretic framework. Invoking the rate in the joint optimization problem provides a fairness criterion for the power control algorithm. In other words, we need to guarantee that the obtained SIRs at equilibrium are enough for all transmitters to establish a communication link with their corresponding receivers at the required data rate.

1.5 Outline

The dissertation is organized as follows: Chapter 2 is devoted to studying and modifying the algorithms in [8] in a fast/slow flat-fading wireless channels. Small-scale fading is studied using three models: Rayleigh, Rician and Nakagami channel models. In Chapter 3 we apply statistical learning theory to overcome the lack of prior knowledge of the channel model, where we learn the utility function class under a slow flat-fading channel model. As an example to validate our analysis, we show a successful application of distribution-free learning theory when the channel is modeled as a Rayleigh slow flat-fading channel. Chapter 4 presents a distributed algorithm of optimizing the outage probability for CDMA system users in an interference-limited fading wireless channel. In Chapter 5 we propose a new power control algorithm that results in a low transmit powers compared to existing algorithms. Chapter 6 presents a study of game-theoretic jointly power and rate control for wireless data. In Chapter 7, we conclude our results in this dissertation and give our thoughts of possible related future work.

1.6 Contributions

In this section we list our contributions in this dissertation as follows:

- Non-cooperative power control games (NPG) and non-cooperative power control games with pricing (NPGP) in [8] were extended for a realistic wireless channels

- A distribution-free learning theory was applied to evaluate the performance of NPG and NPGP in slow flat-fading wireless channels
- We combined distribution-free learning theory and game theory to study the performance of NPG and NPGP algorithms for wireless data
- Successful NPG was proposed to minimize the outage probability in an interference limited multicell CDMA network
- Successful new utility function was proposed for wireless CDMA uplink which results in a very low power compared to existing ones.
- Introducing the first practical game-theoretic joint power and rate control for wireless data.

Chapter 2

Game Theoretic Power Control Algorithms in Flat-Fading Channels

We consider in this chapter a game-theoretic power control algorithm for wireless data in a fading channel. This algorithm depends on an average utility function that assigns a numerical value to the quality of service (QoS) a user gains by accessing the channel. We also study the performance of the game-theoretic power algorithms introduced by [8] for wireless data in the realistic channels: **(a1)** Fast flat-fading channel and **(a2)** Slow flat-fading channel. The fading coefficients under both **(a1)** and **(a2)** are studied for three appropriate small-scale channel models that are used in CDMA cellular systems: A Rayleigh channel, a Rician channel and a Nakagami channel. Our results show that in a non-cooperative power control game (NPG) the best policy for all users in the cell is to target a fixed signal-to-interference and noise ratio (SIR) similar to what was shown in [8]. The difference, however, is that the target SIR in fading channels should be much higher than that in a nonfading channel.

The remaining of this chapter is organized as follows: In Section 2.1 we present the utility function and the system model studied in this chapter. In Section 2.2 we evaluate the performance of the system using the channel models mentioned above. Non-cooperative power control game (NPG) and Non-cooperative power control game with pricing (NPGP) are discussed in Sections 2.3 and 2.4, respectively. Then we establish the existence and uniqueness of Nash equilibrium points for NPG and NPGP under the assumed channel models in Section 2.5. Simulation results are outlined in Section 2.6. Finally, we summarize our results in Section 2.7.

2.1 Utility Function and System Model

We use the concept of a utility function to quantify the level of satisfaction a player can get by choosing an action from its strategy profile given the other players' actions, that is, a utility function maps the player's preferences into the real line. A formal definition of utility functions is available from [29].

Definition 2.1.1. *A function u that assigns a numerical value to the elements of the action set $A, u : A \to \mathbb{R}$ is a utility function if for all $a,\ b \in A$, action a is at least as preferred compared to b if $u(a) \geq u(b)$.*

In a cellular CDMA system there are a number of users sharing a spectrum and the air interface as a common radio resource. Henceforth, each user's transmission adds to the interference of all users at the receiver in the base station (BS). Each user desires to achieve a high quality of reception at the BS, i.e., a high SIR, by using the minimum possible amount of power to extend the battery's life.

The goal of each user to have a high SIR at the BS produces conflicting objectives that make the framework of game theory suitable for studying and solving the problem.

In this chapter we consider a single-cell DS-CDMA (direct sequence code division multiple access) system with N users, where each user transmits frames (packets) of M bits with L information bits ($M-L$ are parity bits) [8]. The rate of transmission is R bits/sec for all users. Let P_c represents the average probability of correct reception of all bits in the frame at the BS, in other words, P_c refers the average frame (packet) correct reception rate, and p represents the average transmit power level. As we know, P_c depends on the SIR, the channel characteristics, the modulation format, the channel coding, etc.

A suitable utility function for a CDMA system is given by (see [8] and references therein):

$$u = \frac{L\,R}{M\,p}\,P_c \tag{2.1.1}$$

where u thus represents the number of information bits received successfully at the BS per joule of expanded energy. With the assumption of no error correction, the *random* packet correct reception rate $\tilde{P}_c$, where $P_c = E[\tilde{P}_c]$, is then given as $\tilde{P}_c = \prod_{l=1}^{M}(1 - \tilde{P}_e(l))$, where $\tilde{P}_e(l)$ is the *random* bit error rate (BER) of the lth bit at a given SIR γ_i (c. f. (2.2.11), (2.2.27) and (2.2.45)).

We are assuming that all users in a cell are using the same modulation scheme, namely non-coherent binary frequency shift Keying (BFSK), and that they are transmitting at the same rate R.

2.2 Evaluation of The Performance

In this Section we find closed-form formulas of the average BERs and the average utility functions under the six assumed channel models. We then use these formulas to study the existence and uniqueness of Nash equilibrium points in Section 2.5.

The SIR γ_i at the receiver for the ith user is given by [15]:

$$\gamma_i = \frac{W}{R} \frac{p_i\, h_i\, \alpha_i^2}{\sum_{k \neq i}^{N} p_k\, h_k\, \alpha_k^2 + \sigma^2}, \tag{2.2.1}$$

where α_i is the path fading coefficient between ith user and the BS and it is a random variable that changes independently from bit to bit in a fast flat-fading channel **(a1)**. On the other hand, it changes independently from packet/frame to packet/frame in a slow flat-fading channel **(a2)**. In both cases: **(a1)** and **(a2)**, the fading coefficients among the different users are assumed to be independent. The parameter W is the spread spectrum bandwidth, p_k is the transmitted power of the kth user, h_k is the path gain between the BS and the kth user, and σ^2 is the variance of the AWGN (additive-white-Gaussian-noise) that represents the thermal noise in the receiver. For simplicity let us express the interference from all other users (i.e., all but user i) as x_{-i}, where

$$x_{-i} = \sum_{k \neq i}^{N} p_k\, h_k\, \alpha_k^2 \tag{2.2.2}$$

therefore (2.2.1) may be written as:

$$\begin{aligned} \gamma_i &= \frac{W}{R} \frac{p_i\, h_i}{x_{-i} + \sigma^2}\ \alpha_i^2 \\ &:= \gamma_i^{'} \alpha_i^2 \end{aligned} \tag{2.2.3}$$

For a given α_i and x_{-i}, the conditioned BER, $\tilde{P}(e|\gamma_i) = \tilde{P}(e|\gamma_i, x_{-i})$ (the dependence on x_{-i} comes through γ_i), of the ith user using non coherent BFSK is given by [15]:

$$\tilde{P}(e|\gamma_i, x_{-i}) = \frac{1}{2}\, e^{-\frac{\gamma_i}{2}} \tag{2.2.4}$$

The average BER and average utility functions for this modulation scheme are evaluated in this chapter for the following channel models: Rayleigh fast/slow flat-fading channel, Rician fast/slow flat-fading channel and Nakagami fast/slow flat-fading channel.

2.2.1 Rayleigh Flat-Fading Channel

In this case α_i is modeled as a Rayleigh random variable with a probability density function (PDF) given by (see Appendix B for the description of channel models):

$$f^{\alpha_i}(\omega) = \frac{\omega}{\sigma_r^2}\, e^{-(1/2\sigma_r^2)\omega^2}, \quad i = 1, 2, \cdots, N \tag{2.2.5}$$

In all following calculations, and as a consequence of the multiplicative channel model of small and large scale fading, it is assumed that $\sigma_r^2 = 1/2$. Using (2.2.3) and (2.2.5) the PDF of γ_i for a given x_{-i} is defined as:

$$f^{\gamma_i|x_{-i}}(\omega) = \frac{1}{\gamma_i'}\, e^{-\omega/\gamma_i'} \tag{2.2.6}$$

Rayleigh Fast Flat-Fading Channel

For the lth bit in the frame, we can rewrite the SIR (2.2.3) and the interference (2.2.2) for the ith user as follows:

$$\gamma_i(l) = \frac{W}{R}\frac{p_i\, h_i\, \alpha_i^2(l)}{x_{-i}(l) + \sigma^2} \tag{2.2.7}$$

$$x_{-i}(l) = \sum_{k\neq i}^{N} p_k\, h_k\, \alpha_k^2(l) \tag{2.2.8}$$

Assuming that both $\{\alpha_i(l)\}_{l=1}^{M}$ and $\{x_{-i}(l)\}_{l=1}^{M}$ are iid (identically independent distributed) random variables, and of course $\alpha_i(l)$ and $x_{-i}(l)$ are independent random variables. Henceforth, the averaged correct reception of all frame (packet) bits at the BS P_c is given as $(1 - P_e)^M$, where P_e is *averaged* BER for each bit in the frame, that is $P_e = E[\tilde{P}_e]$. We will calculate the averaged P_e next.

We can find the conditional error probability $\tilde{P}(e|x_{-i})$ by taking the average of

(2.2.4) with respect to $f^{\gamma_i|x_{-i}}(\omega)$:

$$\begin{aligned}
\tilde{P}(e|x_{-i}) &= E\left[\tilde{P}(e|\gamma_i, x_{-i})\right] \\
&= \int_0^\infty \tilde{P}(e|\omega, x_{-i})\, f^{\gamma_i|x_{-i}}(\omega) d\omega \\
&= \frac{1}{2\gamma_i^{'}} \int_0^\infty e^{-(\frac{2+\gamma_i^{'}}{2\gamma_i^{'}})\omega} d\omega \\
&= \frac{1}{2+\gamma_i^{'}} \qquad (2.2.9)
\end{aligned}$$

Notice that we dropped the bit index l because the average BER does not depend on l. For large SIR, (2.2.9) behaves as:

$$\tilde{P}(e|x_{-i}) \approx \frac{1}{\gamma_i^{'}} = \frac{x_{-i} + \sigma^2}{\frac{W}{R}\, p_i\, h_i} \qquad (2.2.10)$$

Now, we can find the averaged BER P_e by taking the expectation of (2.2.10):

$$\begin{aligned}
P_e &= E\left[\tilde{P}(e|x_{-i})\right] = \frac{E[x_{-i}] + \sigma^2}{\frac{W}{R}\, p_i\, h_i} \\
&= \frac{1}{\overline{\gamma_i}} \qquad (2.2.11)
\end{aligned}$$

where $\overline{\gamma_i}$ is the ratio of the mean of the received power from user i to the mean of the interference at the receiver as given by:

$$\overline{\gamma_i} = \frac{W}{R} \frac{p_i\, h_i}{\sum_{k\neq i}^{N} p_k\, h_k + \sigma^2} \qquad (2.2.12)$$

Therefore, the average utility function of the ith user is given by:

$$\boxed{u_i = \frac{L\ R}{M\ p_i}\ (1 - \frac{1}{\overline{\gamma_i}})^M} \qquad (2.2.13)$$

Rayleigh Slow Flat-Fading Channel

In a slow flat-fading channel model, α_i of the ith user is assumed to change independently for each packet/frame, that is $\alpha_i(1) = \alpha_i(2), \cdots, \alpha_i(M)$. Also, it is assumed that α_i and α_k are independent for all $i \neq k$. The averaged frame correct reception P_c is therefore given as the expectation of $(1 - \tilde{P}(e|\gamma_i, x_{-i}))^M$ with respect to the random variables γ_i and x_{-i}. This suggests rewriting (2.1.1) as:

$$u_i(\mathrm{p}|\gamma_i, x_{-i}) = \frac{L\ R}{M\ p_i}\ (1 - e^{-\gamma_i/2})^M \tag{2.2.14}$$

Where $\mathrm{p} = (p_1, p_2, \cdots, p_N)$ is the vector of transmit powers of all users. Note that $\tilde{P}_e$ was replaced by $2\tilde{P}_e$ to give the utility function $u_i(\mathrm{p}|\gamma_i, x_{-i})$ this property: $u_i(\mathrm{p}|\gamma_i, x_{-i}) \to 0$ as $p_i \to 0$ and $u_i(\mathrm{p}|\gamma_i, x_{-i}) \to 0$ as $p_i \to \infty$ [8]. One can evaluate $u_i(\mathrm{p}|x_{-i})$ as follows:

$$\begin{aligned}
u_i(\mathrm{p}|x_{-i}) &= \int_0^\infty u_i(\mathrm{p}|\omega, x_{-i})\, f^{\gamma_i|x_{-i}}(\omega)\, d\omega \\
&= \int_0^\infty \frac{L\ R}{M\ p_i}\ (1 - e^{-\omega/2})^M\ \frac{1}{\gamma_i^{'}}\ e^{-\omega/\gamma_i^{'}}\, d\omega \\
&= \frac{L\ R}{M\ p_i\, \gamma_i^{'}} \sum_{k=0}^{M} (-1)^k \binom{M}{k} \int_0^\infty\ e^{-(\frac{k}{2} + \frac{1}{\gamma_i^{'}})\,\omega}\ d\omega \\
&= \frac{L\ R}{M\ p_i} \sum_{k=0}^{M} \binom{M}{k}\ \frac{2\,(-1)^k}{k\,\gamma_i^{'} + 2}
\end{aligned} \tag{2.2.15}$$

For $\gamma_i^{'} \gg 1$, (2.2.15) can be approximated by:

$$u(\mathrm{p}|x_{-i}) \approx \frac{L\ R}{M\ p_i} \left(1 + \frac{1}{\gamma_i^{'}}\ \sum_{k=1}^{M} \binom{M}{k}\ \frac{2\,(-1)^k}{k} \right) \tag{2.2.16}$$

Averaging (2.2.16) with respect to x_{-i} we obtain the average utility function for high SIR:

$$\begin{aligned} u_i &= E\left[u_i(\mathrm{p}|x_{-i})\right] \\ &\approx \frac{L\ R}{M\ p_i}\left(1+\frac{E[x_{-i}]+\sigma^2}{\frac{W}{R}\,p_i\ h_i}\ \sum_{k=1}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k}\right) \\ &= \frac{L\ R}{M\ p_i}\left(1+\frac{1}{\gamma_i}\ \sum_{k=1}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k}\right) \\ &\boxed{u_i\ \approx\ \frac{L\ R}{M\ p_i}\ \left(1-\frac{\beta}{\gamma_i}\right)} \end{aligned} \qquad (2.2.17)$$

where $\beta = -\sum_{k=1}^{M}\binom{M}{k}\ \frac{2\,(-1)^k}{k} > 0$.

2.2.2 Rician Flat-Fading Channel

In this case, α_i is modeled as a Rician random variable with PDF given by (see Appendix B):

$$f^{\alpha_i}(\omega) = \frac{\omega}{\sigma_r^2}\,e^{(-\frac{\omega^2+s^2}{2\sigma_r^2})}\,I_0(\frac{\omega\ s}{\sigma_r^2}) \qquad (2.2.18)$$

Similarly to the Rayleigh case, we need to find the PDF of γ_i (see (2.2.3)) for fixed x_{-i} (see (2.2.2)):

$$f^{\gamma_i|x_{-i}}(\omega) = \frac{e^{-s^2}}{\gamma_i'}\ \ e^{-\omega/\gamma_i'}\ I_0(2s\sqrt{\frac{\omega}{\gamma_i'}}) \qquad (2.2.19)$$

where we assumed that $\sigma_r^2 = 1/2$ as we mentioned earlier.

Rician Fast Flat-Fading Channel

Similar to the Rayleigh fast flat-fading case, the averaged frame correct reception is given by $P_c = (1 - P_e)^M$, where P_e can be found as follows:

$$\begin{aligned}\tilde{P}(e|x_{-i}) &= \int_0^\infty \tilde{P}(e|\omega, x_{-i}) f^{\gamma_i|x_{-i}}(\omega) d\omega \\ &= \frac{e^{-s^2}}{2\gamma_i^{'}} \int_0^\infty e^{-\omega(\frac{1}{2}+\frac{1}{\gamma_i^{'}})} I_0\left(2s\sqrt{\frac{\omega}{\gamma_i^{'}}}\right) d\omega \end{aligned} \quad (2.2.20)$$

using the fact that $I_0(\zeta)$ can be written as:

$$I_0(\zeta) = \sum_{n=0}^{\infty} \frac{(\frac{\zeta}{2})^{2n}}{(n!)^2} \quad (2.2.21)$$

and substituting (2.2.21) in (2.2.20), and after few mathematical manipulations we obtain:

$$\tilde{P}(e|x_{-i}) = \frac{1}{2+\gamma_i^{'}} \; e^{s^2(-1+\frac{2}{2+\gamma_i^{'}})} \quad (2.2.22)$$

At high SIR ($\gamma_i^{'} \gg 1$), $\tilde{P}(e|x_{-i})$ may be approximated as:

$$\tilde{P}(e|x_{-i}) \approx \frac{1}{\gamma_i^{'}} e^{-s^2} = \frac{x_{-i} + \sigma^2}{\frac{W}{R} p_i \, h_i} \; e^{-s^2} \quad (2.2.23)$$

In order to find the final average error rate P_e, we need to find $\mu_{x_{-i}}$ the mean of x_{-i}.

$$\begin{aligned}\mu_{x_{-i}} &= E[x_{-i}] = E\left[\sum_{k\neq i}^{N} \alpha_k^2 p_k \, h_k\right] \\ &= \sum_{k\neq i}^{N} p_k \, h_k \, E[\alpha_k^2] = (1+s^2) \sum_{k\neq i}^{N} p_k \, h_k \end{aligned} \quad (2.2.24)$$

where we used the fact that [15]

$$E[\alpha_k^n] = (2\sigma_r^2)^{n/2} \, e^{(-\frac{s^2}{2\sigma_r^2})} \frac{\Gamma((2+n)/2)}{\Gamma(n/2)} \; 1F1[(2+n)/2, n/2; s^2/2\sigma_r^2] \quad (2.2.25)$$

where $\Gamma(.)$ is the Gamma function, and $1F1[a, b; y]$ is the confluent hypergeometric function [20]. By substituting for $\sigma_r^2 = 1/2$ and $n = 2$ in (2.2.25) we obtain the result in (2.2.24). Note that we have used the following special case of the confluent hypergeometric function $1F1[a, b; y]$ in calculating (2.2.24):

$$1F1[2, 1; s^2] = (1 + s^2)\, e^{s^2} \tag{2.2.26}$$

Finally, to obtain P_e we replace x_{-i} in (2.2.23) by $\mu_{x_{-i}}$, that is

$$\begin{aligned} P_e &\approx \frac{e^{-s^2}(\mu_{x_{-i}} + \sigma^2)}{\frac{W}{R} h_i\, p_i} \\ &= \frac{1}{\gamma_i^s} \end{aligned} \tag{2.2.27}$$

where

$$\gamma_i^s = \frac{\frac{W}{R} h_i\, p_i \;\; e^{s^2}}{(1 + s^2)\sum_{k \neq i}^{N} h_k\, p_k + \sigma^2} \tag{2.2.28}$$

Then, the utility function of the ith user is given by

$$\boxed{u_i = \frac{L\ R}{M\ p_i}\,(1 - \frac{1}{\gamma_i^s})^M} \tag{2.2.29}$$

Rician Slow Flat-Fading Channel

Following the same argument of the Rayleigh slow flat-fading case, we find the average P_c or equivalently the average utility function of the ith user as follows.

$$\begin{aligned} u_i(\mathrm{p}|x_{-i}) &= \int_0^\infty u_i(\mathrm{p}|\omega, x_{-i}) f^{\gamma_i|x_{-i}}(\omega) d\omega \\ &= \int_0^\infty \frac{L\ R}{M\ p_i}\,(1 - e^{-\omega/2})^M\ \frac{e^{-s^2}}{\gamma_i'}\ e^{-\omega/\gamma_i'}\ I_0(2s\sqrt{\frac{\omega}{\gamma_i'}})\ d\omega \end{aligned} \tag{2.2.30}$$

By substituting (2.2.21), factorizing $(1 - e^{-\gamma_i/2})^M$, and after few mathematical manipulations we obtain:

$$u_i(\mathrm{p}|x_{-i}) = \frac{L\,R}{M\,p_i} \sum_{k=0}^{M} (-1)^k \binom{M}{k}\ \frac{2}{k\,\gamma_i' + 2}\ e^{s^2(-1 + \frac{2}{k\,\gamma_i' + 2})} \tag{2.2.31}$$

For large SIR, $\gamma_i^{'} \gg 1$ the above equation can be approximated by:

$$u_i(\mathrm{p}|x_{-i}) \approx \frac{L\,R}{M\,p_i}\left[1 + \frac{e^{-s^2}}{\gamma_i^{'}}\sum_{k=1}^{M}(-1)^k\binom{M}{k}\frac{2}{k}\right] \tag{2.2.32}$$

Averaging (2.2.32) with respect to x_{-i} we obtain the final approximate averaged utility function of the ith user in the following form:

$$u_i \approx \frac{L\,R}{M\,p_i}\left[1 + \frac{1}{\gamma_i^s}\sum_{k=1}^{M}(-1)^k\binom{M}{k}\frac{2}{k}\right] \tag{2.2.33}$$

$$\boxed{u_i \approx \frac{L\,R}{M\,p_i}\left[1 - \frac{\beta}{\gamma_i^s}\right]}$$

with γ_i^s given by (2.2.28).

2.2.3 Nakagami Flat-Fading Channel

In this case α_i is modeled as a Nakagami random variable with PDF given by (see Appendix B) [15]:

$$f^{\alpha_i}(\omega) = \frac{2m^m}{\Gamma(m)\Omega^m}\,\omega^{2m-1}\,e^{(-\frac{m}{\Omega})\omega^2}, \tag{2.2.34}$$

Note that by setting $m = 1$ the Nakagami PDF reduces to the Rayleigh PDF. In the following calculations it is assumed that $\Omega = 1$. Then the PDF of γ_i for fixed x_{-i} is given by:

$$f^{\gamma_i|x_{-i}}(\omega) = \frac{1}{\Gamma(m)}\left(\frac{m}{\gamma_i^{'}}\right)^m \omega^{m-1}\,e^{-(\frac{m}{\gamma_i^{'}})\,\omega} \tag{2.2.35}$$

Nakagami Fast Flat-Fading Channel

Here, we find the conditioned error probability $\tilde{P}(e|x_{-i})$ by taking the average of (2.2.4) with respect to $f^{\gamma_i|x_{-i}}(\omega)$:

$$\begin{aligned}\tilde{P}(e|x_{-i}) &= \int_0^\infty \tilde{P}(e|\omega, x_{-i})\, f^{\gamma_i|x_{-i}}(\omega) d\omega \\ &= \frac{1}{2\Gamma(m)} \left(\frac{m}{\gamma_i^{'}}\right)^m \int_0^\infty \omega^{m-1}\; e^{-(\frac{\gamma_i^{'}+2m}{2\gamma_i^{'}})\,\omega} d\omega \\ &= \frac{1}{2}\left(\frac{2m}{2m+\gamma_i^{'}}\right)^m \end{aligned} \tag{2.2.36}$$

For fixed m and $\gamma_i^{'} \gg 1$, (2.2.36) can be rewritten as:

$$\tilde{P}(e|x_{-i}) \approx \frac{1}{2}\left(\frac{2m}{\gamma_i^{'}}\right)^m \tag{2.2.37}$$

To find the *average* P_e, we need to find the mean of $(x_{-i}+\sigma^2)^m$. Here, x_i is the sum of independent random variables, each distributed according to a Gamma density function. This makes the evaluation of $(x_{-i}+\sigma^2)^m$ a tedious mathematical job. In such a case it is easier to find an approximate density function of x_i. To do this, we recall Esseen's inequality which estimates the deviation of the exact PDF of a sum of independent variables from the normal PDF.

Theorem 2.2.1. *[21] let $Y_1, \cdots, Y_N$ be independent random variables with $E[Y_j] = 0$, $E[|Y_j|^3] < \infty \quad (j = 1, \cdots, N)$. Let*

$$\sigma_j^2 := E[Y_j^2], \quad B_N := \sum_{j=1}^N \sigma_j^2, \;\; L_N := B_N^{-3/2} \sum_{j=1}^N E[|Y_j|^3]$$

Let $\psi_K(z)$ be the c.f. (cumulative distribution) of the random variable $B_N^{-1/2}\sum_{j=1}^N Y_j$. Then

$$|\psi_N(z) - e^{-z^2/2}| \le 16\, L_N\; |z|^3\; e^{-z^2/3} \tag{2.2.38}$$

Proof : *See [21].* □

Let us define $\tilde{Y}_k := p_k h_k \alpha_k^2$ and $Y_k := \tilde{Y}_k - p_k h_k$. By simple calculations we find that $\tilde{Y}_k, (k = 1, \cdots, N)$ are Gamma distributed random variables, such that

$$f^{\tilde{Y}_k}(\omega) = \frac{(m/p_k h_k)^m}{\Gamma(m)} \; \omega^{m-1} \, e^{-(m/p_k h_k)\omega}$$

and $E[\tilde{Y}_k] = p_k h_k$, which means that $Y_k, (k = 1, \cdots, N)$ are zero mean random variables. The values of $\sigma_k^2 = E[Y_k^2]$ are $(p_k h_k)^2/m, \forall k = 1, \cdots, N$, and therefore, $B_N = \frac{1}{m}\sum_{k=1}^{N}(p_k h_k)^2$. It is fairly simple to find out that the third moment $E[|Y_k|^3] = E[Y_k^3] = \frac{2(p_k h_k)^3}{m^2}$ $(Y_k \geq 0)$. Then,

$$L_N = \frac{2\sum_{k=1}^{N}(p_k h_k)^3}{\sqrt{m}\;(\sum_{k=1}^{N}(p_k h_k)^2)^{3/2}}.$$

For large N, L_N has a very small value, i.e., $L_N << 1$. By examining (2.2.38) for small values of z, L_N takes care of righthand side of the inequality, making it very small. For large values of z, on the other hand, the exponential term $e^{-z^2/3}$ will decrease the bound and make it approach zero. In conclusion, we can approximate x_{-i} as a Gaussian random variable with mean $\zeta_{x_{-i}}$ and variance $\overline{\sigma}^2_{x_{-i}}$ as given by:

$$\begin{aligned}
\zeta_{x_{-i}} &= E[x_{-i}] = E\left[\sum_{k \neq i}^{N} \alpha_k^2 p_k \, h_k\right] \\
&= \sum_{k \neq i}^{N} p_k \, h_k \, E[\alpha_k^2] \\
&= \sum_{k \neq i}^{N} p_k \, h_k
\end{aligned} \tag{2.2.39}$$

and

$$\begin{aligned}\overline{\sigma}_{x_{-i}}^2 &= E[x_{-i}^2] - \zeta_{x_{-i}}^2 \\ &= E\left[\sum_{l\neq i}^{N}\sum_{k\neq i}^{N} p_l h_l p_k\, h_k\, \alpha_l^2\, \alpha_k^2\right] - \zeta_{x_{-i}}^2 \\ &= \frac{1}{m}\sum_{k\neq i}^{N}(p_k\, h_k)^2 \end{aligned} \tag{2.2.40}$$

where (2.2.40) was obtained using the fact that α_k and α_l are statistically independent for all $k \neq l$. Thus, we can write $\pi(x_{-i})$, the PDF of x_{-i}, as follows:

$$\pi(x_{-i}) = \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_{-i}}}\, e^{-\frac{(x_{-i}-\zeta_{x_{-i}})^2}{2\overline{\sigma}_{x_{-i}}^2}} \tag{2.2.41}$$

where $x_{-i} \geq 0$. Averaging (2.2.37) over $\pi(x_{-i})$ we obtain the average error probability P_e for high SIR as follows:

$$\begin{aligned} P_e &\approx \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_0^\infty \left(x_{-i}+\sigma^2\right)^m \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_{-i}}}\, e^{-\frac{(x_{-i}-\zeta_{x_{-i}})^2}{2\overline{\sigma}_{x_{-i}}^2}}\, dx_{-i} \\ &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_{\sigma^2}^\infty y^m \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_{-i}}}\, e^{-\frac{(y-(\zeta_{x_{-i}}+\sigma^2))^2}{2\overline{\sigma}_{x_{-i}}^2}}\, dy \\ &\approx \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\, p_i\, h_i}\right)^m \int_0^\infty y^m \frac{1}{\sqrt{2\pi}\overline{\sigma}_{x_{-i}}}\, e^{-\frac{(y-(\zeta_{x_{-i}}+\sigma^2))^2}{2\overline{\sigma}_{x_{-i}}^2}}\, dy \end{aligned} \tag{2.2.42}$$

where we used the change of variable $y = x_{-i} + \sigma^2$ and the last approximation in (2.2.42) was based on the fact that $\sigma^2 \ll 1$. By examining (2.2.42) one can see that it represents the mth moment of a random variable normally distributed with mean

$$\zeta_y = \zeta_{x_{-i}} + \sigma^2$$

and variance

$$\sigma_y^2 = \overline{\sigma}_{x_{-i}}^2.$$

Therefore, the average P_e is given by:

$$\begin{aligned} P_e &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\,p_i\,h_i}\right)^m E\left[y^m\right] \\ &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\,p_i\,h_i}\right)^m E\left[((y-\zeta_y)+\zeta_y)^m\right] \\ &= \frac{1}{2}\left(\frac{2m}{\frac{W}{R}\,p_i\,h_i}\right)^m \sum_{k=0}^{m}\binom{m}{k}\,\zeta_y^{m-k}\,C_k \\ &= \frac{1}{2}\left(\frac{2\,m\,\zeta_y}{\frac{W}{R}\,p_i\,h_i}\right)^m \sum_{k=0}^{m}\binom{m}{k}\,\frac{C_k}{\mu_y^k} \\ &= 2^{m-1}\left(\frac{m}{\overline{\gamma_i}}\right)^m \sum_{k=0}^{m}\binom{m}{k}\,\frac{C_k}{\zeta_y^k} \end{aligned} \tag{2.2.43}$$

where $\overline{\gamma_i}$ is given in (2.2.12), and C_k is the kth central moment given by [15]:

$$C_k = \begin{cases} 1.3\cdots(k-1)\,\overline{\sigma}_{x-i}^{k} & k \text{ even} \\ 0 & k \text{ odd} \end{cases}$$

By splitting up the summation in (2.2.43), we obtain:

$$\begin{aligned} \sum_{l=0}^{m}\binom{m}{l}\frac{C_l}{\zeta_y^l} &= 1+\binom{m}{2}\frac{\overline{\sigma}_{x-i}^2}{(\sigma^2+\sum_{k\neq i}^{N}p_k\,h_k)^2}+\cdots \\ &+ \binom{m}{m'}\frac{1.3\cdots(m'-1)\overline{\sigma}_{x-i}^{m'-1}}{(\sigma^2+\sum_{k\neq i}^{N}p_k\,h_k)^{m'}} \end{aligned} \tag{2.2.44}$$

where $m' = m$ if m is even and $m' = m-1$ if m is odd. Since $\overline{\sigma}_x^2$ (see (2.2.40)) is very small compared to ζ_{x-i} (see (2.2.39)), we can approximate the summation by its leading term, namely 1. Therefore the average P_e at high SIR behaves as:

$$P_e \approx 2^{m-1}\left(\frac{m}{\overline{\gamma_i}}\right)^m. \tag{2.2.45}$$

Therefore the utility function of the ith user is given as:

$$\boxed{u_i = \frac{L\,R}{M\,p_i}\left(1-2^{m-1}\left(\frac{m}{\overline{\gamma_i}}\right)^m\right)^M}. \tag{2.2.46}$$

Notice that if we set $m = 1$, we obtain the same performance as in the Rayleigh slow flat-fading case.

Nakagami Slow Flat-Fading Channel

As described earlier, $u_i(\mathrm{p}|x_{-i})$ can be determined as follows:

$$\begin{aligned} u_i(\mathrm{p}|x_{-i}) &= \int_0^\infty u_i(\mathrm{p}|\omega, x_{-i}) f^{\gamma_i|x_{-i}}(\omega) d\omega \\ &= \int_0^\infty \frac{L\ R}{M\ p_i}\ (1 - e^{-\omega/2})^M\ \frac{1}{\Gamma(m)}\ \left(\frac{m}{\gamma_i^{'}}\right)^m\ \omega^{m-1}\, e^{-(\frac{m}{\gamma_i^{'}})\,\omega}\ d\omega \end{aligned} \tag{2.2.47}$$

By factoring $(1 - e^{-\gamma_i/2})^M$ and using the identity $\int_0^\infty y^n e^{-a\,y}\, dy = \frac{\Gamma(n+1)}{a^{n+1}}$ we obtain:

$$u_i(\mathrm{p}|x_{-i}) = \frac{L\,R}{M\,p_i}\sum_{k=0}^{M}(-1)^k \binom{M}{k} \left(\frac{2\,m}{k\,\gamma_i^{'} + 2\,m}\right)^m \tag{2.2.48}$$

For fixed m and high SIR, $\gamma_i^{'} \gg 1$, (2.2.48) may be approximated as:

$$u_i(\mathrm{p}|x_{-i}) \approx \frac{L\,R}{M\,p_i}\left[1 + (\frac{1}{\gamma_i^{'}})^m \sum_{k=1}^{M}(-1)^k \binom{M}{k} \left(\frac{2\,m}{k}\right)^m\right]. \tag{2.2.49}$$

Averaging (2.2.49) with respect to the distribution of x_{-i} and using the same argument as in (2.2.42), (2.2.43) and (2.2.44), we end up with the final approximate averaged utility function given by:

$$\begin{aligned} u_i &\approx \frac{L\,R}{M\,p_i}\left[1 + (\frac{1}{\overline{\gamma}_i})^m \sum_{k=1}^{M}(-1)^k \binom{M}{k} \left(\frac{2\,m}{k}\right)^m\right] \\ &\boxed{u_i \approx \frac{L\,R}{M\,p_i}\left[1 - \xi\ (\frac{1}{\overline{\gamma}_i})^m\right]} \end{aligned} \tag{2.2.50}$$

where $\xi = -\sum_{k=1}^{M}(-1)^k \binom{M}{k} \left(\frac{2\,m}{k}\right)^m > 0$.

2.3 Non-Cooperative Power Control Game (NPG)

Let $\mathcal{N} = \{1, 2, \cdots, N\}$ represent the index set of the users currently served in the cell and $\{P_j\}_{j\in\mathcal{N}}$ represents the set of strategy spaces of all users in the cell. Let $G = [\mathcal{N}, \{P_j\}, \{u_j(.)\}]$ denote a noncooperative game, where each user, basing on local information, chooses its power level from a convex set $P_j = [p_{j-\min}, p_{j-\max}]$ and where $p_{j-\min}$ and $p_{j-\max}$ are respectively the minimum and the maximum power levels in the jth user strategy space. With the assumption that the power vector $\mathrm{p} = [p_1, p_2, \cdots, p_N]$ is the result of NPG, the utility of user j is given as [8]:

$$u_j(\mathrm{p}) = u_j(p_j, \mathrm{p}_{-j}) \tag{2.3.1}$$

where p_j is the power transmitted by user j, and p_{-j} is the vector of powers transmitted by all other users. The right side of (2.3.1) emphasizes the fact that user j can just control his own power. We can rewrite (2.1.1) for user j as:

$$u_j(p_j, \mathrm{p}_{-j}) = \frac{L\,R}{M\,p_j}\,P_c(\gamma_j) \tag{2.3.2}$$

The formal expression for the NPG is given in [8] as:

$$G: \quad \max_{p_j \in \mathrm{P}_j} u_j(p_j, \mathrm{p}_{-j}), \ \text{ for all } \ j \in \mathcal{N} \tag{2.3.3}$$

Where $u_j(p_j, \mathrm{p}_{-j})$ is a continuous function. This game will produce a sequence of power vectors that converges to a point where all users are satisfied with the utility level they obtained. This operating point is called a Nash equilibrium operating point of NPG. In the next subsection, we define the Nash equilibrium point and describe its physical interpretation.

2.3.1 Nash Equilibrium (NE) in NPG

The resulting power vector of NPG is called a Nash equilibrium power vector.

Definition 2.3.1. *[8] A power vector* $\mathrm{p} = [p_1, p_2, \cdots, p_N]$ *is a Nash equilibrium of the* NPG defined above *if for every* $j \in \mathcal{N}, u_j(p_j, \mathrm{p}_{-j}) \geq u_j(p_j^{'}, \mathrm{p}_{-j})$ *for all* $p_j^{'} \in \mathrm{P}_j$.

One interpretation of NE point is that no user has incentive to modify its power level unilaterally, in other words, NE is the best operating point from the user perspective. If we multiply the power vector p by a constant $0 < \lambda < 1$ we may obtain higher utilities for all users, as was the case in nonfading channel. This means that the Nash equilibrium is not Pareto efficient, that is, the resulting p is not the most desired social operating point. In order to reach a Pareto dominant Nash point, a pricing technique was introduced in [8]. We discus this modified NPG game in the following Section.

2.4 Non-Cooperative Power Control Game with Pricing (NPGP)

In NPGP each user maximizes the difference between his/her own utility function and a pricing function. This approach aims to allow more efficient use of the system resources within the cell, where each user is made aware of the cost for the aggressive use of resources and of the harm done to other users in the cell. The pricing function discussed here is a linear pricing function, i.e., it is a pricing factor multiplied by the transmit power. The base station broadcasts the pricing factor to help the users currently in the cell reach a Nash equilibrium that improves the aggregate utilities of the users at lower equilibrium power levels than those of the pure NPG. In other words, the resulting power vector of NPGP is Pareto dominant compared with the resulting power vector of NPG, but is still not Pareto optimal in the sense that we can multiply the resulting power vector of NPGP by a constant $0 < \lambda < 1$ to obtain higher utilities for all users. Let $G_c = [\mathcal{N}, \{\mathrm{P}_j\}, \{u_j^c(.)\}]$ represent an N-

player noncooperative power control game with pricing (NPGP), where the utilities are given by [8]:

$$u_j^c(\mathrm{p}) = u_j(\mathrm{p}) - cp_j \text{ for all } j \in \mathcal{N} \tag{2.4.1}$$

where c is a positive number, chosen to obtain the best possible performance improvement. Therefore, NPGP with a linear pricing function may be expressed as:

$$G_c : \quad \max_{p_j \in \mathrm{P}_j} \{u_j(\mathrm{p}) - c\,p_j\} \text{ for all } j \in \mathcal{N} \tag{2.4.2}$$

2.5 Existence and Uniqueness of NE Point

In this Section we establish the existence and uniqueness of Nash equilibrium points in both NPG and NPGP games under the assumed channel models. We also show that the strategy space defined in [8] should be modified in order to guarantee the existence of Nash equilibrium points under the considered channel models. Let I_i and I_i^s be the effective interference that user i needs to cope with in both Rayleigh and Nakagami channels and Rician channels, respectively. Let $p_i^{\max}$ be the maximizing transmit power level. In the following Lemmas we establish the existence and uniqueness of Nash equilibrium point of NPG in the channel models mentioned above.

Lemma 2.5.1. **a)** *In NPG under Rayleigh fast flat-fading channel with the average utility function u_i given in (2.2.13), the existence of a Nash equilibrium point is guaranteed if the strategy space is modified to $P_i = \{p_i : \overline{\gamma}_i \in (\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max})\}$, where*

$$\overline{\gamma}_{i-\min} = M + 1 - \sqrt{\frac{M(M+1)}{2}}$$

and

$$\overline{\gamma}_{i-\max} = M + 1 + \sqrt{\frac{M(M+1)}{2}}$$

The best response vector of all users $r^1(p) = (r_1^1(p), r_2^1(p), \cdots, r_N^1(p))$*, where*

$$r_i^1(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} = (M+1)\, I_i, \quad I_i = \frac{R\,(\sum_{k \neq i}^{N} h_k p_k + \sigma^2)}{W\, h_i} \tag{2.5.1}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

b) *Under Rayleigh slow flat-fading channel with the average utility function* u_i *given in (2.2.17), the existence of a Nash equilibrium point is guaranteed if the strategy space is modified to* $P_i = \{p_i : \overline{\gamma_i} \in (1, 3\,\beta)\}$*. The best response vector of all users* $r^2(p) = (r_1^2(p), r_2^2(p), \cdots, r_N^2(p))$*, where*

$$r_i^2(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} = 2\,\beta\, I_i, \tag{2.5.2}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

Proof : **a)** In this and following proofs we take advantage of the classical results of game theory, where the existence of a Nash equilibrium point is guaranteed if the utility function is quasiconcave (see Appendix A for the definition of quasiconcave and quasiconvex notations) and optimized on a convex strategy space.

Thus, to prove the existence of a Nash equilibrium point, it is enough to prove that the utility function u_i is concave (a concave function on some set is also a quasiconcave function on the same set) in p_i given p_{-i} on the interval $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max}\right)\}$. Let us find the first- and second-order derivatives with respect to p_i as follows:

$$\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\,(\frac{M+1}{\overline{\gamma_i}} - 1)\,(1 - \frac{1}{\overline{\gamma_i}})^{M-1}, \tag{2.5.3}$$

then

$$\frac{\partial^2 u_i}{\partial p_i^2} = \frac{L\,R\,(1-\frac{1}{\overline{\gamma_i}})^M\,[M^2 + M(3 - 4\,\overline{\gamma_i}) + 2(-1+\overline{\gamma_i})^2]}{M\,p_i^3(-1+\overline{\gamma_i})^2} \tag{2.5.4}$$

Therefore,

$$\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\ \overline{\gamma_i} \in \left(\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max}\right) \tag{2.5.5}$$

where

$$\overline{\gamma}_{i-\min} = M + 1 - \sqrt{\frac{M(M+1)}{2}}$$

and

$$\overline{\gamma}_{i-\max} = M + 1 + \sqrt{\frac{M(M+1)}{2}}$$

This implies that the strategy space should be modified to:

$$\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max}\right)\}$$

to guarantee the concavity of the utility function, and therefore guarantee the existence of Nash equilibrium point.

To prove the uniqueness of a Nash equilibrium point we need to prove that $r^1(\mathrm{p})$ is a standard function. By setting (2.5.3) to zero we find that the maximizing transmit power level that lies in the convex strategy space P_i is given by (2.5.1). See Section A.2 for the definition of a standard vector function before proceeding in the proof.

To prove that $r^1(\mathrm{p})$ is a standard vector function we then need to check the three conditions in the definition A.2.1.

The proof of positivity is trivial, since $\mathrm{P}_i \subset \mathbb{R}^+$ and $r_i^1(\mathrm{p}_{-i}) \in P_i, \forall\, i \in \mathcal{N}$, where $r_i^1(\mathrm{p}_{-i}) = r_i^1(\mathrm{p})$. Also, it is obvious that $p_i^{\max}(\mathrm{p}) > p_i^{\max}(\hat{\mathrm{p}})$ for all i if $\mathrm{p} > \hat{\mathrm{p}}$ by examining (2.5.1), henceforth the monotonicity of $r^1(\mathrm{p})$ is satisfied. To prove scalability, it is enough to prove that $p_i^{\max}(\mathrm{p}_{-i})$ is a scalable function. Let us rewrite equation (2.5.1) as follows:

$$p_i^{\max}(\mathrm{p}_{-i}) = \frac{R\,(M+1)\,(\,\sum_{k\neq i}^{N} h_k p_k + \sigma^2)}{W\,h_i} \tag{2.5.6}$$

then

$$p_i^{\max}(\delta \mathrm{p}_{-i}) = \frac{R\,(M+1)\,(\,\delta\sum_{k\neq i}^{N} h_k p_k + \sigma^2)}{W\,h_i}, \tag{2.5.7}$$

while

$$\delta p_i^{\max}(\mathrm{p}_{-i}) = \frac{\delta\,R\,(M+1)\,(\,\sum_{k\neq i}^{N} h_k p_k + \sigma^2)}{W\,h_i} \tag{2.5.8}$$

From (2.5.7) and (2.5.8), it is clear that $\delta p_i^{\max}(\mathrm{p}_{-i}) > p_i^{\max}(\delta\,\mathrm{p}_{-i})$, therefore $r^1(\mathrm{p})$ is a standard vector function, and by this we conclude the proof of uniqueness of the Nash equilibrium point.

In the following lemmas we omit the proof of existence and/or uniqueness as they are similar to those of lemma 2.5.1.

b) We need to find the first- and second-order derivatives of u_i with respect to p_i:

$$\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\left(\frac{2\,\beta}{\overline{\gamma_i}} - 1\right), \tag{2.5.9}$$

then,

$$\frac{\partial^2 u_i}{\partial p_i^2} = \frac{2\,L\,R}{M\,p_i^3}\left(1 - \frac{3\,\beta}{\overline{\gamma_i}}\right), \tag{2.5.10}$$

which means that

$$\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\ \overline{\gamma_i} \in (1, 3\,\beta) \tag{2.5.11}$$

so the strategy space should have the following form:

$$\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in (1, 3\,\beta)\}$$

to guarantee the concavity of u_i and then to guarantee the existence of Nash point.

□

Lemma 2.5.2. **a)** *In NPG under Rician fast flat-fading channel with the average utility function u_i given in (2.2.29), the existence of a Nash equilibrium point is guaranteed if the strategy space is modified to $P_i = \{p_i : \gamma_i^s \in \left(\gamma_{i-\min}^s, \gamma_{i-\max}^s\right)\}$, where $\gamma_{i-\min}^s = M + 1 - \sqrt{\frac{M(M+1)}{2}}$ and $\gamma_{i-\max}^s = M + 1 + \sqrt{\frac{M(M+1)}{2}}$. The best response vector of all users $r^3(p) = (r_1^3(p), r_2^3(p), \cdots, r_N^3(p))$, where*

$$r_i^3(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$\begin{aligned} p_i^{\max} &= (M+1)\, I_i^s, \\ I_i^s &= \frac{R\,(\,(1+s^2)\sum_{k\neq i}^{N} h_k p_k + \sigma^2)}{W\, h_i\, e^{s^2}} \end{aligned} \tag{2.5.12}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

b) *Under Rician slow flat-fading channel with the average utility function u_i given in (2.2.33), the existence of a Nash equilibrium point is guaranteed if the strategy space is modified to $P_i = \{p_i : \gamma_i^s \in (1, 3\,\beta)\}$. The best response vector of all users $r^4(p) = (r_1^4(p), r_2^4(p), \cdots, r_N^4(p))$, where*

$$r_i^4(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} = 2\,\beta\, I_i^s, \tag{2.5.13}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

Lemma 2.5.3. **a)** *In NPG under Nakagami fast flat-fading channel with the average utility function u_i given in (2.2.46) with $m = 2$, the existence of a Nash equilibrium point is guaranteed if the strategy space is modified to $P_i = \{p_i : \overline{\gamma}_i \in (\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max})\}$, where*

$$\overline{\gamma}_{i-\min} = 2\sqrt{2 + 5M - \sqrt{M\,(8 + 17M)}}$$

and

$$\overline{\gamma}_{i-\max} = 2\sqrt{2 + 5M + \sqrt{M\,(8 + 17M)}}$$

The best response vector of all users $r^5(p) = (r_1^5(p),\, r_2^5(p),\, \cdots,\, r_N^5(p))$, where

$$r_i^5(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} = \sqrt{8 + 16M}\, I_i, \tag{2.5.14}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

b) *Under Nakagami slow flat-fading channel with the average utility function u_i given in (2.2.50), a Nash equilibrium point is guaranteed if the strategy space is modified to $P_i = \{p_i : \overline{\gamma}_i \in (1, \sqrt{6\,\xi})\}$. The best response vector of all users $r^6(p) = (r_1^6(p),\, r_2^6(p),\, \cdots,\, r_N^6(p))$, where*

$$r_i^6(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} = \sqrt{3\zeta}\; I_i, \tag{2.5.15}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

Proof : **a)** We find the first- and second-order derivatives of u_i in (2.2.46) after setting $m = 2$ with respect to p_i as follows:

$$\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\,(\frac{16M+8}{\overline{\gamma}_i^2} - 1)\,(1 - \frac{8}{\overline{\gamma}_i^2})^{M-1}, \tag{2.5.16}$$

then

$$\frac{\partial^2 u_i}{\partial p_i^2} = \frac{1}{M\,p_i^3(-8+\overline{\gamma}_i^2)^2}\,(2\,L\,R\,(1-\frac{2}{\overline{\gamma}_i^2})^M[64(1+M)(2M+1)-8(2+5M)\,\overline{\gamma}_i^2+\overline{\gamma}_i^4]) \tag{2.5.17}$$

which implies that

$$\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\ \overline{\gamma_i} \in \left(\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max}\right), \tag{2.5.18}$$

where

$$\overline{\gamma}_{i-\min} = \sqrt{2+5M-\sqrt{M\,(8+17M)}}$$

and

$$\overline{\gamma}_{i-\max} = \sqrt{2+5M+\sqrt{M\,(8+17M)}}$$

Henceforth, the strategy space should have the following form:

$$\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max}\right)\}$$

to guarantee that u_i is strict concave, and thus the existence of a Nash equilibrium.

b) The first- and second-order derivatives of u_i after setting $m = 2$ with respect to p_i are given by:

$$\frac{\partial u_i}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\left(\frac{3\,\xi}{\overline{\gamma}_i^2} - 1\right), \tag{2.5.19}$$

and

$$\frac{\partial^2 u_i}{\partial p_i^2} = \frac{2\,L\,R}{M\,p_i^3}\left(1 - \frac{6\,\xi}{\overline{\gamma}_i^2}\right), \tag{2.5.20}$$

therefore,

$$\frac{\partial^2 u_i}{\partial p_i^2} < 0, \forall\ \overline{\gamma_i} \in \left(1, \sqrt{6\,\xi}\right) \tag{2.5.21}$$

As a result, the strategy space should be modified to:

$$\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \left(1, \sqrt{6\,\xi}\right)\} \tag{2.5.22}$$

to guarantee the strict concavity of u_i and then the existence of a Nash equilibrium point is guaranteed.

□

Now, we turn to the existence and uniqueness of Nash equilibrium point of NPGP under the assumed channel models.

Lemma 2.5.4. **a)** *In NPGP under Rayleigh fast flat-fading channel model with utility function $u_i^c = u_i - c\,p_i$, where u_i is given in (2.2.13), a Nash equilibrium point existence is guaranteed if the strategy space is: $P_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max}\right)\}$, where*

$$\overline{\gamma}_{i-\min} = M + 1 - \sqrt{\frac{M(M+1)}{2}}$$

and

$$\overline{\gamma}_{i-\max} = M + 1$$

The best response vector $r^7(p) = (r_1^7(p), r_2^7(p), \cdots, r_N^7(p))$ of all users, where

$$r_i^7(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} \approx \frac{-6\,2^{1/3}\,a + 2^{2/3}(27\,b_i + \sqrt{108\,a^3 + 729\,b_i^2}\ )^{2/3}}{6\,(27\,b_i + \sqrt{108\,a^3 + 729\,b_i^2}\ )^{1/3}}$$
$$a = \frac{L\,R}{M\,c}, \quad b_i = \frac{(M+1)\,L\,R\,I_i}{M\,c} \tag{2.5.23}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

b) *Under Rayleigh slow flat-fading channel model with utility function* $u_i^c = u_i - c\, p_i$*, where* u_i *is given in (2.2.17), a Nash equilibrium point existence is guaranteed if the strategy space is the following:* $P_i = \{p_i : \overline{\gamma_i} \in (1, 2\,\beta)\}$*. The best response vector of all users* $r^8(p) = (r_1^8(p), r_2^8(p), \cdots, r_N^8(p))$*, where*

$$r_i^8(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$\begin{aligned} p_i^{\max} &= \frac{-6\,2^{1/3}\,a + 2^{2/3}(27\,b_i + \sqrt{108\,a^3 + 729\,b_i^2}\,)^{2/3}}{6\,(27\,b_i + \sqrt{108\,a^3 + 729\,b_i^2}\,)^{1/3}} \\ a &= \frac{L\,R}{M\,c}, \quad b_i = \frac{2\,L\,R\,\beta\,I_i}{M\,c} \end{aligned} \tag{2.5.24}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique. Note that we have equality in (2.5.24) because there was no approximation in obtaining $p_i^{\max}$*.*

Proof : **a)** Let us find the maximizing power $p_i^{\max}$ in terms of the SIR $\overline{\gamma_i}$ as follows:

$$\frac{\partial u_i^c}{\partial p_i} = \frac{L\,R}{M\,p_i^2}\,(\frac{M+1}{\overline{\gamma_i}} - 1)\,(1 - \frac{1}{\overline{\gamma_i}})^{M-1} - c = 0, \tag{2.5.25}$$

then

$$p_i^{\max} = \sqrt{\frac{L\,R}{M\,c}\,(\frac{M+1}{\overline{\gamma_i}} - 1)\,(1 - \frac{1}{\overline{\gamma_i}})^{M-1}} \tag{2.5.26}$$

For $p_i^{\max}$ to be feasible, i.e., real and positive, we need to enforce the following condition on the strategy space: $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in (1, M+1)\}$. On the other hand, to guarantee the concavity of the utility function u_i^c, we must have $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in \big(\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max}\big)\}$, where

$$\overline{\gamma}_{i-\min} = M + 1 - \sqrt{\frac{M(M+1)}{2}}$$

and

$$\overline{\gamma}_{i-\max} = M + 1 + \sqrt{\frac{M(M+1)}{2}}$$

Therefore, to fulfill both conditions, the strategy space should be the intersection of the two sets, that is $\mathrm{P}_i = \{p_i : \overline{\gamma_i} \in (\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max})\}$, where

$$\overline{\gamma}_{i-\min} = M + 1 - \sqrt{\frac{M(M+1)}{2}}$$

and

$$\overline{\gamma}_{i-\max} = M + 1$$

Since $\overline{\gamma_i} >> 1$ in the convex strategy space P_i given above, one can approximate $p_i^{\max}$, the solution of (2.5.25), as the feasible solution of the following equation:

$$p_i^3 + \frac{L\,R}{M\,c}\,p_i - \frac{(M+1)\,L\,R\,I_i}{M\,c} = 0 \qquad (2.5.27)$$

This equation has only one real and positive solution which is given in (2.5.23). It is fairly easy to prove that $r^7(\mathrm{p})$ with $p_i^{\max}$ as given in (2.5.23) is a standard vector function. Therefore, the Nash equilibrium point is unique.

□

In the following lemmas we omit the proof of existence and/or uniqueness if it can be argued the same way as in lemma 2.5.4.

Lemma 2.5.5. **a)** *In NPGP under Rician fast flat-fading channel model with utility function $u_i^c = u_i - c\,p_i$, where u_i is given in (2.2.29), a Nash equilibrium point existence is guaranteed if the strategy space is the following: $P_i = \{p_i : \gamma_i^s \in (\gamma_{i-\min}^s, \gamma_{i-\max}^s)\}$, where $\gamma_{i-\min}^s = M+1-\sqrt{\frac{M(M+1)}{2}}$ and $\gamma_{i-\max}^s$ $= M+1$. The best response vector of all users $r^9(p) = (r_1^9(p), r_2^9(p), \cdots, r_N^9(p))$, where*

$$r_i^9(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} \approx \frac{-6\, 2^{1/3}\, a + 2^{2/3}(27\, b_i + \sqrt{108\, a^3 + 729\, b_i^2}\ )^{2/3}}{6\,(27\, b_i + \sqrt{108\, a^3 + 729\, b_i^2}\ )^{1/3}}$$
$$a = \frac{L\, R}{M\, c}, \quad b_i = \frac{(M+1)\, L\, R\, I_i^s}{M\, c} \tag{2.5.28}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

b) *Under Rician slow flat-fading channel model with utility function* $u_i^c = u_i - c\, p_i$*, where* u_i *is given in (2.2.33), a Nash equilibrium point existence is guaranteed if the strategy space is the following:* $P_i = \{p_i : \gamma_{i-\min}^s \in (1, 2\,\beta)\}$*. The best response vector of all users* $r^{10}(p) = (r_1^{10}(p), r_2^{10}(p), \cdots, r_N^{10}(p))$*, where*

$$r_i^{10}(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} = \frac{-6\, 2^{1/3}\, a + 2^{2/3}(27\, b_i + \sqrt{108\, a^3 + 729\, b_i^2}\ )^{2/3}}{6\,(27\, b_i + \sqrt{108\, a^3 + 729\, b_i^2}\ )^{1/3}}$$
$$a = \frac{L\, R}{M\, c}, \quad b_i = \frac{2\, L\, R\, \beta\, I_i^s}{M\, c} \tag{2.5.29}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

Lemma 2.5.6. **a)** *In NPGP under Nakagami fast flat-fading channel model with utility function* $u_i^c = u_i - c\, p_i$*, where* u_i *is given in (2.2.46), a Nash equilibrium point existence is guaranteed if the strategy space is the following:* $P_i = \{p_i : \overline{\gamma_i} \in \left(\overline{\gamma}_{i-\min}, \overline{\gamma}_{i-\max}\right)\}$*, where*

$$\overline{\gamma}_{i-\min} = 2\sqrt{2 + 5M - \sqrt{M\,(8 + 17M)}}$$

and

$$\overline{\gamma}_{i-\max} = 2\sqrt{2 + 4M}$$

The best response vector of all users $r^{11}(p) = (r_1^{11}(p), r_2^{11}(p), \cdots, r_N^{11}(p))$*, where*

$$r_i^{11}(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} \approx \sqrt{\frac{L\,R}{2\,M\,c}}\sqrt{-1+\sqrt{1+\frac{4\,(8+16M)\,I_i^2\,M\,c}{L\,R}}} \tag{2.5.30}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

b) *Under the Nakagami slow flat-fading channel model with utility function* $u_i^c = u_i - c\,p_i$*, where* u_i *is given in (2.2.50), a Nash equilibrium point existence is guaranteed if the strategy space is the following:* $P_i = \{p_i : \overline{\gamma_i} \in \big(1, \sqrt{3\,\xi}\big)\}$*. The best response vector of all users* $r^{12}(p) = (r_1^{12}(p), r_2^{12}(p), \cdots, r_N^{12}(p))$*, where*

$$r_i^{12}(p) = \min(p_i^{\max}, p_{i-\max}),$$

and

$$p_i^{\max} = \sqrt{\frac{L\,R}{2\,M\,c}}\sqrt{-1+\sqrt{1+\frac{12\,\xi\,I_i^2\,M\,c}{L\,R}}} \tag{2.5.31}$$

is a standard vector function, therefore by [2], the Nash equilibrium point is unique.

Proof : **a)** The maximizer transmit power $p_i^{\max}$ is the feasible solution of $\frac{\partial u_i}{\partial p_i} - c = 0$, where $\frac{\partial u_i}{\partial p_i}$ is given in (2.5.16), this results in a polynomial of degree $2M+4$. It is tedious to find a closed-form for the feasible solution of this polynomial. By assuming $\overline{\gamma_i} >> \sqrt{8}$, the maximizer transmit power level $p_i^{\max}$ can be a approximated by the feasible solution of the following equation.

$$p_i^4 + \frac{L\,R}{M\,c}\,p_i^2 - \frac{(8+16M)\,L\,R\,I_i^2}{M\,c} = 0 \tag{2.5.32}$$

The only feasible solution of the equation above is as given by (2.5.30).

b) The maximizer transmit power level $p_i^{\max}$ is the feasible solution of the following equation.

$$p_i^4 + \frac{L\,R}{M\,c}\,p_i^2 - \frac{3\xi\,L\,R\,I_i^2}{M\,c} = 0 \tag{2.5.33}$$

The only feasible solution of the equation above is as given by (2.5.31). It is simple to check that $r^{12}(\mathrm{p})$ with the maximizer power in (2.5.31) satisfies all the conditions of a standard vector function. Henceforth, the Nash equilibrium point is unique.

□

Observing lemmas 2.5.1-2.5.3, we see that the maximizing SIR $\gamma_i^{\max}$ is the same for all users : $\gamma_i^{\max} = M + 1$, under fast Rayleigh and fast Rician fading channels. On the other hand $\gamma_i^{\max} = 2\,\beta$, under slow Rayleigh and slow Rician fading channels, while $\gamma_i^{\max} = \sqrt{8 + 16M}$ under fast Nakagami fading channels, and $\gamma_i^{\max} = \sqrt{3\,\xi}$ under slow Nakagami fading channels. For nonfading channels it was shown in [8] that $\gamma_i^{\max} = 12.4$. This implies, as expected, that users in a fading channels have to target higher SIRs in order to overcome the fading effect as compared to the nonfading channels.

Next, we introduce an algorithm that converges to the Nash equilibria of NPG and NPGP. We need to keep in mind that the strategy space denoted by P_i in the algorithm depends on the channel model. Since it was proved in [2] that synchronous and asynchronous algorithms with standard best response function converge to the same fixed point, we focus on asynchronous algorithms in this chapter as they represent the best model of distributed algorithms. Our algorithm is the same as in [8] except that the strategy space is modified to the forms given in lemmas 2.5.1-2.5.3 in order to guarantee the existence of Nash equilibria.

Assume user j updates its power level at time instances that belong to a set T_j, where $T_j = \{t_{j1}, t_{j2}, \cdots\}$, with $t_{jk} < t_{jk+1}$ and $t_{j0} = 0$ for all $j \in \mathcal{N}$. Let

$T = \{t_1, t_2, \cdots\}$ where $T = T_1 \bigcup T_2 \bigcup \cdots \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{p}$ to be the smallest power vector in the modified strategy space $\mathrm{P} = \mathrm{P}_1 \bigcup \mathrm{P}_2 \bigcup \cdots \bigcup \mathrm{P}_N$.

Algorithm 2.5.1. *Consider a non-cooperative game G as given in (2.3.3) and generate a sequence of power vectors as follows:*

1. *Set the power vector at time* $t = 0$*:* $p(0) = \underline{p}$*, let* $k = 1$
2. *For all* $j \in \mathcal{N}$*, such that* $t_k \in T_j$*:*
   (a) *Given* $p(t_{k-1})$*, calculate* $p_j^{\max}(t_k) = arg \max_{p_j \in P_j} u_j(p_j, p_{-j}(t_{k-1}))$
3. *If* $p(t_k) = p(t_{k-1})$ *stop and declare the Nash equilibrium power vector as* $p(t_k)$*, else let* $k := k + 1$ *and go to 2.*

The following algorithm finds the best pricing factor c for NPGP. We need to keep in mind that the strategy space should be chosen according to lemmas 2.5.4-2.5.6.

Algorithm 2.5.2. 1. *Set* $c = 0$ *and broadcast* c *to all users currently in the cell.*

2. *Use Algorithm 2.5.1 to obtain* u_j^c *for all* $j \in \mathcal{N}$ *at equilibrium.*
3. *Increment* $c := c + \Delta c$*, where* Δc *is a positive constant, and broadcast* c *to all users, and then go to 2*
   (a) *If* $u_j^{c+\Delta c} \geq u_j^c$ *for all* $j \in \mathcal{N}$ *go to 3, else stop and declare the best pricing factor*$c = c_{Best}$

2.6 Simulation Results

We study the effects of time-varying (fast and slow fading) channels, on the equilibrium utilities and powers which are the outcomes of the NPG and NPGP algorithms

(algorithms 2.5.1 and 2.5.2) studied in [8]. We use the same definition of utility function (see (2.1.1)) with P_c modified to fit the channel model in the following cases: Fast/Slow Rician fading channel model, fast/slow Rayleigh fading channel model and fast/slow Nakagami fading channel. The system studied is a single-cell DS-CDMA cellular mobile system with 9 stationary users using the same data rate R and the same modulation scheme, non-coherent BFSK. The system parameters used in this study are given in Table 2.1. The distances between the 9 users and the BS are d $= [310, 460, 570, 660, 740, 810, 880, 940, 1000]$ in meters. The path attenuation between user j and the BS using the simple path loss model [16] is $h_j = 0.097/d_j^4$, where 0.097 approximates the shadowing effect. In a fast fading channel, P_c is given by:

$$P_c(\breve{\gamma}_j) = (1 - \frac{1}{\breve{\gamma}_j})^M \text{ for all } j = 1, 2, \cdots, N \tag{2.6.1}$$

where $\breve{\gamma}_j = \overline{\gamma_j}$ in a Rayleigh channel, $\breve{\gamma}_j = \gamma_j^s$ in a Rician channel and $\breve{\gamma}_j = 2^{1-m}\,(m/\overline{\gamma_j})^{-m}$ in a Nakagami channel. Fig. 2.1 shows that under Rayleigh, Rician, or Nakagami fast flat-fading channels with spreading gain $W/R = 100$, users do not reach a desired Nash equilibrium point, since all users except for the nearest user to the BS have reached the highest power level in the strategy space. More clearly, in Fig. 2.2 one can see that users obtain very low utilities as a result of NPG compared to deterministic path gains. Fig. 2.3 and Fig. 2.4 show that with the same parameters as in Fig. 2.1 and Fig. 2.2 but with spreading gain $W/R = 1000$ the results are better since a Nash equilibrium was reached, and comparable to that of deterministic channel gains. The equilibrium utilities and equilibrium powers of the NPGP are shown in Fig. 2.5 and Fig. 2.6, respectively. Results show that the Pareto improvement over NPG for Rayleigh, Rician, and Nakagami channels was similar to the case of deterministic channel gains.

Next we present the effects of a slow flat-fading channel **(a2)** on the equilibrium utilities and powers which are the outcomes of NPG algorithm 2.5.1 and compare

Table 2.1: the values of parameters used in the simulations.

L, number of information bits	64
M length of the codeword	80
W, spread spectrum bandwidth	10^6, 10^7 Hz
R, data rate	10^4 bits/sec
σ^2, AWGN power at the BS	5×10^{-15}
N, number of users in the cell	9
s^2, specular component	1
W/R, spreading gain	100, 1000
m, fading figure	2
$p_{i-\max}$, maximum power in ith user's strategy space	2

them to those obtained under **(a1)** as one can see in Fig. 2.7-Fig. 2.12. As expected, these figures, illustrate that users can achieve better performance in slow flat-fading channels than that in fast flat-fading ones.

2.7 Summary

We presented the noncooperative power control game (NPG) and noncooperative power control game with pricing (NPGP) introduced in [8] for realistic channel models. We studied the impact of power statistical variation in Rayleigh, Rician, and Nakagami fast/slow flat-fading channels on the powers and utilities vectors at equilibrium. The results showed that the equilibrium was reached in both games only at higher processing gains ($W/R > 100$). Utilities for Rayleigh, Rician, and Nakagami flat-fading channel gains at equilibrium are lower (at higher equilibrium power vector) than the utilities for deterministic channel gains. However, in order to overcome the fading effects, the SIRs obtained at equilibrium must be higher for all users at equilibrium in the Rician, Rayleigh, and Nakagami flat-fading channels than under deterministic channels.

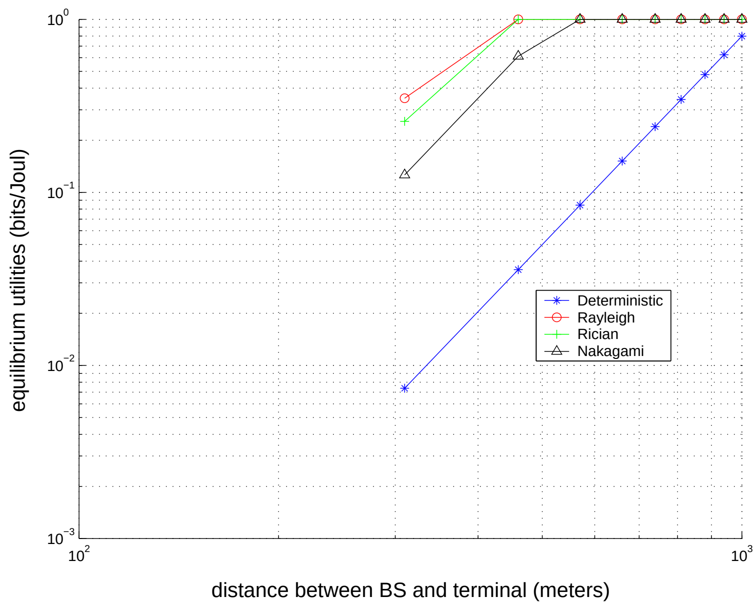

Figure 2.1: Equilibrium powers of NPG for Rician flat-fading channel gain (+), Rayleigh flat-fading channel gain (o), Nakagami flat-fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 100$.

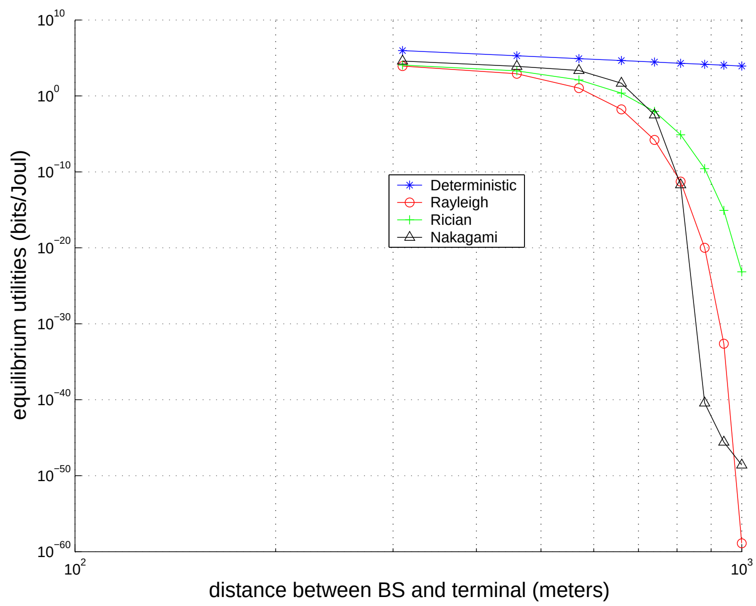

Figure 2.2: Equilibrium utilities of NPG for Rician flat-fading channel gain (+), Rayleigh flat-fading channel gain (o), Nakagami flat-fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 100$.

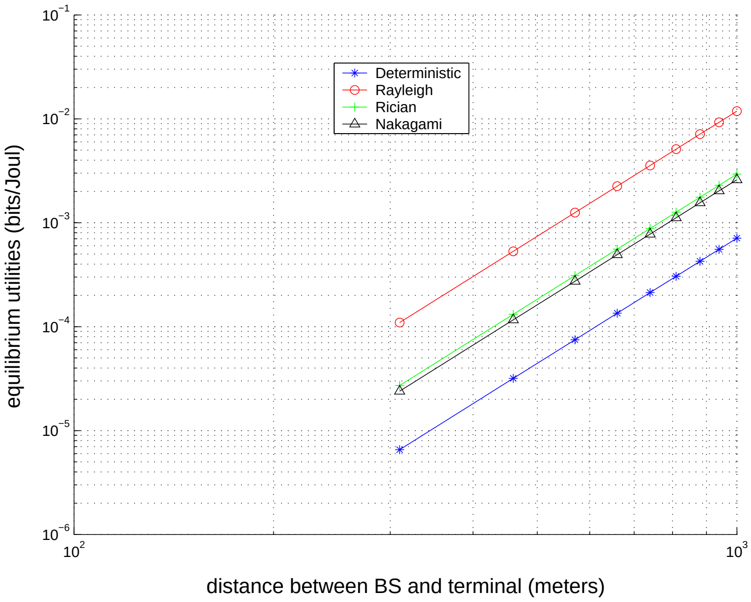

Figure 2.3: Equilibrium powers of NPG for Rician flat-fading channel gain (+), Rayleigh flat-fading channel gain (o), Nakagami flat-fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 1000$.

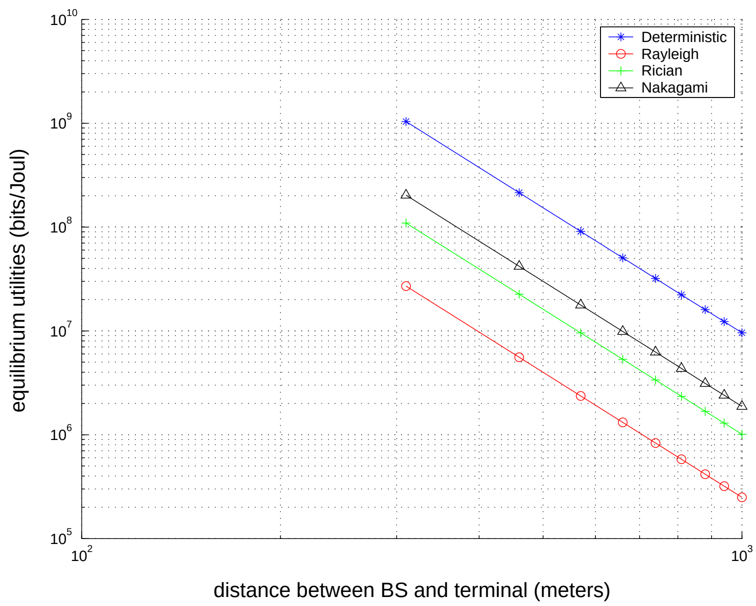

Figure 2.4: Equilibrium utilities of NPG for Rician flat-fading channel gain (+), Rayleigh flat-fading channel gain (o), Nakagami flat-fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 1000$.

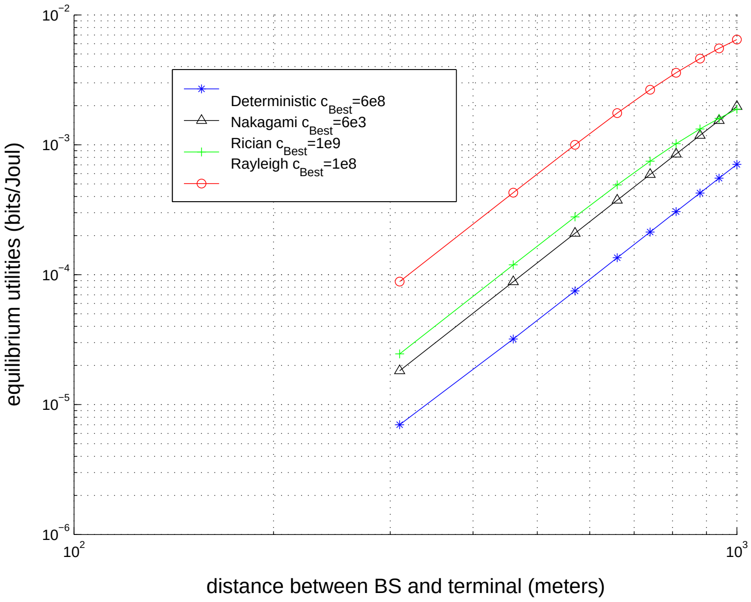

Figure 2.5: Equilibrium powers of NPGP for Rician flat-fading channel gain (+), Rayleigh flat-fading channel gain (o), Nakagami flat-fading ($\triangle$) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 1000$.

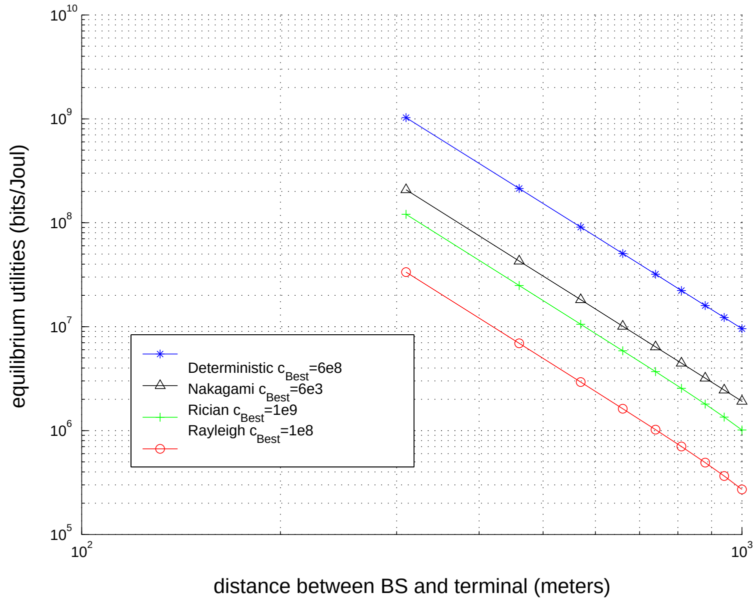

Figure 2.6: Equilibrium utilities of NPGP for Rician flat-fading channel gain (+), Rayleigh flat-fading channel gain (o), Nakagami flat-fading (△) and deterministic channel gain (*) versus the distance of a user from the BS in meters with $W/R = 1000$.

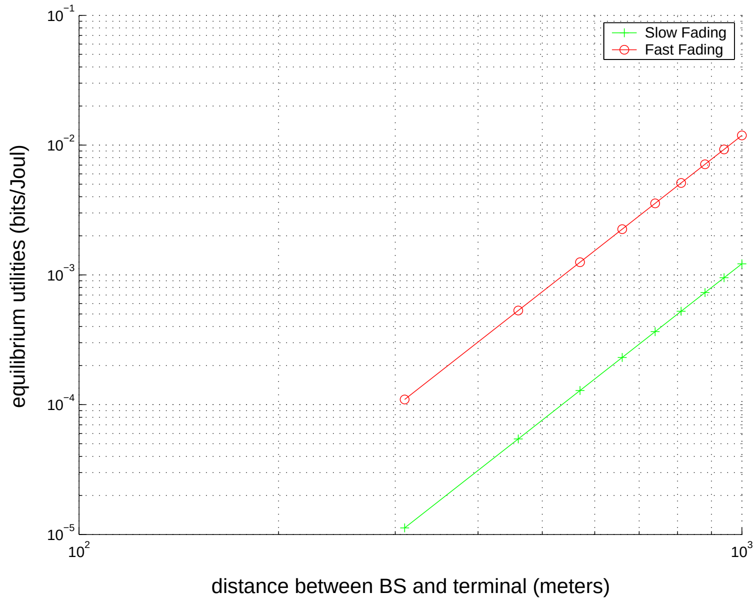

Figure 2.7: Equilibrium powers of NPG for Rayleigh fast flat-fading channel gain (o) and slow flat-fading channel (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

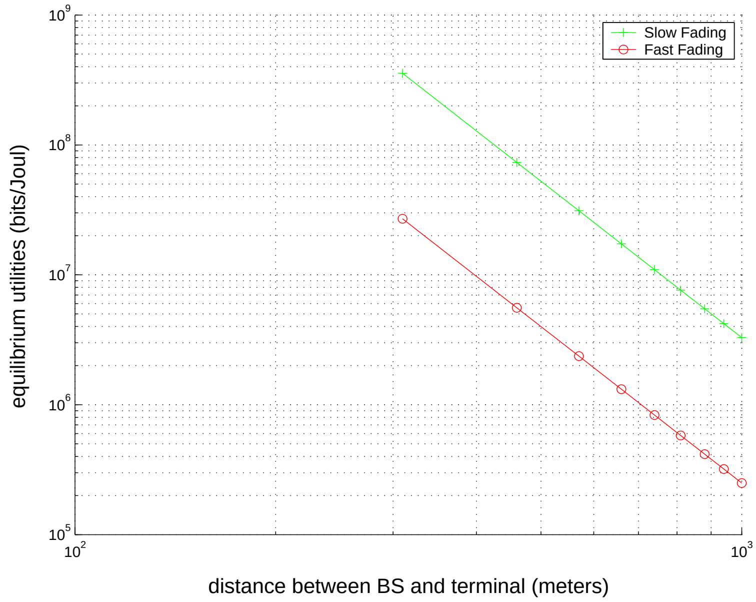

Figure 2.8: Equilibrium utilities of NPG for Rayleigh fast flat-fading channel gain (o) and slow flat-fading channel (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

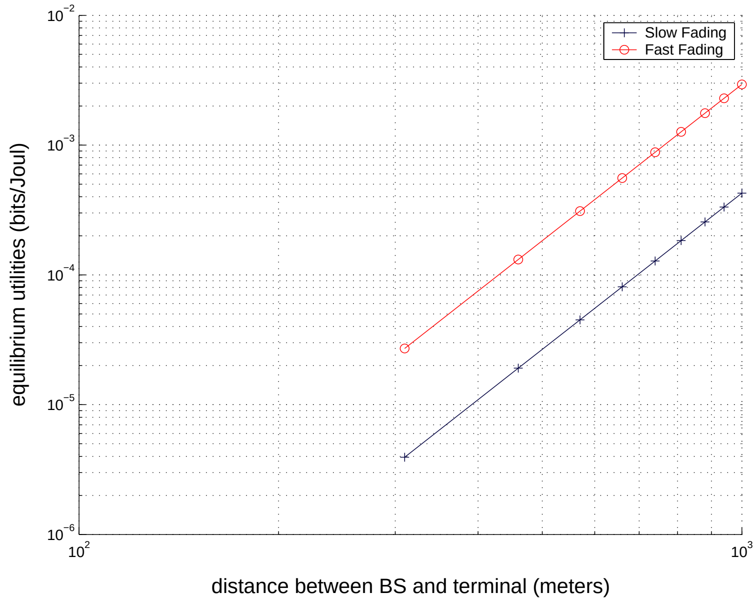

Figure 2.9: Equilibrium powers of NPG for Rician fast flat-fading channel gain (o) and slow flat-fading channel (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

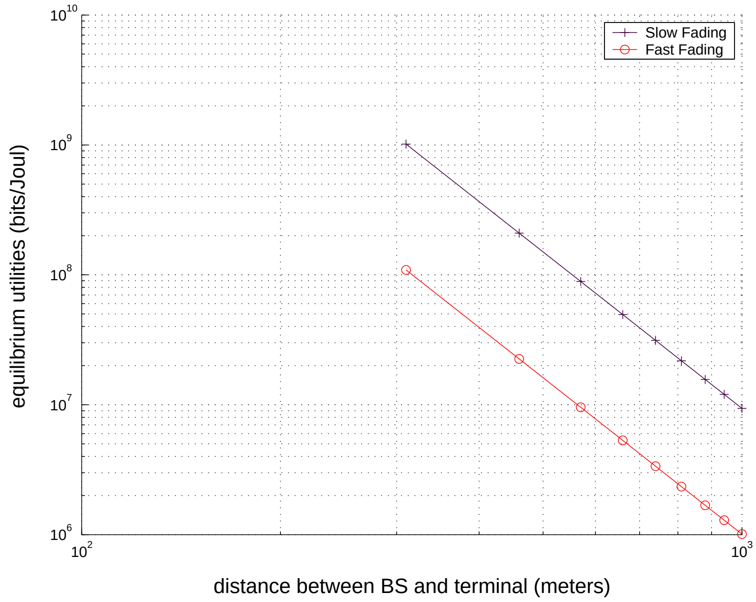

Figure 2.10: Equilibrium utilities of NPG for Rician fast flat-fading channel gain (o) and slow flat-fading channel (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

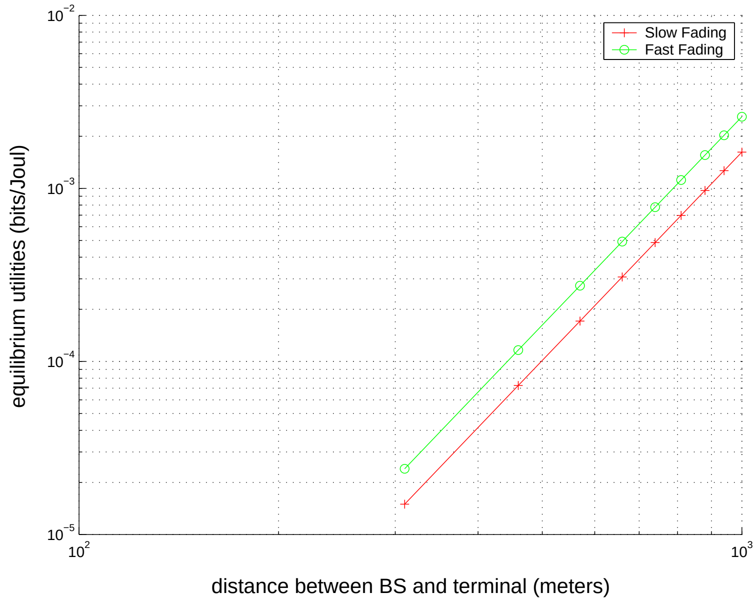

Figure 2.11: Equilibrium powers of NPG for Nakagami fast flat-fading channel gain (o) and slow flat-fading channel (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

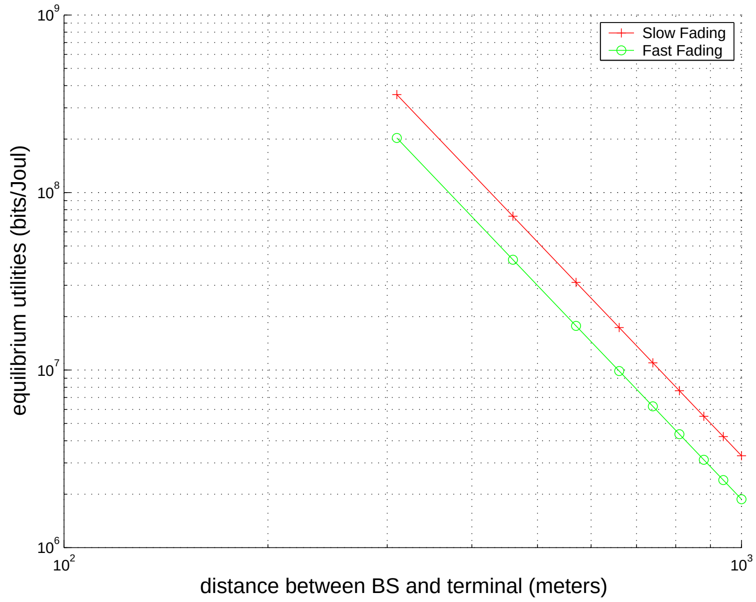

Figure 2.12: Equilibrium utilities of NPG for Nakagami fast flat-fading channel gain (o) and slow flat-fading channel (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

Chapter 3

Machine Learning Theory and Game Theory

In this Chapter we use machine learning theory to evaluate the performance of game theoretic power control algorithms for wireless data in arbitrary channels, i.e., whenever no presumed channel model is available. To show the validity of applying machine learning theory in this context, we studied a slow flat-fading channel, and more specifically, we simulated the case of Rayleigh flat-fading channel for the same setup as [8]. With the help of a relatively small number of training samples, the results suggest the learnability of the utility function classes defined by changing the users power (adjusted parameter) for each user's utility function.

The remaining of this Chapter is organized as follows: In Section 3.1 we describe the utility function and the system used in this Chapter, while in Section 3.2 we describe the two power control algorithms, namely NPG and NPGP. A brief discussion of *distribution-free* learning theory is presented in Section 3.3. The application of *distribution-free* learning theory to NPG and NPGP under a slow flat-fading channel model is introduced in Section 3.4. Discussion of a Rayleigh slow flat-fading channel and simulation results are outlined in Section 3.5. Finally, the summary of this Chapter is presented in Section 3.6.

3.1 Utility Function and System Model

Since we are studying the same setup as in [8], the utility function will be the same as in Chapter 2. For convenience however, we re-introduce the system model and the utility function as follows: Suppose we have a single-cell system with N users, where each user transmits packets of M total bits with L information bits and with power p Watts per bit. The rate of transmission is R bits/sec for all users. Let $P_c(\gamma)$ represent the probability of correct reception of all bits in the frame at the BS at a given SIR γ. The channel is assumed to be slow flat-fading channel (wireless channel changes independently from packet to packet) in addition to AWGN at the receiver in the BS. All users are using BFSK as a modulation scheme. And the utility function is given by:

$$u = \frac{L\,R}{M\,p}\,P_c \tag{3.1.1}$$

With the assumption of no error correction, perfect detection, and that each bit is experiencing independent noise. For user i, $P_c = E[(1 - \tilde{P}(e|\gamma_i, x_i))^M]$ where $\tilde{P}(e|\gamma_i, x_i)$ is the bit error rate (BER) at a given SIR γ_i and given by (2.2.4) , and x_i is given by (2.2.2).

3.2 NPG and NPGP

Both NPG and NPGP games in this Chapter are the same ones in Chapter 2. In order to avoid repetition we only introduce the formal expressions of these games as follows:

$$\text{NPG}: \quad \max_{p_j \in \mathrm{P}_j} u_j(p_j, \mathrm{p}_j), \ \forall \ j \in \mathcal{N} \tag{3.2.1}$$

As we said in Chapter 2, usually Nash equilibrium point of NPG is not efficient, to obtain a Pareto dominant over pure NPG, NPGP was proposed in [8]. NPGP with linear pricing function can be expressed as:

$$\text{NPGP}: \max_{p_j \in \mathrm{P}_j} \{u_j(\mathrm{p}) - c \, p_j\}, \ \forall \ j \in \mathcal{N} \tag{3.2.2}$$

In the next Section we present a brief discussion of distribution-free learning theory, where we focus on the learnability of the utility function class, depending on learning samples received at the BS.

3.3 Distribution-Free Learning

Distribution-free learning theory enables us to evaluate the performance of game theoretic power control algorithms for wireless data without the need to have a prior knowledge of the channel model. Of course, this may only be done under the condition that the utility function class is "learnable." The learnability of the utility function class is highly dependent on the channel model as will be apparent shortly. See Appendix C for a brief description of some of the vocabulary of machine learning theory that we need in this Chapter.

If a function (concept) class has a finite P-dimension (VC-dimension), then such function (concept) class is said to be a distribution-free learnable [23], that is we can learn the target function (concept) using the learning samples drawn according to an unknown probability measure. The learning problem under study is as follows: assume X is a given set (in this work we take $X = [0, \infty]$, i.e., the space of the coefficients α_i) , and $\mathcal{U}$ (called the utility function class) is a family of functions such that $u : \mathrm{X} \to [0,1]\ \forall u \in \mathcal{U}$. It should be noted here that limiting ourselves to the interval $[0,1]$ does not necessarily mean that the function $u \in [0,1]$, but rather that it is bounded [24]. Suppose $\mathcal{P}$ represent the set of all probability laws (represent all possible channel models) on X. Assume that $u \in \mathcal{U}$ is the utility function (can be thought of as a random variable since it is a function of α_i) that corresponds to the learning multisample $\mathrm{x} = [x_1, x_2, \cdots, x_n] \in X^n$ (fading coefficients), drawn according to the probability law $F^n \in \mathcal{P}$ (channel model), with the assumption that F^n is differentiable with PDF f^n. Then the average utility function U (the function we need to learn) is given by:

$$U \quad := \quad E_F(u) = \int_{\mathrm{X}} u(\mathrm{x}) f^n(\mathrm{x}) d\mathrm{x} \tag{3.3.1}$$

while the empirical utility function U_{emp} is given by:

$$U_{emp} := \frac{1}{n} \sum_{l=1}^{n} u(\mathrm{x}_l) \tag{3.3.2}$$

For $\epsilon > 0$, define $\delta(n, \epsilon, P)$ as follows [23]:

$$\delta(n, \epsilon, P) := P^n \left\{ \mathrm{x} \in X^n : \sup_{u \in \mathcal{U}} |U_{emp} - U| > \epsilon \right\},\ n = 1, 2, \cdots \tag{3.3.3}$$

where P^n denotes the product probability on X^n, and define

$$\delta^*(\epsilon, n) := \sup_{P \in \mathcal{P}} \delta(n, \epsilon, P)$$

The family of function classes $\mathcal{U}$ has the property of *distribution-free uniform convergence of empirical means* if $\delta^*(n, \epsilon) \to 0$ as $n \to \infty$ for each $\epsilon > 0$ [23]. Which is a result of the following theorem whose proof may be found in [23].

Theorem 3.3.1. *Suppose the family* $\mathcal{U}$ *has a finite P-dimension with value equal to* d*. Consider* $0 < \epsilon < e/(2 \log_2 e) \approx 0.94$*. Then*

$$\delta^*(n, \epsilon) \leq 8 \left(\frac{16e}{\epsilon} \ln \frac{16e}{\epsilon} \right)^d \exp(-n\ \epsilon^2/32)\ \ \forall n$$

Therefore $\mathcal{U}$ *has the property of distribution-free uniform convergence of empirical means.*

One can see from the above theorem that the learnability of $\mathcal{U}$ is highly dependent on the P-dimension (d) for a given accuracy (ϵ), where a large value of d could lead to a prohibitive sample complexity (n) to achieve the accuracy with confidence $1 - \delta^*(n, \epsilon)$. In the next Section we study the learnability of the utility function class defined under NPG and NPGP by evaluating the P-dimension with the assumption that the channel is modeled as a slow flat-fading channel.

3.4 Application to NPG and NPGP in a Slow Flat-Fading Channel

In this Section we apply *distribution-free learning* theory where the channel is modeled as a slow flat-fading channel. Using this model, the SIR (γ_i) of the ith user is given by [15]:

$$\gamma_i = \frac{W}{R} \frac{p_i\, h_i\, \alpha_i^2}{\sum_{k \neq i}^{N} p_k\, h_k\, \alpha_k^2 + \sigma^2} \tag{3.4.1}$$

where W is the spread-spectrum bandwidth, R is the data rate (bits/sec), p_k is the transmitted power (the adjusted parameter) of the kth user, h_k is the path gain between the BS and the kth user, σ^2 is the variance of the AWGN channel, and α_k is a slow flat-fading coefficient of the path between the BS and kth user. For both NPG and NPGP, it is assumed that each user knows the background noise and the interference from other users at each time instance he updates his transmit power level. This allows the user to adjust his own parameter (power) to obtain the maximum possible utility function. This then enables us to write (3.4.1) in the form:

$$\gamma_i = C_i\, p_i\, \alpha_i^2, \tag{3.4.2}$$

where

$$C_i = \frac{W}{R} \frac{h_i}{\sum_{k \neq i}^{N} p_k\, h_k\, \alpha_k^2 + \sigma^2}$$

A simple interpretation of (3.4.2) is that the interference and the background noise are considered constant at each time instance the user adjusts its power (see Algorithm 2.5.1). Suppose that each user is using noncoherent binary shift keying (BFSK) to transmit each data bit, i.e, $P_e = \frac{1}{2}\, e^{-\gamma/2}$. The channel is assumed to be a slow flat-fading channel, in other words, the fading coefficient α_i is constant for each frame time interval. This enables us to write $P_c(\gamma_i) = (1 - P_e(\gamma_i))^M$. So, we can rewrite (3.1.1) for the ith user in the following form:

$$u_i = \frac{L\ R}{M\ p_i}\ (1 - e^{-\gamma_i/2})^M \tag{3.4.3}$$

Where P_e was replaced by $2P_e$ to give the utility function (u_i) the properties that $u_i \to 0$ as $p_i \to 0$ and $u_i \to 0$ as $p_i \to \infty$ [8]. Let us split up the ith utility function into the following functions $\forall i \in \mathcal{N}$:

$$f_1 = 1 - e^{-\gamma_i/2}, \tag{3.4.4}$$

$$f_2 = f_1^M, \tag{3.4.5}$$

$$f_3 = \frac{1}{p_i}, \tag{3.4.6}$$

and finally

$$f_4 = f_2\ f_3 \tag{3.4.7}$$

Notice that $u_i = \frac{LR}{M} f_4$. Now, we need to find the first-order partial derivative of $\{f_k\}$ with respect to p_i and α_i in order to show the learnability of u_i in (3.4.3).

$$\frac{\partial f_1}{\partial \alpha_i} = \frac{\partial f_1}{\partial \gamma_i}\frac{\partial \gamma_i}{\partial \alpha_i} = (-f_1 + 1)(C_i\, p_i\, \alpha_i) \tag{3.4.8}$$

$$\frac{\partial f_1}{\partial p_i} = \frac{\partial f_1}{\partial \gamma_i}\frac{\partial \gamma_i}{\partial p_i} = (-\frac{1}{2} f_1 + \frac{1}{2})(C_i\, \alpha_i^2) \tag{3.4.9}$$

$$\frac{\partial f_2}{\partial \alpha_i} = \frac{\partial f_2}{\partial f_1}\frac{\partial f_1}{\partial \alpha_i} = M\, f_1^{M-1}(-f_1 + 1)(C_i\, p_i\, \alpha_i) \tag{3.4.10}$$

$$\frac{\partial f_2}{\partial p_i} = \frac{\partial f_2}{\partial f_1}\frac{\partial f_1}{\partial p_i} = M\, f_1^{M-1}(-\frac{1}{2}f_1 + \frac{1}{2})(C_i\, \alpha_i^2) \tag{3.4.11}$$

$$\frac{\partial f_3}{\partial \alpha_i} = 0 \tag{3.4.12}$$

$$\frac{\partial f_3}{\partial p_i} = -f_3^2 \tag{3.4.13}$$

$$\frac{\partial f_4}{\partial \alpha_i} = f_3\frac{\partial f_2}{\partial \alpha_i} + f_2\frac{\partial f_3}{\partial \alpha_i} = M\ f_3\, f_1^{M-1}(-f_1 + 1)(C_i p_i\, \alpha_i) \tag{3.4.14}$$

$$\begin{aligned}\frac{\partial f_4}{\partial p_i} &= f_3\frac{\partial f_2}{\partial p_i} + f_2\frac{\partial f_3}{\partial p_i} \\ &= f_3\ M\, f_1^{M-1}(-\frac{1}{2}f_1 + \frac{1}{2})(C_i\, \alpha_i^2) - f_2\ f_3^2\end{aligned} \tag{3.4.15}$$

As we can see from the above first-order partial derivatives, f_1, f_2, f_3, f_4 are a Pfaffian chain of length $q = 4$ and of degree at most $D = 2$ in α_i and p_i. The importance of this observation is that our problem at hand turns out to be similar to a neural network architecture in which the shaping function is a polynomial ($\gamma_i = C_i\, p_i\, \alpha_i^2$) and the activation functions ($\{f_k\}_{k=1}^4$) are Pfaffian chain. For such neural network architecture, the upper bound of the P-dimension (d) is given by ([23], Theorem 10.8):

$$\begin{aligned} d \ \leq\ & 2l(l(q+1)^2/2 + \log_2 Q + (2(q+1)+1)\log_2 l \\ +\ & (q+2)\log_2(2(Q+D)) + \log_2(2e)) \end{aligned} \tag{3.4.16}$$

Where d is the P-dimension of the function class $\mathcal{U}$, Q is degree of γ_i in p_i, α_i and f_1, f_2, f_3, f_4, and l is the number of adjustable parameters of each user (in the case under study the parameters are p_i and C_i, that is $l = 2$). Substituting the numerical values of D, q, and l, we obtain $d \leq 247$. These results can be extended to NPGP in a straight-forward manners to obtain the same values of D, q and l. Henceforth, the utility function classes defined under NPG and under NPGP have the same upper bound on the P-dimension.

3.5 Discussion of Rayleigh Slow Flat-Fading Channel and Simulation Results

As a specific case of slow flat-fading channel model we present results for the case where α_i is modeled as a Rayleigh random variable (as an example of the validity of the results in the previous Section) with PDF given by:

$$p(\alpha_i) = \frac{\alpha_i}{\sigma_r^2}\ \exp(-\alpha_i^2/2\sigma_r^2), \quad i = 1, 2, \cdots, N \tag{3.5.1}$$

In the following calculations again it is assumed that $\sigma_r^2 = 1/2$. Using the results of Section 2.2.1, the average utility function of the ith user is given by:

$$U_i(\text{p}) \ = \ \frac{L\ R}{M\ p_i}\left(1 + \frac{1}{\overline{\gamma_i}}\ \sum_{k=1}^{M} \binom{M}{k}\ \frac{2\,(-1)^k}{k}\right) \tag{3.5.2}$$

where $\overline{\gamma_i}$ is the ratio of the mean of the received power of the ith user to the mean of the interference at the BS and given by:

$$\overline{\gamma_i} = \frac{W}{R}\frac{p_i\, h_i}{\sum_{k\neq i}^{N} p_k\, h_k + \sigma^2} \tag{3.5.3}$$

While the empirical value of the utility function U_{emp_i} is given by:

$$U_{emp_i} = \frac{1}{n}\sum_{l=1}^{n} u_i(\breve{\alpha}_l), \tag{3.5.4}$$

where $\breve{\alpha}_l = [\alpha_{l,1}, \alpha_{l,2}, \cdots, \alpha_{l,N}]$. The system studied is a single-cell with 9 stationary users ($N = 9$) using the same data rate R and the same modulation scheme, noncoherent BFSK. The system parameters used in this study are given in Table 2.1. The

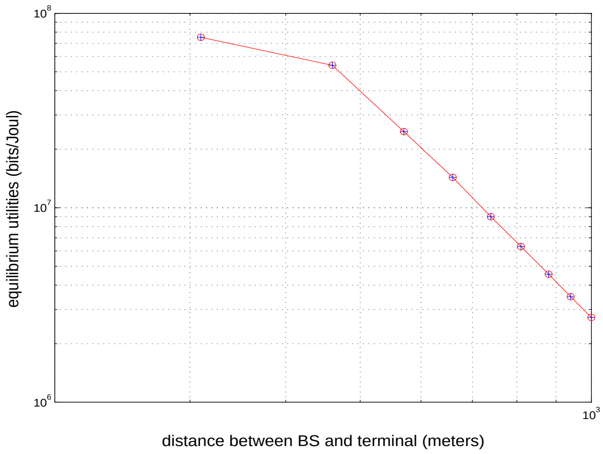

Figure 3.1: Equilibrium utilities of NPG for Rayleigh slow flat-fading channel by using (3.5.2) (o) and by simulation with samples drawn according to Rayleigh distribution (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

distances between the 9 users and the BS are d $= [310,\ 460, 570,\ 660, 740,\ 810, 880,\ 940, 1000]$. The path attenuation between user j and the BS using the simple path loss model [16] is $h_j = 0.097/d_j^4$. Using (3.4.16) and Theorem 3.3.1 with accuracy $\epsilon = 0.04$ and confidence $1 - \delta^*(\epsilon, n) \approx 0.99$ (see theorem 3.3.1 for the definition of $\delta^*(\epsilon, n)$) the sample complexity required was $n \leq 47000$.

Fig. 3.1 and Fig. 3.2 show, respectively, the equilibrium utilities and the equilibrium powers (o) obtained by NPG using the average utility function in (3.5.2) compared to the empirical values obtained by simulating the Rayleigh slow flat-fading channel (+). In the simulation, the sample complexity (the number of samples drawn from the channel according to a Rayleigh distribution) was $47,000$ as mentioned above. NPG was run for each sample from the channel, then the empirical means of the equilibrium utilities were calculated according to (3.5.4). As one can see, the figures show that the empirical results (+) fit the results obtained by averaging with respect to the known distribution (Rayleigh distribution in our case) (o). This shows the learnability of the utility function classes $\mathcal{U}$ with reasonable sample complexity.

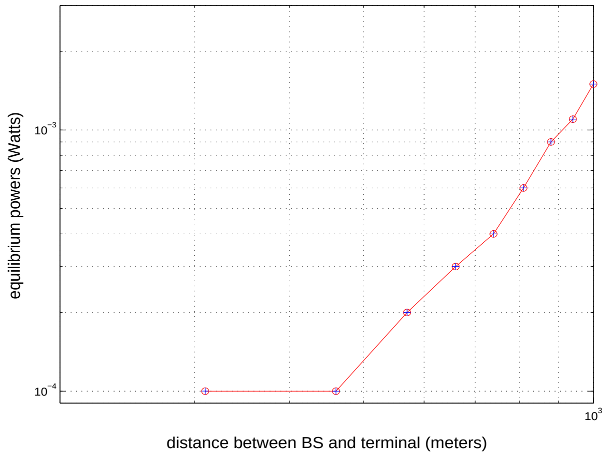

Figure 3.2: Equilibrium powers of NPG for Rayleigh slow flat-fading channel by using (3.5.2) (o) and by simulation with samples drawn according to Rayleigh distribution (+) versus the distance of a user from the BS in meters with $W/R = 1000$.

3.6 Summary

In this Chapter we studied a noncooperative power control game (NPG) and a noncooperative power control game with pricing (NPGP) introduced in [8]-[7] using more realistic channels similar to those in [22]. We proposed the use of *distribution-free* learning theory to evaluate the performance of game theoretic power control algorithms for wireless data CDMA cellular systems in arbitrary channels. We studied in detail the case when the channel is modeled as a slow flat-fading channel. We evaluated an upper bound for the P-dimension of the utility function class and presented simulation results for the Rayleigh case, which showed the learnability of the utility class function defined by adjusting the power for each user.

Chapter 4

Outage Probability In Interference Limited Wireless Fading Channels

In this Chapter we consider a game-theoretic power control algorithm in interference-limited fading wireless channels, and propose a distributed (non-cooperative) algorithm to optimize the induced Rayleigh and Nakagami fading outage probability by maximizing the certainty equivalent margin (CEM). We then find a closed-form of the outage probability in Nakagami flat-fading channels with fading figure $m = 2$ (see appendix B), then prove that the problem of minimizing the fading induced outage probability for both Rayleigh and Nakagami channels is equivalent to the problem of maximizing CEM in term of power allocation. We then propose a distributed game theoretic power control algorithm, and using this non-cooperative game, we prove that the best policy in interference limited fading wireless channels is for all users to send at the *minimum* power in their corresponding strategy spaces. This result considerably simplifies the implementation of the proposed game.

In this Chapter the problem posed in [12] to optimize the outage probability and the certainty-equivalent margin for Rayleigh wireless fading channels, is solved in a non-centralized manner. We also provide a simpler proof (compared to that of [12]) of the tight relationship between the two problems: Minimizing the fading induced outage probability, and maximizing the CEM. We then present a closed-form of the outage probability in a Nakagami flat-fading channel. Using the outage probability formula we prove that both problems mentioned above remain equivalent in Nakagami flat-fading channels.

The remaining of this Chapter is organized as follows: In Section 4.1 we present the system setup used in this Chapter. Section 4.2 is devoted to evaluating a closed-form of the induced outage probability in an interference-limited Nakagami flat-fading channel. The tight relationship between the optimization of the outage probability and the optimization of the certainty-equivalent margin is emphasized in 4.3. Non-cooperative power control game (NPG) is discussed in Section 4.4. Simulation results are outlined in Section 4.5, and the summary of this Chapter is presented in Section 4.6.

4.1 System Model

The system we are investigating is the same as that studied in [12], where the solutions for optimizing the system outage probability and the system certainty-equivalent margin in Rayleigh flat-fading channels were obtained based on a centralized power control algorithm. In this Chapter however, we propose a non-centralized power control game theoretic-algorithm for the same system in Rayleigh and Nakagami flat-fading channels. For convenience, we cast the problem as follows: Suppose we have N Tx/Rx pairs in a cellular mobile system. The ith transmitter is supposed to send messages at a power level p_i from his convex strategy space P_i to the ith receiver. A transmitter-receiver pair does not necessarily indicate physically separated receivers [12]. The received power level at the ith receiver from the kth transmitter is given by:

$$G_{i,k}F_{i,k}p_k \tag{4.1.1}$$

where $G_{i,k} > 0$ is the path gain from the kth transmitter to the ith receiver. This gain may represent spreading gain and/or cross correlation between codes in CDMA systems. It can also represent coding gain, log-normal shadowing, and antenna gains. $F_{i,k},\ \ i,k = 1,2,...,N$ are exponentially id (independent distributed) random variables with mean equal to 1 to represent the statistical power variation in a wireless Rayleigh flat-fading channel. On the other hand in a Nakagami flat-fading channel, $F_{i,k},\ \ i,k = 1,2,...,N$ are Gamma id random variables with mean also equal to 1. This means that the power received from the kth user at the ith receiver is exponentially (Gamma) distributed with expected value

$$\mathrm{E}\left\{G_{i,k}F_{i,k}p_k\right\} = G_{i,k}p_k \tag{4.1.2}$$

In interference-limited fading channels, the background AWGN is assumed to be negligible compared to the interference power from the users. Henceforth, the signal-to-interference ratio of the ith user at the corresponding receiver is given by:

$$SIR_i = \frac{G_{i,i}F_{i,i}p_i}{\sum_{k\neq i}^{N} G_{i,k}F_{i,k}p_k}. \tag{4.1.3}$$

SIR_i is thus a random variable, a ratio of an exponentially (Gamma) distributed random variable to a summation of independent exponentially (Gamma) distributed random variables with different means. The outage probability O_i is defined as the probability that the SIR of an active user i, will go below a threshold SIR_{th}, so that for user i:

$$\begin{aligned} O_i &= \Pr\{SIR_i \leq SIR_{th}\} \\ &= \Pr\{G_{i,i}F_{i,i}p_i \leq SIR_{th} \sum_{k\neq i}^{N} G_{i,k}F_{i,k}p_k\} \end{aligned} \tag{4.1.4}$$

This probability was evaluated in [12] for a Rayleigh flat-fading channel and is given by:

$$O_i = 1 - \prod_{k \neq i}^{N} \frac{1}{1 + SIR_{th} \frac{G_{i,k} p_k}{G_{i,i} p_i}} \quad (4.1.5)$$

The outage probability of the system, O is defined as:

$$O = \max_{1 \leq i \leq N} O_i \quad (4.1.6)$$

Thus O plays the role of a figure of merit of the cellular system and the power control algorithm. The certainty-equivalent margin of the ith user is defined as the ratio of his/her certainty-equivalent SIR to the corresponding threshold SIR. Mathematically,

$$CEM_i = \frac{SIR_i^{ce}}{SIR_{th}} \quad (4.1.7)$$

where, the certainty-equivalent SIR (SIR_i^{ce}) of the ith user is defined as the ratio of his/her mean received power at the corresponding receiver to the mean of the interference from the other users in the system, i.e.,

$$SIR_i^{ce} = \frac{G_{i,i} p_i}{\sum_{k \neq i}^{N} G_{i,k} p_k} \quad (4.1.8)$$

The certainty-equivalent SIR of the system SIR^{ce} is defined as:

$$SIR^{ce} = \min_{1 \leq i \leq N} SIR_i^{ce} \quad (4.1.9)$$

Therefore, the certainty-equivalent margin of the system CEM is given by:

$$CEM = \min_{1 \leq i \leq N} CEM_i \quad (4.1.10)$$

The CEM plays the role of yet another cellular system figure of merit for the power control algorithm and the complete system.

4.2 Outage Probability in an Interference Limited Nakagami Flat-Fading Channel

In this Section we derive a closed-form of the outage probability in an interference limited Nakagami flat-fading channels.

Let $\alpha_{i,k}$ be the fading coefficient between the kth transmitter and the ith receiver, then $\alpha_{i,k}$ has a Nakagami PDF (see Appendix B):

$$f^{\alpha_{i,k}}(\omega) = \frac{2\, m^m}{\Gamma(m)\, \Omega^m}\; \omega^{2m-1}\; \exp(-\frac{m}{\Omega}\omega^2) \tag{4.2.1}$$

The following lemma presents a closed-form of the outage probability in a flat-fading Nakagami channels.

Lemma 4.2.1. *In a Nakagami flat-fading channel with* $\Omega = 1$ *and* $m = 2$*, the outage probability of the* i*th user* O_i *is given by:*

$$O_i \;=\; 1 - \left(N - \sum_{k\neq i}^{N} \left[\frac{1}{1 + \frac{SIR_{th}\, G_{i,k}\, p_k}{G_{i,i}\, p_i}}\right]\right) \prod_{k\neq i}^{N} \left[\frac{1}{1 + \frac{SIR_{th}\, G_{i,k}\, p_k}{G_{i,i}\, p_i}}\right]^2 \tag{4.2.2}$$

Proof : For simplicity, define $Y_{i,k} := G_{i,k}F_{i,k}p_k$, where $F_{i,k} = \alpha_{i,k}^2$, and hence $Y_{i,k}$ is a Gamma distributed random variable, that is:

$$f^{Y_{i,k}}(\omega) = \frac{\xi_{i,k}^m}{\Gamma(m)}\; \omega^{m-1}\; \exp(-\xi_{i,k}\, \omega) \tag{4.2.3}$$

where $\xi_{i,k} = \frac{m}{\Omega\, G_{i,k}\, p_k}$. In the following calculations we set $m = 2$ and $\Omega = 1$. Henceforth, the outage probability O_i of the ith user is given by:

$$\begin{aligned} O_i &= \Pr\{Y_{i,i} \le SIR_{th} \sum_{k\neq i}^{N} Y_{i,k}\} \\ &= \int_0^{X_{i,k}} f^{Y_{i,i}}(\omega) d\omega \end{aligned} \tag{4.2.4}$$

where we defined $X_{i,k} := SIR_{th} \sum_{k\neq i}^{N} Y_{i,k}$. As one can see, $X_{i,k}$ is the sum of independent Gamma distributed random variables with different means. To simplify the problem in (4.2.4), we first find the conditional outage probability $O_i^c(x_{i,k}) = \Pr(O_i|X_{i,k} = x_{i,k})$ where $x_{i,k}$ is a realization of the random variable $X_{i,k}$. Therefore, we have the following:

$$\begin{aligned} O_i^c(x_{i,k}) &= \xi_{i,i}^2 \int_0^{x_{i,k}} \omega \ \exp(-\xi_{i,i}\ \omega) d\omega \\ &= 1 - (1 + \xi_{i,i}\ x_{i,k})\ \exp(-\xi_{i,i}\, x_{i,k}). \end{aligned} \tag{4.2.5}$$

Hence, outage probability O_i can be found as:

$$\begin{aligned} O_i &= \int_0^{\infty} O_i^c(\omega)\ f^{X_{i,k}}(\omega) d\omega \\ &= E\{O_i^c(X_{i,k})\} \\ &= E\{1 - (1 + \xi_{i,i}\ X_{i,k})\ \exp(-\xi_{i,i}\, X_{i,k})\}. \end{aligned} \tag{4.2.6}$$

Using the following intermediate equations:

$$\int_0^{\infty} \omega^n \ \exp(-a\,\omega) d\omega = \frac{\Gamma(n+1)}{a^{n+1}}, \tag{4.2.7}$$

$$E\left\{\exp(-\xi_{i,i}\, SIR_{th}\, Y_{i,k})\right\} = \left[\frac{1}{1 + \frac{SIR_{th}\, G_{i,k}\, p_k}{G_{i,i}\, p_i}}\right]^2, \tag{4.2.8}$$

and

$$E\{Y_{i,k}\ \exp(-\xi_{i,i}\, SIR_{th}\, Y_{i,k})\} = G_{i,k}\, p_k \left[\frac{1}{1 + \frac{SIR_{th}\, G_{i,k}\, p_k}{G_{i,i}\, p_i}}\right]^3 \tag{4.2.9}$$

we end up with the following formula of the outage probability O_i,

$$\begin{aligned} O_i &= 1 - \prod_{k\neq i}^{N} \left[\frac{1}{1 + \frac{SIR_{th}\, G_{i,k}\, p_k}{G_{i,i}\, p_i}}\right]^2 - \sum_{k\neq i}^{N} \frac{SIR_{th}\, G_{i,k}\, p_k}{G_{i,i}\, p_i} \left[\frac{1}{1 + \frac{SIR_{th}\, G_{i,k}\, p_k}{G_{i,i}\, p_i}}\right]^3 \\ &\times \prod_{l\neq i, l\neq k}^{N} \left[\frac{1}{1 + \frac{SIR_{th}\, G_{i,l}\, p_l}{G_{i,i}\, p_i}}\right]^2 \end{aligned} \tag{4.2.10}$$

To make (4.2.10) look like (4.2.2), we substitute the following equations in (4.2.10):

$$\prod_{l\neq i,l\neq k}^{N}\left[\frac{1}{1+\frac{SIR_{th}\,G_{i,l}\,p_l}{G_{i,i}\,p_i}}\right]^2=\left(1+\frac{SIR_{th}\,G_{i,k}\,p_k}{G_{i,i}\,p_i}\right)^2\prod_{l\neq i}^{N}\left[\frac{1}{1+\frac{SIR_{th}\,G_{i,l}\,p_l}{G_{i,i}\,p_i}}\right]^2$$

and

$$\begin{aligned}\sum_{k\neq i}^{N}\frac{SIR_{th}\,G_{i,k}\,p_k}{G_{i,i}\,p_i}\left[\frac{1}{1+\frac{SIR_{th}\,G_{i,k}\,p_k}{G_{i,i}\,p_i}}\right] &= \sum_{k\neq i}^{N}\left(1-\left[\frac{1}{1+\frac{SIR_{th}\,G_{i,k}\,p_k}{G_{i,i}\,p_i}}\right]\right) \quad (4.2.11)\\ &= \left(N-1-\sum_{k\neq i}^{N}\left[\frac{1}{1+\frac{SIR_{th}\,G_{i,k}\,p_k}{G_{i,i}\,p_i}}\right]\right)\end{aligned}$$

which concludes the proof. □

4.3 Relation Between Outage Probability and Certainty-Equivalent Margin

4.3.1 Rayleigh Flat Fading channel

In this subsection we present the following proposition first introduced in [12], and provide our own simpler proof.

Proposition 4.3.1. *The problems of minimizing the fading-induced outage probability O_i and maximizing the certainty equivalent CEM_i of the ith user in a Rayleigh wireless fading channel are equivalent in terms of power allocation.*

Proof : First, recall that the problem under study is to minimize the outage probability of the system in a distributive fashion, i.e., each user transmits at a power level that minimizes his/her outage probability. Mathematically,

$$\min_{p_i\in P_i} O_i = \min_{p_i\in P_i} 1-\prod_{k\neq i}^{N}\frac{1}{1+SIR_{th}\frac{G_{i,k}p_k}{G_{i,i}p_i}} \quad (4.3.1)$$

and this in turn is equivalent to

$$\max_{p_i \in P_i} \prod_{k \neq i}^{N} \frac{1}{1 + SIR_{th} \frac{G_{i,k} p_k}{G_{i,i} p_i}} \tag{4.3.2}$$

or

$$\min_{p_i \in P_i} \prod_{k \neq i}^{N} \left(1 + SIR_{th} \frac{G_{i,k} p_k}{G_{i,i} p_i} \right) \tag{4.3.3}$$

Using the monotonicity of the Logarithmic function, minimizing (4.3.3) is equivalent to solving the following problem:

$$\min_{p_i \in P_i} \sum_{k \neq i}^{N} \log \left(1 + SIR_{th} \frac{G_{i,k} p_k}{G_{i,i} p_i} \right). \tag{4.3.4}$$

Now, using the inequality $\log(x) \leq x - 1$, (4.3.4) can be bounded as:

$$\begin{aligned} \min_{p_i \in P_i} \quad & \textstyle\sum_{k \neq i}^{N} \log \left(1 + SIR_{th} \frac{G_{i,k} p_k}{G_{i,i} p_i} \right) \\ \leq \quad & \min_{p_i \in P_i} \sum_{k \neq i}^{N} SIR_{th} \frac{G_{i,k} p_k}{G_{i,i} p_i}. \end{aligned} \tag{4.3.5}$$

Finally, it is fairly simple to see that the right side of the inequality in (4.3.5) is exactly the same as:

$$\max_{p_i \in P_i} \frac{G_{i,i} p_i}{SIR_{th} \sum_{k \neq i}^{N} G_{i,k} p_k} = \max_{p_i \in P_i} CEM_i \tag{4.3.6}$$

□

4.3.2 Nakagami Flat Fading Channel

Similar to Rayleigh flat-fading channels, we have the following proposition for Nakagami flat-fading channels.

Proposition 4.3.2. *The problems of minimizing the fading induced outage probability O_i and maximizing the certainty equivalent CEM_i of the ith user in a Nakagami wireless fading channel are equivalent in terms of power allocation.*

Proof : For simplicity, let us define $H_{i,k} := SIR_{th}\frac{G_{i,k}\,p_k}{G_{i,i}\,p_i}$. Minimizing the outage probability O_i given in (4.2.2) is equivalent to the following problem:

$$\max_{p_i \in P_i}\left\{\left(N - \sum_{k\neq i}^{N}\left[\frac{1}{1+H_{i,k}}\right]\right)\prod_{k\neq i}^{N}\left[\frac{1}{1+H_{i,k}}\right]^2\right\} \tag{4.3.7}$$

or

$$\begin{aligned}
&\max_{p_i \in P_i}\left\{\ln\left(N - \sum_{k\neq i}^{N}\left[\frac{1}{1+H_{i,k}}\right]\right) + 2\sum_{k\neq i}^{N}\ln\left(\frac{1}{1+H_{i,k}}\right)\right\}\\
=\ &\max_{p_i \in P_i}\left\{\ln\left(1 + \sum_{k\neq i}^{N}\left(1 - \frac{1}{1+H_{i,k}}\right)\right) + 2\sum_{k\neq i}^{N}\ln\left(1 - \frac{H_{i,k}}{1+H_{i,k}}\right)\right\}\\
\leq\ &\max_{p_i \in P_i}\left\{\sum_{k\neq i}^{N}(1 - \frac{1}{1+H_{i,k}}) - 2\sum_{k\neq i}^{N}\frac{H_{i,k}}{1+H_{i,k}}\right\}\\
=\ &\max_{p_i \in P_i}\left\{\sum_{k\neq i}^{N} -\frac{H_{i,k}}{1+H_{i,k}}\right\}
\end{aligned} \tag{4.3.8}$$

where we used the inequality $\ln(x) \leq x - 1$ again to get the inequality in (4.3.8). Now one can see that the problem in (4.3.8) is equivalent to:

$$\max_{p_i \in P_i}\left\{\sum_{k\neq i}^{N}\frac{1+H_{i,k}}{H_{i,k}}\right\} = \max_{p_i \in P_i}\left\{N - 1 + \sum_{k\neq i}^{N}\frac{G_{i,i}\,p_i}{SIR_{th}\,G_{i,k}\,p_k}\right\} \tag{4.3.9}$$

and since $N-1$ is a constant common for all users, therefore (4.3.9) is equivalent to:

$$\min_{p_i \in P_i}\left\{\frac{\sum_{k\neq i}^{N} SIR_{th}\,G_{i,k}\,p_k}{G_{i,i}\,p_i}\right\}. \tag{4.3.10}$$

Finally, this problem is clearly equivalent to the problem of maximizing CEM_i, and by this we conclude the proof of proposition 4.3.2 □

In the next Section we present a simple non-cooperative game $G2$, and show that $G2$ results in an optimal power allocation to maximize the system certainty-equivalent margin CEM, minimize the system outage probability O, and minimize the total transmitted power.

4.4 Power Control Algorithm to Optimize The Outage Probability

In this Section we introduce a non-cooperative power control game, in which user i attempts to find the optimal transmit power level p_i from his/her strategy space P_i that enables him/her to obtain a maximum possible certainty equivalent margin CEM_i^*, instead of maximizing CEM_i directly as given below:

$$\begin{aligned} G1 \quad &: \quad \max_{p_i \in P_i} \; CEM_i, \\ &= \quad \max_{p_i \in P_i} \; \frac{G_{i,i} p_i}{SIR_{th} \; \sum_{k \neq i}^{N} G_{i,k} p_k}, \forall\, i = 1, 2, ..., N \end{aligned} \tag{4.4.1}$$

The reason for avoiding maximizing CEM_i directly is that the game $G1$ in (4.4.1) has an objective function CEM_i which is linear in p_i given the power vector p_{-i} of all users except for the ith user. This will lead user i to send at the maximum power in his/her strategy space, as will all users. This selfish act will result in all users having very small CEMs. Due to this reason we propose the following non-cooperative power control game $G2$ defined as follows:

$$\begin{aligned} &G2: \quad \max_{p_i \in P_i} \; CEM_i^*(n), \; n = 1, 2, \cdots \\ &\text{subject to} \\ &p_i(n) = \min \left(p_{i-\max}, \frac{CEM_i^*(n) \; SIR_{th} \; \sum_{k \neq i}^{N} G_{i,k} p_k}{G_{i,i}} \right) \end{aligned} \tag{4.4.2}$$

Seemingly, game $G2$ is a multistage game where in the nth stage, user i transmits at a power level $p_i(n)$ that enables him/her to attain a constant $CEM_i^*(n)$ (e.g. $CEM_i^*(n) = 1$), then if transmit power level $p_i(n)$ is feasible, i.e., $p_i(n) < p_{i-\max}$, user i seeks a higher value of CEM_i^* at the $(n+1)$th stage of the game. Mathematically speaking, user i sets $CEM_i^*(n+1) > CEM_i^*(n)$ and finds $p_i(n+1)$ such that $CEM_i^*(n+1) = \frac{G_{i,i} p_i(n+1)}{SIR_{th} \sum_{k \neq i}^{N} G_{i,k} p_k}$ and so forth until he/she is satisfied with the value of $CEM_i^*(n)$. However, it turns out that game $G2$ is a one shot game, that is, it has a Nash equilibrium point with all users able to attain their maximum CEM_i^* in the first stage ($n = 1$) as we show in the following lemma. The lemma also guarantees the existence, uniqueness, and optimality of the Nash equilibrium point:

Lemma 4.4.1. *The non-cooperative game $G2$ with strategy space $P_i = [p_{i-\min}, p_{i-\max}]$ for user i and with $p_{i-\min} > 0$, has a unique Nash equilibrium operating point. Also, this Nash equilibrium point is optimal in the sense that it corresponds to the minimum total transmitted power of all users. Users in game $G2$ will attain their maximum possible CEM_i^* in the first trial.*

Proof : At each time instance, user i updates his/her transmit power p_i in order to satisfy the following equation:

$$CEM_i^* = \frac{G_{i,i} p_i}{SIR_{th} \sum_{k \neq i}^{N} G_{i,k} p_k}, \tag{4.4.3}$$

for a target CEM_i^*. Exploiting the linearity of equation (4.4.3), we rewrite it in a matrix form as follows:

$$\mathrm{A} \; \mathrm{p} = \mathrm{p}, \tag{4.4.4}$$

where the entries of the matrix A are given by:

$$\mathrm{A}(i,j) = \begin{cases} \frac{SIR_{th} \, CEM_i^* \, G_{i,j}}{G_{i,i}}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$$

And p = $(p_1, p_2, ..., p_N)$ is the vector of transmit powers of all users. Since $G_{i,j} > 0$, matrix A is a nonnegative irreducible matrix. By the Perron-Frobenius theorem, the largest eigenvalue in magnitude of matrix A is real and positive [3]. Therefore, if it happens that 1 is this eigenvalue of A, then the solution p will be the eigenvector that corresponds to eigenvalue 1. Otherwise, the only solution of equation (4.4.4) is the trivial solution p = 0. Since p = 0 is not a feasible solution, each user will transmit at the lowest power level in his/her strategy space. Therefore, the Nash equilibrium point of game $G2$ is unique and corresponds to an optimal point that minimizes the total transmitted power of all users.

Equation (4.4.4) holds true for any $CEM_i^* \geq 1$ including the maximum value that users can attain at the equilibrium point. This implies that the cellular users will attain their maximum possible CEM_i^* in their first trial, i.e., $G2$ is a one-shot game. □

It is easy to notice that if user i increases his power unilaterally to improve his/her CEM_i and O_i, at least one other user in the network will be harmed.

Lemma 4.4.1 implies that in interference-limited wireless Rayleigh and Nakagamai flat-fading channels, the best policy is for all users to transmit at the minimum power in their corresponding strategy spaces. The question that may arise is therefore: How do we guarantee the quality of service (QoS), e.g. SIR at the BS? The following lemma answers this question.

Lemma 4.4.2. *If users in an interference-limited wireless fading channels are not able to attain a satisfactory QoS (SIR) by transmitting at their minimum power vector* $p_{\min} = (p_{1-\min}, p_{2-\min}, ..., p_{N-\min})$ *in their corresponding strategy spaces, they will not be able to attain a satisfactory QoS by transmitting at any other power vector* $p^l = (p_1^l, p_2^l, ..., p_N^l)$ *larger than* $p_{\min}$*,* $p^l > p_{\min}$ *component wise.*

Proof : Let $\mathcal{N} = (1, 2, ..., N)$ be the indexing set of all active users in the cell. Suppose $CEM_i^{\min}$ is the CEM value user i attained by assuming that all users are transmitting at the minimum powers in their corresponding strategy spaces, that is, $\forall i \in \mathcal{N}$:

$$CEM_i^{\min} := \frac{G_{i,i}\, p_{i-\min}}{SIR_{th} \sum_{k\neq i}^{N} G_{i,k}\, p_{k-\min}}, \tag{4.4.5}$$

and suppose that CEM_i^l is the value of CEM the ith user attains assuming all users transmitting at p^l, henceforth, $\forall i \in \mathcal{N}$ we have:

$$CEM_i^l := \frac{G_{i,i}\, p_i^l}{SIR_{th} \sum_{k\neq i}^{N} G_{i,k}\, p_k^l} \tag{4.4.6}$$

To prove lemma 4.4.2, it is enough to prove that there is a subset of users $\mathcal{K}_N \subset \mathcal{N}$, such that:

$$CEM_n^l \leq CEM_n^{\min}, \forall n \in \mathcal{K}_N \tag{4.4.7}$$

Suppose this is not true, therefore

$$CEM_i^l > CEM_i^{\min}. \tag{4.4.8}$$

Define $d_i^{l,\min}$ as:

$$\begin{aligned} d_i^{l,\min} &:= CEM_i^l - CEM_i^{\min} \\ &= \frac{1}{SIR_{th}} \left[\frac{G_{i,i}\, p_i^l}{\sum_{k\neq i}^{N} G_{i,k}\, p_k^l} - \frac{G_{i,i}\, p_{i-\min}}{\sum_{k\neq i}^{N} G_{i,k}\, p_{k-\min}} \right]. \end{aligned} \tag{4.4.9}$$

Without loss of generality, $\forall i \in \mathcal{N}$ let $p_i^l = \delta_i\, p_{i-\min}$, where $\delta_i > 1$. Then, (4.4.9) can be written as:

$$d_i^{l,\min} = \frac{G_{i,i}\, p_{i-\min}}{SIR_{th}} \left[\frac{\delta_i}{\sum_{k\neq i}^{N} \delta_k\, G_{i,k}\, p_{k-\min}} - \frac{1}{\sum_{k\neq i}^{N} G_{i,k}\, p_{k-\min}} \right] \tag{4.4.10}$$

The inequality in (4.4.8) implies that $d_i^{l,\min} > 0$ for all $i \in \mathcal{N}$, therefore

$$\delta_i \sum_{k\neq i}^{N} G_{i,k}\, p_{k-\min} > \sum_{k\neq i}^{N} \delta_k\, G_{i,k}\, p_{k-\min} \tag{4.4.11}$$

For simplicity define $x_{i,k} := G_{i,k}\, p_{k-\min}$, then (4.4.11) can be expressed as:

$$\delta_i \sum_{k\neq i}^{N} x_{i,k} > \sum_{k\neq i}^{N} \delta_k\, x_{i,k} \tag{4.4.12}$$

If we set $i = 1$ in the above equation we obtain the following result:

$$\delta_1 > \delta_m, m = 2, 3, ..., N, \tag{4.4.13}$$

while if we set $i = 2$, we obtain:

$$\delta_2 > \delta_m, m = 1, 3, ..., N, \tag{4.4.14}$$

From equation (4.4.13), we have $\delta_1 > \delta_2$, while from (4.4.14) we find $\delta_2 > \delta_1$, leading to a contradiction. This proves the statement in equation (4.4.7), and concludes the proof of lemma 4.4.2. □

4.5 Simulation Results

The cellular system is assumed to have $N = 50$ Tx/Rx pairs. The path gains $G_{i,k}$ were generated according to a uniform distribution on the interval $[0, 0.001]$ for all $i \neq k \in \mathcal{N}$ and $G_{i,i} = 1\ \forall i \in \mathcal{N}$. Game $G2$ was run for different values of the threshold signal-to-interference ratios (SIR_{th}) in the interval $[3, 10]$.

In Fig. 4.1 we show the system certainty-equivalent margin values (CEM) resulting from game $G2$ versus the threshold signal-to-interference ratios, SIR_{th} under both Rayleigh and Nakagami flat-fading channels. While in Fig. 4.2, we present the resulting system outage probability, O of a Rayleigh fading channel($*$) and a Nakagami fading channel ($\circ$) versus the threshold SIR (SIR_{th}) compared to the minimum bound $1/(1 + CEM)$ (solid line) and the upper bound $1 - e^{-1/CEM}$ (dashed line) derived in [12]. In this figure, as one can see in the Rayleigh case, the upper bound overlaps with the equilibrium outage probability which is the output of game $G2$.

The results shown in Fig. 4.1 and Fig. 4.2 happened to be very close to the results obtained using Perron-Frobenius theorem [12] (in Rayleigh fading channel) but at a lower power allocation. This is obvious from tables 4.1 - 4.4. In these tables we show CEM_i and O_i for the first 10 users as evaluated by the Perron-Frobenius theorem in [12] and as equilibrium outcomes of the NPG game $G2$ of both channel models: Rayleigh flat-fading channel and Nakagami flat-fading channel. The averages of CEM_i and O_i presented in the tables are calculated for all users in the system. We observed that the average value of CEM obtained through NPG game $G2$ was higher than that obtained by the Perron-Frobenius theorem for all values of SIR_{th}. The average value of O obtained through NPG game $G2$ was sometimes lower and sometimes higher than that obtained by Perron-Frobenius theorem. By examining tables 4.1 - 4.4, one can see that by transmitting at Perron-Frobenius eigenvector (power vector), many users attain lower CEM values than they obtained when all users transmitted at the minimum power vector. This agrees with what was proved in lemma 4.4.2. Finally, notice that in Fig. 4.2 results show that users can achieve better performance in a Nakagami fading channel with ($m = 2$) than in a Rayleigh channel. This was expected since a Nakagami fading channel with fading figure $m = 2$ represents a less severe fading channel than a Rayleigh fading channel which is a Nakagami fading channel with fading figure $m = 1$.

4.6 Summary

We proved the tight relationship between the two optimization problems: minimizing the system outage probability and maximizing the system certainty-equivalent margin under Rayleigh and Nakagami flat-fading wireless channels. A closed-form of the outage probability under Nakagami flat-fading channel was also provided. We then proposed an asynchronous distributed non-cooperative power control game-theoretic

Table 4.1: Equilibrium values of CEM_i and O_i for the first 10 users using in a Rayleigh flat-fading channel Perron-Frobenius theorem and the NPG game $G2$ introduced in this Chapter at $SIR_{th} = 3$

Results using Perron-Frobenius theorem [12]				**Results of game $G2$**		
i	p_i	CEM_i	O_i	p_i	CEM_i	O_i
1	0.1292	13.7107	0.0703	0.0100	14.9809	0.0645
2	0.1162	13.7107	0.0703	0.0100	16.6629	0.0582
3	0.1476	13.7107	0.0703	0.0100	13.0747	0.0736
4	0.1482	13.7107	0.0703	0.0100	12.9549	0.0742
5	0.1290	13.7107	0.0703	0.0100	14.9275	0.0647
6	0.1192	13.7107	0.0703	0.0100	16.3274	0.0594
7	0.1297	13.7107	0.0703	0.0100	15.0442	0.0643
8	0.1312	13.7107	0.0703	0.0100	14.6970	0.0657
9	0.1327	13.7107	0.0703	0.0100	14.4091	0.0670
10	0.1361	13.7107	0.0703	0.0100	14.2906	0.0675
average		13.7107	0.0703	**average**	13.7823	0.0704

algorithm to optimize the system certainty-equivalent margin and the system outage probability. Using the proposed non-cooperative game $G2$, we proved that the best power allocation in interference limited Rayleigh and Nakagami fading wireless channels is the minimum power vector in the total strategy spaces of active users in the system. Power was more effectively and more simply allocated according to this proposed non-centralized algorithm than the centralized algorithm in [12].

Table 4.2: Equilibrium values of CEM_i and O_i for the first 10 users in a Rayleigh flat-fading channel using Perron-Frobenius theorem and the NPG game $G2$ introduced in this Chapter at $SIR_{th} = 10$

Results using Perron-Frobenius theorem [12]				**Results of game $G2$**		
i	p_i	CEM_i	O_i	p_i	CEM_i	O_i
1	0.1439	4.0985	0.2159	0.0100	4.0243	0.2194
2	0.1541	4.0985	0.2159	0.0100	3.7254	0.2347
3	0.1266	4.0985	0.2159	0.0100	4.5414	0.1971
4	0.1497	4.0985	0.2159	0.0100	3.8603	0.2275
5	0.1375	4.0985	0.2159	0.0100	4.1930	0.2116
6	0.1378	4.0985	0.2159	0.0100	4.1877	0.2118
7	0.1096	4.0985	0.2159	0.0100	5.3148	0.1711
8	0.1514	4.0985	0.2159	0.0100	3.8264	0.2293
9	0.1398	4.0985	0.2159	0.0100	4.1414	0.2139
10	0.1384	4.0985	0.2159	0.0100	4.1889	0.2118
average		4.0985	0.2159	**average**	4.1255	0.2156

Table 4.3: Equilibrium values of CEM_i and O_i for the first 10 users in a Nakagami flat-fading channel using Perron-Frobenius theorem and the NPG game $G2$ introduced in this Chapter at $SIR_{th} = 3$

Results using Perron-Frobenius theorem				**Results of game $G2$**		
i	p_i	CEM_i	O_i	p_i	CEM_i	O_i
1	0.1368	13.7415	0.0580	0.0100	14.2272	0.0561
2	0.1501	13.7415	0.0580	0.0100	12.9034	0.0618
3	0.1305	13.7415	0.0580	0.0100	14.6840	0.0543
4	0.1360	13.7415	0.0580	0.0100	14.2485	0.0560
5	0.1270	13.7415	0.0580	0.0100	15.0958	0.0528
6	0.1539	13.7415	0.0580	0.0100	12.5887	0.0633
7	0.1499	13.7415	0.0580	0.0100	12.8494	0.0620
8	0.1402	13.7415	0.0580	0.0100	13.7151	0.0581
9	0.1577	13.7415	0.0580	0.0100	12.2655	0.0649
10	0.1552	13.7415	0.0580	0.0100	12.5932	0.0633
average		13.7415	0.0580	**average**	13.8343	0.0581

Table 4.4: Equilibrium values of CEM_i and O_i for the first 10 users in a Nakagami flat-fading channel using Perron-Frobenius theorem and the NPG game $G2$ introduced in this Chapter at $SIR_{th} = 10$

Results using Perron-Frobenius theorem				**Results of game $G2$**		
i	p_i	CEM_i	O_i	p_i	CEM_i	O_i
1	0.1539	4.0590	0.1906	0.0100	3.7011	0.2078
2	0.1432	4.0590	0.1906	0.0100	3.9928	0.1936
3	0.1384	4.0590	0.1906	0.0100	4.1278	0.1876
4	0.1553	4.0590	0.1906	0.0100	3.6718	0.2094
5	0.1378	4.0590	0.1906	0.0100	4.1369	0.1872
6	0.1305	4.0590	0.1906	0.0100	4.3997	0.1766
7	0.1593	4.0590	0.1906	0.0100	3.5804	0.2143
8	0.1553	4.0590	0.1906	0.0100	3.6847	0.2087
9	0.1456	4.0590	0.1906	0.0100	3.9305	0.1965
10	0.1483	4.0590	0.1906	0.0100	3.8569	0.2000
average		4.0590	0.1906	**average**	4.0948	0.1903

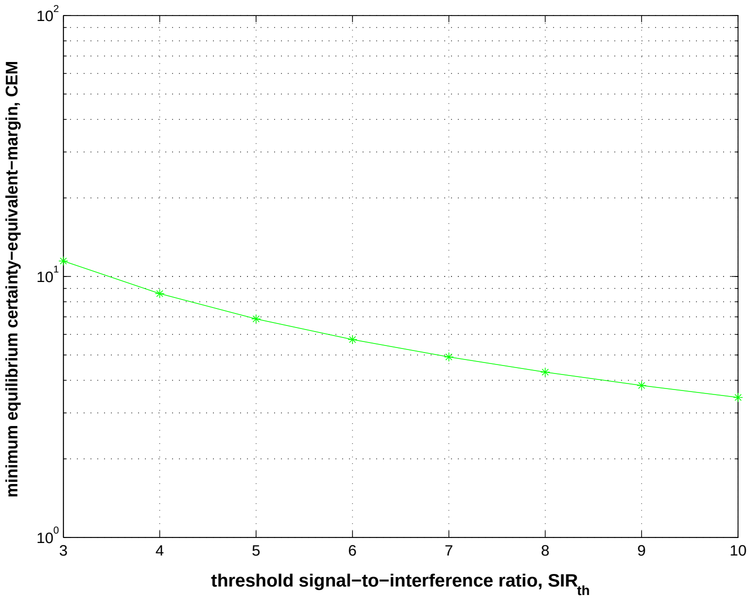

Figure 4.1: Minimum equilibrium certainty-equivalent-margin in Rayleigh and Nakagami fading channels versus the threshold signal-to-interference ratio.

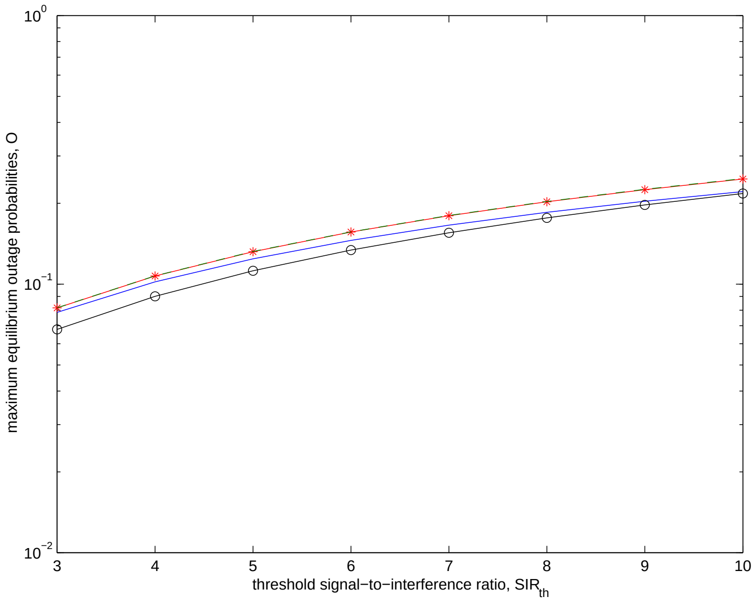

Figure 4.2: Maximum equilibrium Rayleigh fading induced outage probability ($*$), the lower bound of the outage probability $\frac{1}{1+CEM}$ (solid line), the upper bound $1 - e^{-1/CEM}$ (dashed line) and the maximum outage probability in a Nakagami channel ($\circ$) versus the threshold signal-to-interference ratio.

Chapter 5

New Power Control Game Theoretic Algorithms

In Chapters 2 and 3 we used a utility function (number of bits received correctly at receiver per one Joule expanded from the battery) that was originally proposed by [5]. One limitation of this utility function is difficulty of writing the optimizing transmit power in terms of the interference from other users and quantities of interest for the users (e.g. desired SIR) in a closed formula. Expressing the optimizing transmit power in a compact and closed formula leads to a fast algorithm that implement the game, since a user just needs to plug in the local measurement broadcasted from the BS in his formula to find the optimizer transmit power. Because otherwise, the user at each instance he/she updates his/her transmit power based on local information from the BS he/she needs to search through all his power strategy space to find the optimizer. For this reason, we use a different utility function that we describe shortly.

In this Chapter, we study a game-theoretic distributed power control algorithm for wireless data in code division multiple access (CDMA) communication systems.

In [26], the authors proposed a cost function which is the difference between a utility function and a pricing function. The proposed utility function is proportional to the capacity of the channel, while the pricing function is linear in the user's transmit power.

The existence and uniqueness of a Nash equilibrium point was established. Moreover, the authors in [26] offered different schemes of pricing and two methods of updating the user's transmit power. One limitation however of the cost function in [26] is that under market-based pricing, and if the users desire SIRs such that their utility factors are the same, the transmit power level will increase as the mobile comes closer to the BS as we will show in the sequel. We propose a target function composed of the difference between a utility function and a pricing function. The pricing function used is a linear function of the transmit power with a slope (pricing factor) chosen in a market-based mechanism, where the base station (BS) decides the pricing factor and communicates it to the users in the cell. Finally, we establish the existence and uniqueness of a Nash equilibrium operating point in an S-modular games framework.

Our work in this Chapter was in fact inspired by the work in [26] for two reasons: firstly, the target function we propose takes care of the limitation mentioned above, and secondly, the proposed target function attains the optimum transmit power at a lower value than the cost function in [26].

The remaining of this Chapter is organized as follows: In Section 5.1 we present the system setup used in this Chapter. The existence and uniqueness of Nash equilibrium is established in Section 5.2. A comparison of the performance of our proposed objective function and that of [26] is presented in 5.3. Simulation results are outlined in Section 5.4, and the summary of this Chapter is presented in Section 5.5.

5.1 System Setup

The system under study is an uplink CDMA communication system in a single cell with N active users sharing the radio channel. A user i is at distance d_i meters from the BS with path gain $h_i = 0.097/d_i^4$, where the constant 0.097 is to approximate the effect of log-normal shadowing. The ith user accesses the radio channel by transmitting to the BS at a power level p_i selected from his strategy space (action profile) P_i. The transmission rate of user i is R_i. The proposed target function is the difference between a utility function and a pricing function. The pricing function is a linear function of the user's transmit power level with a slope (pricing factor) evaluated in a market-based mechanism, in other words, it is chosen by the BS and then communicated to the users in the cell. The proposed target function is given by:

$$L_i(p_i, \mathrm{p}_{-i}, \lambda) = u_i \arctan(1 + \gamma_i) - \lambda\, p_i \tag{5.1.1}$$

where $\lambda > 0$ is the pricing factor broadcasted by the BS to all users, u_i is the utility factor of the ith user selected locally based on the desired SIR, which also measures the willingness of user i to pay. Vector p_{-i} is the vector of the transmit powers of all users except the ith user, and γ_i is the signal-to-interference ratio (SIR) defined by:

$$\gamma_i = \frac{G_i\, h_i\, p_i}{\sum_{k \neq i}^{N} h_k\, p_k + \sigma^2} \tag{5.1.2}$$

where $G_i = W/R_i$ is the spreading gain of user i and W is the chip rate (spread spectrum bandwidth) that is common for all users, and σ^2 is the background AWGN power at the receiver in the BS. The optimization problem of each user is the following: Assume $\mathcal{N} = \{1, 2, \cdots, N\}$ is the indexing set of the users currently in the cell. A user i is supposed to find the optimizing transmit power level that maximizes the target function $L_i(p_i, \mathrm{p}_{-i}, \lambda)$ defined in (5.1.1) as given in the following distributed game G_1^{λ}:

$$\mathrm{G}_1^{\lambda} : \quad \max_{p_i \in P_i} \left(u_i \arctan(1 + \gamma_i) - \lambda\, p_i \right), \ \forall\, i \in \mathcal{N} \tag{5.1.3}$$

For the sake of simplicity, we may refer to $L_i(p_i, \mathrm{p}_{-i}, \lambda)$ by only L_i. In the next Section we establish the existence of a Nash equilibrium point of game G_1^λ in the S-modular games framework.

5.2 Existence of Nash Equilibrium

The optimization problem of the ith user defined in game G_1^λ is to find the transmit power level p_i^o from strategy space P_i that maximizes the utility function defined in (5.1.1). To find the maximizing p_i^o and the conditions for attaining p_i^o, we evaluate the first-order partial derivative of the cost function (5.1.1) with respect to p_i as follows:

$$\frac{\partial L_i}{\partial p_i} = \frac{G_i\, u_i\, h_i}{I_{-i}} \, \frac{1}{1 + (1 + \gamma_i)^2} - \lambda \tag{5.2.1}$$

where $I_{-i} := \sum_{k \neq i}^{N} h_k\, p_k + \sigma^2$ is the sum of the interference from other users and the AWGN power at the BS. The maximizing transmit power p_i^o is thus given by:

$$p_i^o = \frac{1}{G_i\, h_i}\, [\nu_i - I_{-i}] \tag{5.2.2}$$

where $\nu_i^2 := \frac{G_i\, u_i\, h_i}{\lambda}\, I_{-i} - I_{-i}^2$. Note that the second-order derivative $\frac{\partial^2 L_i}{\partial p_i^2} < 0, \forall\, i \in \mathcal{N}$, which means that p_i^o is unique when it exists. By subtracting $\frac{1}{G_i} p_i^o$ from both sides of (5.2.2), it can be written as:

$$p_i^o = \frac{1}{(G_i - 1)\, h_i}\, [\nu_i - I] \tag{5.2.3}$$

where $I = \sum_{k=1}^{N} h_k\, p_k + \sigma^2$ is the measure of the number of users requesting admission to the system, i.e., it quantifies the level of demand in the system.

Note 5.2.1. *A necessary condition for the ith user to be active is that:*

$$u_i > \frac{\lambda\, I_{-i}}{G_i\, h_i}\, [1 + (1 + \frac{\gamma_i}{G_i})^2], \forall\, i \in \mathcal{N} \tag{5.2.4}$$

This is fairly simple to see since the ith user is active if only $p_i^o > 0$, $\forall i \in \mathcal{N}$, and this implies that

$$\nu_i > I \Rightarrow \nu_i^2 > I^2$$

and with simple mathematical manipulations we get the result in note 5.2.1. The importance of equation (5.2.4) is that if the minimum required SIR of user i is $\gamma_i = \gamma_i^*$ then he/she can unilaterally choose his/her utility factor $u_i = \frac{\rho_i \lambda I_{-i}}{G_i h_i} [1 + (1 + \frac{\gamma_i^*}{G_i})^2]$, with $\rho_i > 1$ to guarantee attaining this minimum SIR. Note that the utility factor u_i is a function of the interference from all other users I_{-i}. Henceforth, if the I_{-i} is large, i.e., a large number of users are trying to be admitted to the system, user i has to choose u_i large enough to get his/her minimum SIR as long as his needed maximizing transmit power level p_i^o is in his/her strategy space, else he/she may choose not to transmit at all. Next, we propose the best response of user i in game G_1^λ as follows:

Proposition 5.2.1. *For game G_1^λ defined in (5.1.3), the best response of user i, given the power vector of the other users p_{-i} is given by:*

$$r_i(p_{-i}) = min\,(p_i^o, p_{i-max})\,, \forall i \in \mathcal{N} \tag{5.2.5}$$

where p_{i-max} is the maximum allowed power in the ith user's strategy space P_i.

Proof : From equation (5.2.2), p_i^o is the unconstrained maximizer of the target function L_i, i.e., $p_i^o = \underset{p_i \in \mathbb{R}^+}{\text{argmax}}\, L_i$. And since the second-order derivative of L_i with respect to p_i is negative $\forall p_i \in \mathbb{R}^+$, then this maximizer is unique. Now, assume that p_i^o is not feasible, that is, $p_i^o \notin P_i$, then user i will get his/her maximum at p_{i-max} because the target function is increasing on the set $\{p_i : p_i < p_i^o\}$. This implies that $p_i = p_{i-max}$ is the best response of user i given p_{-i}. □

In the following Section we offer a discussion of S-modular games to help in establishing the existence of Nash equilibrium points for game G_1^λ.

5.2.1 S-Modular Games and G_1^λ

We present a brief description of S-modular games in order to establish the existence and uniqueness of a Nash equilibrium operating point. S-modular games were first introduced to the literature by Topkis in 1970 [27] (see Appendix A). The advantage of S-modular games is that the strong conditions of quasiconvexity and quasiconcavity of target functions (cost or utility functions, respectively) are not needed for the establishment of Nash equilibrium points. See Section A.3 for the definition of the notion (NDD) and S-modular games.

The impact of the NDD property in supermodular games is that the utility function of all users has a best response correspondence that converges to a fixed point, which is a Nash equilibrium operating point. The following theorem guarantees the existence of a Nash equilibrium point in S-modular games [27]:

Theorem 5.2.1. *The Nash equilibria set of S-modular game is not empty. Also, the Nash equilibria set has a smallest and largest elements.*

Proof : *The proof can be found in [27].* □

This theorem states that the Nash equilibria set $\mathcal{S}$ has the form, $\mathcal{S} =[\mathrm{p}_s,\mathrm{p}_l]$, where p_s and p_l are the smallest and largest (component wise) power vectors in the equilibria set, i.e. the equilibria set $\mathcal{S}$ is bounded from below and above. However, it does not guarantee that all points in $\mathcal{S}$ are equilibrium points [8], although the equilibrium points set p is a subset of $\mathcal{S}$, i.e. $\mathrm{p} \subset \mathcal{S}$.

In our case, G_1^λ has a parameter λ, the pricing factor, that is outside the control of any user in the CDMA cellular system. Such a parameter is called an exogenous parameter in the S-modular games context, and the corresponding S-modular game is sometimes called a *parameterized game with complementarities*. See Appendix A for the definition of such games. The next theorem describes the behavior of $\mathrm{p}_s(\omega)$

and $\mathrm{p}_l(\omega)$, the smallest and largest elements of the equilibria set $\mathcal{S}$, respectively, with respect to the exogenous parameter ω.

Theorem 5.2.2. *Both* $\mathrm{p}_s(\omega)$ *and* $\mathrm{p}_l(\omega)$ *are nondecreasing with* ω *in a parameterized game with complementarities.*

Proof : The proof can be found in [28]. □

Theorem 5.2.2 tells us that both $\mathrm{p}_s(\lambda)$ and $\mathrm{p}_l(\lambda)$ are non-increasing with λ in the game G_1^λ. This is intuitive since increasing the price decreases the willingness of the users to transmit at higher power levels. The following lemma states an important result:

Lemma 5.2.1. *The game* G_1^λ *defined in (5.1.3) is a supermodular game or a parameterized game with complementarities if each user in the system has the following strategy space* $P_j \subset [p_j^{'}, \infty)$, $\forall j = 1, 2, \cdots, N$, *where* $p_j^{'}$ *is the transmit power level such that* $\gamma_j^{'} = \frac{G_j\, h_j\, p_j^{'}}{I_{-j}} = \sqrt{2}$. *Accordingly, game* G_1^λ *has a Nash equilibria set.*

Proof : First, we prove that the target function $L_j(p_j, \mathrm{p}_{-j}, \lambda)$ has NDD property in (p_j, p_{-j}) , $\forall\ p_j \in P_j$. By evaluating the second-order derivative of $L_j(p_j, \mathrm{p}_{-j}, \lambda)$ with respect to p_j and p_i as follows:

$$\frac{\partial^2 L_j(p_j, \mathrm{p}_{-j}, \lambda)}{\partial p_j\, \partial p_i} = \chi_{j,i}\ (\gamma_j^2 - 2), \forall\, i \neq j \tag{5.2.6}$$

where

$$\chi_{j,i} = \frac{G_j\, u_j\, h_j\, h_i}{I_{-j}^2\, [1 + (1 + \gamma_j)^2]^2}$$

we find that $\chi_{j,i} > 0, \forall p_j \in \mathbb{R}^+$, and the sign of the second-order derivative is determined by $(\gamma_j^2 - 2)$, and this is positive for all $\gamma_j > \gamma_j^{'} = \sqrt{2}$. Second, we prove that L_j has NDD property in (p_j, ω), where we make the change of variable

$\omega = -\lambda$. Then we evaluate the second-order derivative of L_j with respect to p_j and ω as follows:

$$\frac{\partial^2 L_j(p_j, \mathrm{p}_{-j}, \lambda)}{\partial p_j \, \partial \omega} = 1, \quad \forall p_j \in P_j, \forall\, j \in \mathcal{N} \tag{5.2.7}$$

As one can see that $\frac{\partial^2 L_j}{\partial p_j \, \partial \omega} > 0$, $\forall p_j \in P_j$ and according to definition A.3.3, the game G_1^λ is a supermodular game or a parameterized game with complementarities. Thus, by theorem 5.2.1 it has a Nash equilibria set. □

The following theorem guarantees the uniqueness of a Nash equilibrium operating point of game G_1^λ.

Theorem 5.2.3. *If a power control algorithm with a standard best response function has a Nash equilibrium point, then this Nash equilibrium point is unique*

Proof : The proof can be found in [2]. □

Theorem 5.2.3 allows us to state the following lemma:

Lemma 5.2.2. *The game G_1^λ has a unique Nash equilibrium operating point.*

Proof : Lemma 5.2.1 and theorem 5.2.1 show that the game G_1^λ has a Nash equilibrium point, and what we need to prove is that this Nash equilibrium point is unique. Let us denote the vector of best responses of all users by $\vec{r}(\mathrm{p}) = (r_1(\mathrm{p}_{-1}), r_2(\mathrm{p}_{-2}), \cdots, r_N(\mathrm{p}_{-N}))$. By theorem 5.2.3, uniqueness can be guaranteed by proving that the vector of best responses of all users is a standard vector function, or simply a standard function. To prove that $\vec{r}(\mathrm{p})$ is a standard vector function we only need to check the three conditions in definition A.2.1.

The proof of positivity is trivial, since $P_i \subset \mathbb{R}^+$ and $r_i(\mathrm{p}_{-i}) \in P_i$, $\forall\, i \in \mathcal{N}$. The monotonicity results from the definition of the supermodular game that has the NDD property in (p_i, p_{-i}), $\forall\, i \in \mathcal{N}$. To prove the scalability, it is enough to prove that $p_i^o(\mathrm{p}_{-i})$ is a scalable function and then the scalability of $\vec{r}(\mathrm{p})$ comes through. Let us rewrite equation (5.2.2) as follows:

$$p_i^o(\mathrm{p}_{-i}) = \frac{1}{G_i\, h_i}\, \left[\nu_i(\mathrm{p}_{-i}) - I_{-i}(\mathrm{p}_{-i})\right] \tag{5.2.8}$$

then,

$$p_i^o(\delta\, \mathrm{p}_{-i}) = \frac{\delta}{G_i\, h_i}\, \left[\sqrt{\frac{G_i\, u_i\, h_i}{\delta\, \lambda}\, I_{-i}(\mathrm{p}_{-i}) - I_{-i}^2(\mathrm{p}_{-i})} - I_{-i}(\mathrm{p}_{-i})\right],\ \delta > 1 \tag{5.2.9}$$

and obviously,

$$p_i^o(\delta\, \mathrm{p}_{-i}) < \delta\, p_i^o(\mathrm{p}_{-i}) = \frac{\delta}{G_i\, h_i}\left[\sqrt{\frac{G_i\, u_i\, h_i}{\lambda}\, I_{-i}(\mathrm{p}_{-i}) - I_{-i}^2(\mathrm{p}_{-i})} - I_{-i}(\mathrm{p}_{-i})\right] \tag{5.2.10}$$

and this implies the scalability of $\vec{r}(\mathrm{p})$, therefore $\vec{r}(\mathrm{p})$ is a standard function. Since $\vec{r}(\mathrm{p})$ is a standard function, the equilibrium point p is unique by theorem 5.2.3. □

If we scale the resulted NE power vector by a constant $0 < \rho < 1$ we get higher utilities for all users, in other words NE operating point is not Pareto optimal.

Now, we consider an asynchronous power control algorithm which converges to the unique Nash equilibrium point p of game G_1^λ. In this algorithm the users update their powers in the same manner as in [8]. This algorithm generates a sequence of power vectors that converges to $\mathrm{p} = \mathrm{p}_s(\lambda) = \mathrm{p}_l(\lambda)$, the lowest and the largest power vector in $\mathcal{S}$. Assume user j updates its power level at time instances in the set $T_j = \{t_{j1}, t_{j2}, \cdots\}$, with $t_{jk} < t_{jk+1}$ and $t_{j0} = 0$ for all $j \in \mathcal{N}$. Let $T = \{t_1, t_2, \cdots\}$ where $T = T_1 \bigcup T_2 \bigcup \cdots \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{p}$ to be the power vector picked randomly from the total strategy space $P = P_1 \bigcup P_2 \bigcup \cdots \bigcup P_N$.

Algorithm 5.2.1. *Consider the game G_1^λ as given in (5.1.3) and generate a sequence of power vectors as follows:*

1. *Set the power vector at time $t = 0$: $p(0) = \underline{p}$, let $k = 1$*

2. *For all* $j \in \mathcal{N}$*, such that* $t_k \in T_j$*:*

   (a) *Given* $p(t_{k-1})$*, choose* $u_j > \frac{\lambda I_{-j}}{G_j h_j} [1 + (1 + \frac{\gamma_j^*}{G_j})^2]$*, then calculate* $p_j^o(t_k) = \underset{p_j \in P_j}{argmax}\; L_j\,(p_j, p_{-j}(t_{k-1}), \lambda)$

   (b) *Let the transmit power* $p_j(t_k) = r_j(t_k) = min(p_j^o(t_k), p_{j-max})$

3. *If* $p(t_k) = p(t_{k-1})$ *stop and declare the Nash equilibrium power vector as* $p(t_k)$*, else let* $k := k + 1$ *and go to 2.*

5.3 Proposed Target Function Compared to Previous Target Functions

In this Section we compare the performance of our proposed target (utility) function with the performance of the cost function defined in [26].

Lemma 5.3.1. *The cost function of [26]:*

$$J_j(p_j, p_{-j}) = \lambda\, p_j - u_j\, \log(1 + \gamma_j) \tag{5.3.1}$$

when all users choose the same utility factor $u_j = u_i = u,\ \forall\, j, i \in \mathcal{N}$ *in a market-based pricing, will result in a user closer to the BS transmitting at a higher power level than a distant user.*

Proof : In [26] all users are transmitting data at the same rate, that is $R_i = R$ and $G_i = G$, $\forall i \in \mathcal{N}$. We shall use the following indexing: $i < k \Rightarrow h_i > h_k$, i.e., the ith user is closer than the kth user to the BS. The minimizing transmit power of the ith user p_i^* is given by [26]:

$$p_i^* = \frac{G}{(G-1)\, h_i} \left(a_i - \frac{1}{G} \sum_{k=1}^{N} h_k\, p_k \right) \tag{5.3.2}$$

where $a_i = \frac{u_i\, h_i}{\lambda} - \frac{\sigma^2}{G}$. For simplicity define $\beta_1 := \frac{G}{G-1}$ and $\beta_2 := \frac{1}{G-1}\sum_{k=1}^{N} h_k\, p_k$, therefore we have:

$$p_i^* = \beta_1\,(\frac{u_i}{\lambda} - \frac{\sigma^2}{G\,h_i}) - \frac{\beta_2}{h_i} \tag{5.3.3}$$

and

$$p_k^* = \beta_1\,(\frac{u_k}{\lambda} - \frac{\sigma^2}{G\,h_k}) - \frac{\beta_2}{h_k} \tag{5.3.4}$$

By subtracting (5.3.3) from (5.3.4) we get:

$$p_k^* - p_i^* = \frac{\beta_1}{\lambda}\,(u_k - u_i) + (\frac{\beta_1\,\sigma^2}{G} + \beta_2)\,(\frac{1}{h_i} - \frac{1}{h_k}) \tag{5.3.5}$$

Now, if we set $u_k = u_i$ we obtain:

$$p_k^* - p_i^* = (\frac{\beta_1\,\sigma^2}{G} + \beta_2)\,(\frac{1}{h_i} - \frac{1}{h_k}) \tag{5.3.6}$$

Observing (5.3.6), one can see that $(\frac{\beta_1\,\sigma^2}{G} + \beta_2) > 0$, while $(\frac{1}{h_i} - \frac{1}{h_k}) < 0$ and this implies $p_k^* < p_i^*$ □

Lemma 5.3.2. *The proposed cost function $L_j(p_j, p_{-j}, \lambda)$ in (5.1.1) with equal transmission rates for all users ($R_i = R$) does not suffer the limitation described in lemma 5.3.1, i.e. with $u_i = u_k$, $i, k \in \mathcal{N}$ users closer to BS send at a lower power than the distant users.*

Proof : By using the same indexing as above, i.e., $i < k \Rightarrow h_i > h_k$, and

$$(G-1)\,(p_k^o - p_i^o) = \frac{\nu_k - I}{h_k} + \frac{I - \nu_i}{h_i} \tag{5.3.7}$$

we find that it is enough to prove that

$$\frac{\nu_k - I}{h_k} > \left|\frac{I - \nu_i}{h_i}\right| = \frac{\nu_i - I}{h_i}$$

where $|x|$ is the absolute value of x, or equivalently prove that:

$$\frac{\nu_k}{h_k} - \frac{\nu_i}{h_i} > I\,(\frac{1}{h_k} - \frac{1}{h_i}) \tag{5.3.8}$$

to conclude the proof. Using the fact that $I_{-k} > I_{-i}$ and from equation (5.2.2) we find that $\nu_k > \nu_i$, henceforth

$$\frac{\nu_k}{h_k} - \frac{\nu_i}{h_i} > \nu_i \, (\frac{1}{h_k} - \frac{1}{h_i}) > I \, (\frac{1}{h_k} - \frac{1}{h_i})$$

and this holds for all u_i and u_k that satisfy the condition in note 5.2.1. This concludes the proof. □

This allows us to introduce the following remark:

Remark 5.3.1. *A generic game with a target function composed of a utility function $f(\gamma_i)$ (concave on $\mathbb{R}^+$) and a linear pricing function as follows:*

$$L_i^g(p_i, \mathrm{p}_{-i}, \lambda) = u_i \, f(\gamma_i) - \lambda \, p_i, \tag{5.3.9}$$

does not suffer the limitation described in lemma 5.3.1, if with $h_k < h_i$, it satisfies the following condition:

$$\frac{u_k}{u_i} > \frac{\gamma_i \, f^{'}(\gamma_i)}{\gamma_k \, f^{'}(\gamma_k)}, \tag{5.3.10}$$

where $f^{'}(\gamma_i)$ is the derivative of $f(\gamma_i)$ with respect to γ_i.

Proof : First, we need to find the optimizing power p_i^g of user i by evaluating the first-order derivative of $L_i^g(p_i, \mathrm{p}_{-i}, \lambda)$ with respect to p_i and setting it to zero:

$$\frac{\partial \, L_i^g(p_i, \mathrm{p}_{-i}, \lambda)}{\partial \, p_i} = \frac{\gamma_i}{p_i} \, f^{'}(\gamma_i) - \lambda = 0 \tag{5.3.11}$$

Then the optimizing power p_i^g of user i is given by:

$$p_i^g = \frac{u_i}{\lambda} \, \gamma_i \, f^{'}(\gamma_i) \tag{5.3.12}$$

Then the ratio of the optimizing power p_k^g of the kth user to the optimizing power p_i^g of the ith user is given by:

$$\frac{p_k^g}{p_i^g} = \frac{u_k}{u_i} \frac{\gamma_k \, f^{'}(\gamma_k)}{\gamma_i \, f^{'}(\gamma_i)} > 1, \tag{5.3.13}$$

and this is equivalent to the following:

$$\frac{u_k}{u_i} > \frac{\gamma_i\, f'(\gamma_i)}{\gamma_k\, f'(\gamma_k)} \tag{5.3.14}$$

□

5.4 Simulation results

In this Section we compare the performance of game G_1^λ in terms of the resulting Nash power vector, the outcome of algorithm 5.2.1, and the attained SIRs of all users. Then we compare these results with those obtained by the following distributed game, G_2^λ:

$$G_2^\lambda : \quad \min_{p_i \in P_i} \; (\lambda\, p_i - u_i \log(1 + \gamma_i)) \tag{5.4.1}$$

The target function we are minimizing in the game G_2^λ is the cost function in [26]. As we mentioned earlier, we are studying an uplink wireless CDMA link in a single cell. The number of active users in the cell is $N = 9$ with the following distances from the BS, $d = [310, 460, 570, 660, 740, 810, 880, 940, 1000]$ in meters. The system's parameters and their values used in this study are given in Table 5.1.

In Fig. 5.1 and Fig. 5.3 we show the equilibrium powers as a result of games G_1^λ and G_2^λ versus the distance of the users from the BS. The vector of utility factors assigned to the users in these figures is $\vec{u} = [1,\, 2, 5,\, 8, 10,\, 15, 20,\, 30, 42]$ and all users are assumed to transmit at the same transmission rate $R_i = 10^4, \forall i \in \mathcal{N}$. These two figures show that the Nash equilibrium point of game G_1^λ is Pareto dominant with respect to Nash equilibrium point of game G_2^λ. In Fig. 5.2 and Fig. 5.4 we present the resulting SIRs at equilibrium of both games versus the distance of the users from the BS. These two figures show that the Pareto dominance of game G_1^λ was not a compromise on the attained SIRs. Clearly, Fig. 5.2 shows that the farthest 6 users were able to obtain higher SIRs through game G_1^λ than they got through game G_2^λ. While in Fig. 5.4 the same conclusion applies for the farthest 4 users from the BS. In general we can say that game G_1^λ is more fair than game G_2^λ with users far away from the BS.

In the previous figures, results were obtained by assigning the same fixed values of the utility factors u_i, $\forall\, i \in \mathcal{N}$ in both games G_1^λ and G_2^λ. In Fig. 5.5 and Fig. 5.6, we show the results obtained by executing both games with each user unilaterally choosing his utility factor as follows: In game G_1^λ, user i chooses his utility factor u_i such that:

$$u_i = \frac{\rho_i\, \lambda\, I_{-i}}{G_i\, h_i}\; [1 + (1 + \frac{\gamma_i^*}{G_i})^2], \tag{5.4.2}$$

where γ_i^* is the ith minimum desired SIR in $\gamma^* = [5,\, 15, 10,\, 20, 15,\, 15, 15,\, 25, 18]$ and ρ_i is the ith component of $\rho = 20\,\gamma^*$. In game G_2^λ, user i chooses his utility factor u_i as follows [26]:

$$u_i = \frac{\rho_i\, \lambda\, I_{-i}}{G_i\, h_i}\; [1 + \gamma_i^*] \tag{5.4.3}$$

Fig. 5.5 shows that the Pareto dominance of Nash equilibrium point of game G_1^λ with respect to Nash equilibrium point of game G_2^λ is more clearly emphasized than in Fig. 5.1 and Fig. 5.3. The reason for this, is that for the same requirements of the minimum SIRs γ_i^* in both games, users in game G_2^λ choose higher utility factors than users in game G_1^λ as one can see from equations (5.4.2) and (5.4.3). Here we give the values of the utility factors of all users at equilibrium for the two games: Under game G_1^λ, $\vec{u} =$ [0.0012, 0.0173, 0.0272, 0.0979, 0.1159, 0.1664, 0.2318, 0.5043, 0.4641]. While, under game G_1^λ, $\vec{u} =$ [2.6, 112.6, 151.5, 1107.1, 1027.6, 1496.1, 2103.1, 7454.9, 5043.1]. One needs to know that high utility factors imply that users will use high transmit powers to achieve their goals. In Fig. 5.6 we present the attained SIRs by both games at equilibrium and the minimum required SIRs by all users versus the distance between the users and the BS. Under game G_1^λ all users (the closer and

Table 5.1: the values of parameters used in the simulations.

W, spread spectrum bandwidth	10^6, 10^7 Hz
σ^2, AWGN power at the BS	5×10^{-15}
N, number of users in the cell	9
W/R, spreading gain	100, 1000
p_{i-max}, maximum power in P_i, $\forall i \in \mathcal{N}$	1 watt

the distant ones) were able to make a good connection with the BS, while under G_2^{λ} the farthest two users were not able to attain their minimum SIRs.

In Fig. 5.1-Fig. 5.6 we run the simulations under the assumption that all users are sending data at the same transmission rate $R_i = 10^4, \forall\, i \in \mathcal{N}$. In Fig. 5.7 and Fig. 5.8, the simulations were conducted in a more realistic environment, where users are sending data at different transmission rates $\vec{R} = [\, 10^4, 10^3, 10^3, 10^2, 10^1, 10^5, 10^5, 10^5, 10^3]$, but they are assumed to have the same spreading bandwidth $W = 10^7$. In this case each user's choice of the transmit power depends on his/her desired SIR, his distance from the BS, and the transmission rate. A low transmission rate R_i of user i implies high spreading gain G_i and this results in a low level transmit power p_i required to attain some minimum target SIR γ_i^*. In Fig. 5.7, we show the equilibrium powers of both games G_1^{λ} and G_2^{λ} versus the distance of the users from the BS in meters. The high dependence of equilibrium powers at the transmission rates is very clear in both games with large Pareto dominance obtained by game G_1^{λ} with respect to G_2^{λ}. All users with their different transmission rates were able to top their minimum required SIRs through game G_1^{λ}, while 3 users failed to achieve their minimum required SIRs through game G_2^{λ} as one can see in Fig. 5.8.

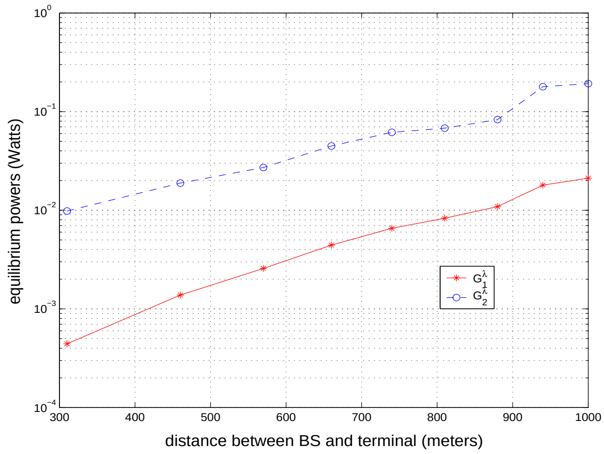

Figure 5.1: Equilibrium powers of the game G_1^λ $(*)$ and equilibrium powers of the game G_2^λ (o) versus the distance between the users and the BS with spreading gain $G = 10^2$ and pricing factor $\lambda = 10^2$.

5.5 Summary

In this Chapter a game-theoretic distributed power control algorithm for wireless data in CDMA communication systems was studied. We proposed a target function

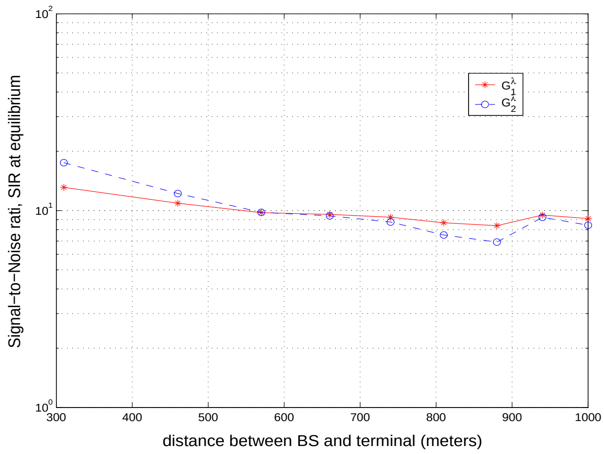

Figure 5.2: Equilibrium SIRs of the game G_1^λ $(*)$ and equilibrium SIRs of the game G_2^λ (o) versus the distance between the users and the BS with spreading gain $G = 10^2$, pricing factor $\lambda = 10^2$ and transmission rates $R_i = 10^4$, $\forall\, i \in \mathcal{N}$.

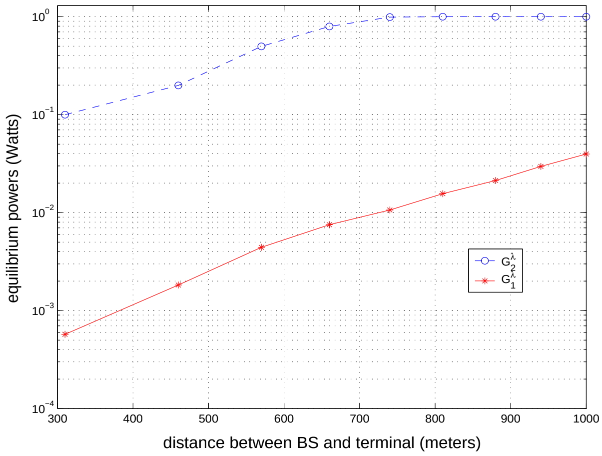

Figure 5.3: Equilibrium powers of the game G_1^λ (∗) and equilibrium powers of the game G_2^λ (o) versus the distance between the users and the BS with spreading gain $G = 10^3$, pricing factor $\lambda = 10^1$ and transmission rates $R_i = 10^4$, $\forall\, i \in \mathcal{N}$.

which is composed of the difference between a utility function and a pricing function. We established the existence and uniqueness of a Nash equilibrium point using the S-modular games framework. Then, we compared the performance of the proposed target function with the performance of the cost function in [26]. We found that the

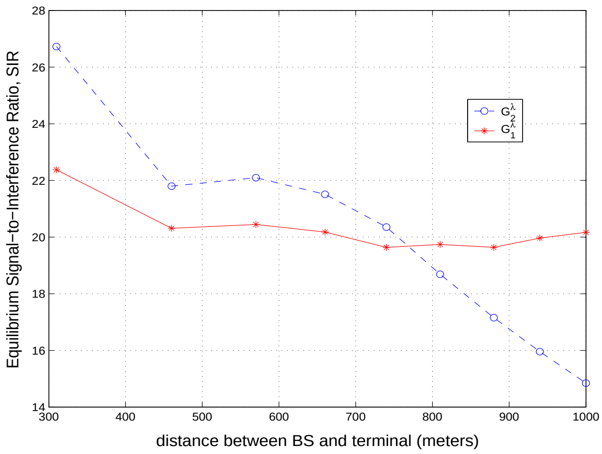

Figure 5.4: Equilibrium SIRs of the game G_1^λ (∗) and equilibrium SIRs of the game G_2^λ (o) versus the distance between the users and the BS with spreading gain $G = 10^3$, pricing factor $\lambda = 10^1$ and transmission rates $R_i = 10^4$, $\forall\, i \in \mathcal{N}$.

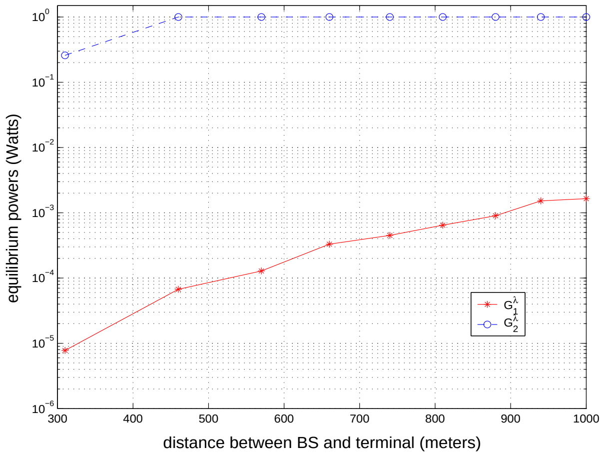

Figure 5.5: Equilibrium powers of the game G_1^λ $(*)$ and equilibrium powers of the game G_2^λ (o) versus the distance between the users and the BS with spreading gain $G = 10^3$, pricing factor $\lambda = 10^1$ and transmission rates $R_i = 10^4$, $\forall\, i \in \mathcal{N}$.

Nash equilibrium point resulting from our proposed target function exhibits Pareto dominance with respect to the Nash equilibrium point of the cost function in [26]. Pareto dominance in the resulting Nash power vector was not achieved at the expense

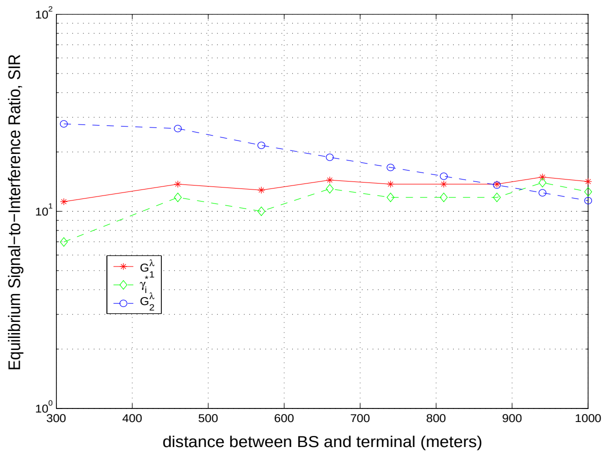

Figure 5.6: Equilibrium SIRs of the game G_1^λ $(*)$, equilibrium SIRs of the game G_2^λ (o) and the minimum desired SIRs of the users $(\diamond)$ versus the distance between the users and the BS with spreading gain $G = 10^3$, pricing factor $\lambda = 10^1$ and transmission rates $R_i = 10^4$, $\forall\, i \in \mathcal{N}$.

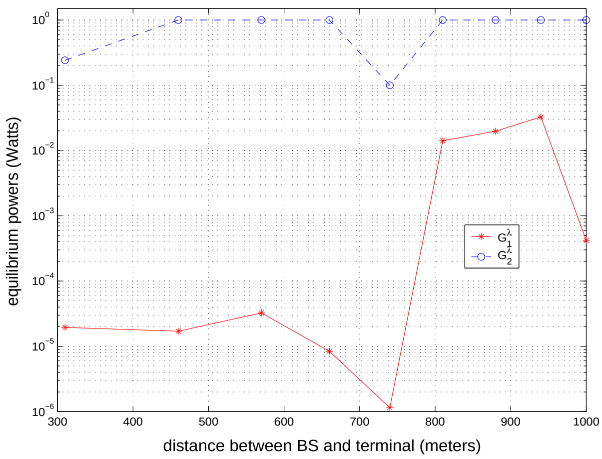

Figure 5.7: Equilibrium powers of the game G_1^λ ($*$) and equilibrium powers of the game G_2^λ (o) versus the distance between the users and the BS with spreading bandwidth $W = 10^7$, pricing factor $\lambda = 10^1$ and transmission rates $\vec{R} = [\ 10^4,\ 10^3, 10^3, 10^2, 10^1,\ 10^5, 10^5,\ 10^5, 10^3]$.

of the attained SIRs.

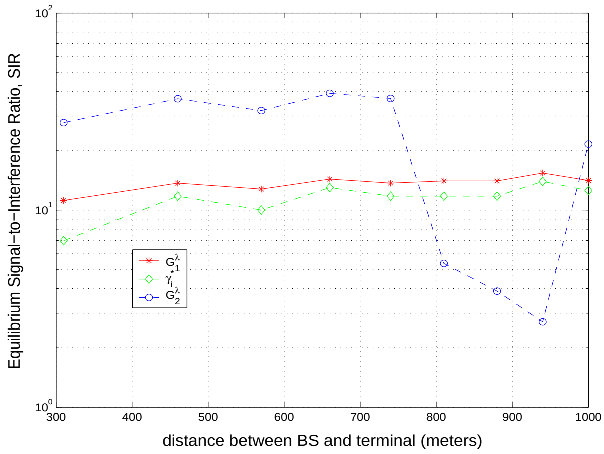

Figure 5.8: Equilibrium SIRs of the game G_1^λ ($*$), equilibrium SIRs of the game G_2^λ (o) and the minimum desired SIRs of the users ($\diamond$) versus the distance between the users and the BS with spreading bandwidth $W = 10^7$, pricing factor $\lambda = 10^1$ and transmission rates $\vec{R} = [10^4,\ 10^3,\ 10^3, 10^2,\ 10^1, 10^5,\ 10^5, 10^5,\ 10^3]$.

Chapter 6

New Distributed Joint Rate and Power Control Games

The new generations of CDMA communications systems (G3, BG3 and G4) and *adhoc* networks are expected to support multirate services (multimedia applications, email, Internet, etc.) in addition to telephone service (fixed rate service) which was the only service offered by G1 and G2. Each user in these new generations of communications systems has different QoSs (e.g. SIR, FER and data rate) that he/she is willing to fulfill by accessing the common radio interface. This establishes a strong need for new algorithms that enable the efficient spectral use of the common radio interface.

The different QoSs needs of each user restrict attention to more realistic algorithms that optimize multiple objectives: power consumption, SIR, FER and data rate. Multi-objective algorithms are important because they give a comprehensive picture of the performance of the system. In other words, sometimes if we just consider algorithms for only optimizing the power consumption as researchers have been doing so far, results may be misleading as the attained SIR may not be enough to support the data rate required by the user who may then prefer not to transmit.

Due to the strong relation between the SIR and the data rate at which a user can send information as was shown by Shannon [25], it is natural to propose joint power and rate control algorithms for wireless data. This idea was conducted and studied in non game-theoretic framework as in [30],[31].

Prior work that have emerged to address this problem have used a centralized algorithm (c.f [33]). Because of the difficulty in implementing the centralized algorithms, and to avoid the extensive number of control signals that cause delays in the system operation, a need for distributed algorithms arose. Game theory was shown to be an appropriate tool for finding power control algorithms such as [5]-[10] and rate flow control algorithms such as [35, 36] and [32, 37]. In this Chapter we show that game theory is also an appropriate tool for finding a distributed algorithms that solve a joint rate and power control optimization problem. In a distributed joint rate and power control algorithm, each user efficiently chooses his transmission rate and power level in an attempt to optimize a target function. This target function maps the level of the quality-of-service (QoS) of the user onto the real line. We are aware of only one paper that addresses the problem of jointly optimizing the rate and power in a game-theoretic distributed fashion [38]. Unfortunately, the authors in [38] forced the transmission rates of all users to be equal to guarantee the uniqueness of Nash equilibrium point. Moreover, the resulting unique Nash equilibrium point is not Pareto optimal (efficient).

To solve the problem of jointly optimizing the rate and the power in a distributed fashion, we propose two layered non-cooperative games that take place at the user level as follows: The first game $G1$ is responsible for allocating the optimal transmission rates for all users, then providing the second game with a vector of constants $\mathcal{M} = (\mathcal{M}_1, \mathcal{M}_2, \cdots, \mathcal{M}_N)$ for a reason that will become clear shortly. The second game $G2$ is responsible for evaluating the optimal transmit power levels that support the resulting Nash equilibrium transmission rates of game $G1$ using $\mathcal{M}$. The target function that sets the rules for the users in $G1$ is the difference between two functions, a utility function and a pricing function. The pricing function is a linear function of the user's transmission rate with a slope (pricing factor) evaluated in a market-based mechanism. In other words, it is chosen by the BS and then communicated to the users in the cell. While in game $G2$, the pricing function is a linear function with a slope $\mathcal{M}_i, \forall i = 1, 2, \cdots, N$ as we describe in the sequel.

The remaining of this Chapter is organized as follows: In Section 6.1 we introduce the system model and our approach to solve the joint rate and power control optimization problem. Existence of Nash equilibria is established in both games in Section 6.2. we present our simulation results in Section 6.3. Finally, the summary of this Chapter is presented in Section 6.4.

6.1 System Model and Our Approach

The setup in this Chapter is as follows: Suppose we have N Tx/Rx pairs (users) in a mobile cellular network. The receivers are not necessarily physically separated. The ith transmitter is supposed to send messages at a power level p_i from its convex strategy space P_i to the ith receiver. User i sends data at a rate r_i from its convex strategy space R_i. The received power level at the kth receiver from the ith transmitter is given as $G_{k,i}p_i$, where $G_{k,i} > 0$ is the path gain from the ith transmitter to the kth receiver. This gain may represent spreading gain and/or cross correlation between codes in CDMA systems. It can also represent coding gain, log-normal shadowing and antenna gains, or any gain that captures the effect of a fading channel. The index set of users in the network is denoted by $\mathcal{N} = \{1, 2, \cdots, N\}$.

The game $G1$ chosen to optimally allocate the transmission rates for the users optimally is given by:

$$G1: \max_{r_i \in R_i} \{L_i(r_i, \mathrm{r}_{-i}, \lambda) = u_i \log(1 + K_i\, r_i) + \beta_i \log(r_i - r_{i-min}) - \lambda\, r_i\} \quad (6.1.1)$$

Where $K_i = (\sum_{k \neq i}^{N} r_k)^{-1}$, $\lambda > 0$ is the pricing factor broadcasted by the BS to all users (market-based pricing scheme), and u_i is the utility factor of the ith user selected locally based on the desired transmission rate, which also measures the willingness of user i to pay. Finally, β_i is a constant selected such that $\beta_i << u_i$, and r_{i-min} is the minimum required transmission rate.

The first term of the utility function is chosen to maximize the transmission rate of user i, while the second term works as a barrier to prevent the ith user's transmission rate from going below the minimum required rate, and also to help the fair allocation of the transmission rates among the users. The goal of the pricing function $\lambda\, r_i$ is to prevent a greedy use of the available radio channel capacity.

The game $G2$ responsible for allocating the transmit power levels that support the resulting Nash equilibrium rates $r_i^o, \forall i = 1, 2, \cdots, N$ of game $G1$ has the following form:

$$G2: \max_{p_i \in P_i} \{g_i(p_i, \mathrm{p}_{-i}, \mathcal{M}_i) = \log(\gamma_i) - \mathcal{M}_i\, p_i\} \quad (6.1.2)$$

where $\mathcal{M}_i$ is given by:

$$\mathcal{M}_i = (I_i \, \exp(r_i^o))^{-1}, \quad (6.1.3)$$

and $I_i = \frac{\sum_{k \neq i}^{N} G_{i,k}\, p_k + \sigma^2}{G_{i,i}}$ is the effective interference that user i needs to overcome. Vectors r_{-i} and p_{-i} are the vectors of the transmission rates and transmit powers of all users except for the ith user, and γ_i is the signal-to-interference ratio (SIR) defined by:

$$\gamma_i = \frac{G_{i,i}\, p_i}{\sum_{k \neq i}^{N} G_{i,k}\, p_k + \sigma^2} \quad (6.1.4)$$

In an application where the spectrum and the power are limited resources, it is recommended to use a spectrally and power efficient modulation technique like M-QAM. An empirical link rate model for M-QAM of user i is given by [34]:

$$r_i = \mu \log(1 + \theta_i \gamma_i) \tag{6.1.5}$$

Where $\theta_i = -1.5/ln(5\,BER_i)$ with BER_i is the target bit-error-rate of user i. The constant μ is related to the base of the logarithm and other system constants (e.g. the channel bandwidth). In this Chapter we use the following approximation of (6.1.5) at high SIR [33]

$$r_i = \log(\gamma_i), \tag{6.1.6}$$

where r_i is normalized by the channel bandwidth with units nats/s/Hz. A user can change his transmission rate by adapting different modulation formats (e.g. 2-QAM, 4-QAM, ...). Therefore, the transmission rate of each user belongs to a discrete set, but in this Chapter we assume that the transmission rates are continuous for simplicity. In the next Section we establish the existence, uniqueness, and optimality of a equilibrium point $(\mathrm{r}^o, \mathrm{p}^o)$ of both games.

6.2 Existence of Nash Equilibrium

In this Section we study in detail both non-cooperative rate control game with pricing (NRGP) $G1$ and non-cooperative power control game with pricing (NPGP) $G2$.

6.2.1 Non-Cooperative Rate Control Game with Pricing (NRGP)

The optimization problem of the ith user defined in game $G1$ is to find the transmission rate r_i^o from the strategy space R_i that maximizes the utility function defined in (6.1.1). To find the maximizing r_i^o we evaluate the first-order partial derivative of the target function (6.1.1) with respect to r_i as follows:

$$\frac{\partial L_i}{\partial r_i} = \frac{u_i\, K_i}{1 + K_i\, r_i} + \frac{\beta_i}{r_i - r_{i-min}} - \lambda \tag{6.2.1}$$

The maximizing transmission rate of user i, r_i^o is thus given by:

$$r_i^o = -\frac{1}{2}B_i + \sqrt{\frac{1}{4}B_i^2 + C_i} \tag{6.2.2}$$

where $B_i = \frac{1}{K_i} - (r_{i-min} + \frac{u_i}{\lambda} + \frac{\beta_i}{\lambda})$ and $C_i = \frac{r_{i-min}}{K_i} - \frac{u_i\, r_{i-min}}{\lambda} + \frac{\beta_i}{K_i\, \lambda}$. Note that the second-order derivative $\frac{\partial^2 L_i}{\partial r_i^2} < 0, \forall\, i \in \mathcal{N}$, which means that L_i is a strictly concave function of r_i. Therefore, L_i is a quasiconcave function optimized on a convex set R_i, and game theory results guarantee the existence of a Nash equilibrium point [29].

Note 6.2.1. *The maximizing transmission rate r_i^o is feasible only if u_i satisfies the following condition:*

$$u_i < \frac{1}{K_i}\left(\lambda + \frac{\beta_i}{r_{i-min}}\right)$$

This results from the fact that C_i should be positive in order to guarantee that r_i^o is feasible (positive). The impact of this upper bound on the utility factor u_i is preventing user i from being very greedy and selfish in using the spectrum, since increasing the utility factor beyond the upper bound will result in a negative transmission rate which is not feasible and hence canceling such greedy user from the game. In the remainder of this Section we prove the uniqueness of this Nash equilibrium point. We first need the following result.

Proposition 6.2.1. *For game $G1$ defined in (6.1.1), the best response of user i, given the transmission rates vector of the other users r_{-i} is given by:*

$$\rho_i(r_{-i}) = min\,(r_i^o, r_{i-max})\,,\, \forall\, i\, \in \mathcal{N} \tag{6.2.3}$$

where r_{i-max} is the maximum allowed transmission rate in the ith user's strategy space R_i.

Proof. First, we define the best response function $\rho_i(\mathrm{r}_{-i})$ of the ith user as the best action that user i can take to attain the maximum pay off given the other users' actions r_{-i}. That is, $\rho_i(\mathrm{r}_{-i}) = \{r_i : L_i(r_i, \mathrm{r}_{-i}) \geq L_i(r_i^{'}, \mathrm{r}_{-i}), \forall r_i^{'} \in R_i\}$, where this set contains only one point.

From equation (6.2.2), r_i^o is the unconstrained maximizer of the target function L_i, i.e., $r_i^o = \underset{r_i \in \mathbb{R}^+}{\text{argmax}} L_i$. And since the second-order derivative of L_i with respect to p_i is negative $\forall r_i \in \mathbb{R}^+$, then this maximizer is unique. Now, assume that r_i^o is not feasible, that is, $r_i^o \notin R_i$, then user i will get his/her maximum at r_{i-max} because the target function is increasing on the set $\{r_i : r_i < r_i^o\}$. This implies that $r_i = r_{i-max}$ is the best response of user i given r_{-i}. □

The following theorem guarantees the uniqueness of the Nash equilibrium operating point of game $G1$.

Theorem 6.2.1. *[2] If a power control algorithm with a standard best response function has a Nash equilibrium point, then this Nash equilibrium point is unique*

Proof. The proof can be found in [2]. □

Theorem 6.2.1 allows us to state the following lemma:

Lemma 6.2.1. *In game $G1$, the best response vector of all users given by:*

$$\rho(r) = (\rho_1(r), \rho_2(r), \cdots, \rho_N(r))$$

is a standard vector function. Therefore, by theorem 6.2.1, game $G1$ has a unique Nash equilibrium point $r^o = (r_1^o, r_2^o, \cdots, r_N^o)$.

The proof can be argued the same way as lemma 5.2.2.

6.2.2 Non-cooperative Power Control Game with Pricing (NPGP)

The optimization problem of user i defined in game $G2$ defined by (6.1.2) is to find the maximizing transmit power level p_i^o of the utility function from the convex strategy space P_i . To find the maximizing p_i^o we evaluate the first-order partial derivative of the target function (6.1.2) with respect to p_i as follows:

$$\frac{\partial g_i}{\partial p_i} = \frac{1}{I_i\,\gamma_i} - \mathcal{M}_i, \tag{6.2.4}$$

By substituting for the value of $\mathcal{M}_i$, the maximizing transmit power level is thus given by:

$$p_i^o = I_i \; \exp(r_i^o) \tag{6.2.5}$$

The transmit power level p_i^o represents the minimal power required to support the optimal transmission rate r_i^o, i.e., there is no wasted transmit power. Note that the second-order derivative of g_i with respect to p_i is $\frac{\partial^2 g_i}{\partial p_i^2} = -1/p_i^2 < 0, \forall\, i \in \mathcal{N}$. Therefore, g_i is a strictly concave function, and using the same argument for L_i in $G1$, there exists a Nash equilibrium point $\mathrm{p}^o = (p_1^o,\, p_2^o,\, \cdots\, ,\, p_N^o)$ in game $G2$. This Nash equilibrium point is Pareto optimal in the sense that it represents the lowest aggregate transmit power levels to support the resulting Nash transmission rates of game $G1$ as we show in lemma 6.2.2.

In what follows we prove the uniqueness of the Nash equilibrium point of game $G2$. We propose the following best response of user i in game $G2$:

Proposition 6.2.2. *For game $G2$ defined in (6.1.2), the best response of user i, given the transmit power levels vector of the other users p_{-i} is given by:*

$$\nu_i(r_{-i}) = min\,(p_i^o, p_{i-max})\,,\, \forall\, i\, \in \mathcal{N} \tag{6.2.6}$$

where p_{i-max} is the maximum transmit power level in the ith user's strategy space P_i.

Proof. The proof is similar to that of proposition 6.2.1. □

Then, the uniqueness of Nash equilibrium operating point can be proved the same way as in game $G1$ since the best response vector of users in $G2$ given as $\nu(\mathrm{r}) = (\nu_1(\mathrm{r}), \nu_2(\mathrm{r}), \cdots, \nu_N(\mathrm{r}))$ is also a standard function.

The following Lemma guarantees Pareto optimality (efficiency) of the equilibrium point $(\mathrm{r}^o, \mathrm{p}^o)$ of both games $G1$ and $G2$.

Lemma 6.2.2. *The Nash equilibrium point (r^o, p^o) of both NRGP and NPGP games $G1$ and $G2$, respectively is Pareto optimal. Mathematically speaking, for $G1$, $\nexists\, r^* = (r_1^*, r_2^*, \cdots, r_N^*) : L_j(r^*) \geq L_j(r^o)$, $\forall\, j \in \mathcal{N}$ and $L_m(r^*) > L_m(r^o)$ for some $m \in \mathcal{N}$, with $r^* > r^o$ component wise. For $G2$, $\nexists\, p^* = (p_1^*, p_2^*, \cdots, p_N^*) : g_j(p^*) \geq g_j(p^o)$, $\forall\, j \in \mathcal{N}$ and $g_n(p^*) > g_n(p^o)$ for some $n \in \mathcal{N}$, with $p^* < p^o$ component wise.*

Proof. We already know from (6.2.1) that

$$f_j(\mathrm{r}^o) \triangleq \frac{u_j\, K_j^o}{1 + K_j^o\, r_j^o} + \frac{\beta_j}{r_j^o - r_{j-min}} - \lambda = 0 \tag{6.2.7}$$

where $K_j^o = (\sum_{k\neq j}^N r_k^o)^{-1}$, therefore (6.2.7) can be written as:

$$f_j(\mathrm{r}^o) = \frac{u_j}{\sum_{k=1}^N r_k^o} + \frac{\beta_j}{r_j^o - r_{j-min}} - \lambda = 0 \tag{6.2.8}$$

Without loss of generality, let $r_k^* = \rho_k\, r_k^o$, $\forall k \in \mathcal{N}$ where $\rho_k > 1, \forall k \in \mathcal{N}$. Then we have the following:

$$L_j(\mathrm{r}^*) \;=\; u_j \log(1 + \frac{\rho_j\, r_j^o}{\sum_{k\neq j}^N \rho_k\, r_k^o}) + \beta_j \log(\rho_j\, r_j^o - r_{j-min}) - \lambda\, \rho_j\, r_j^o \tag{6.2.9}$$

In order to find out how $L_j(\mathrm{r}^*)$ behaves with ρ_j, we need to find the first-order derivative of $L_j(\mathrm{r}^*)$ with respect to ρ_j as follows:

$$\begin{aligned}\frac{\partial L_j(\mathrm{r}^*)}{\partial \rho_j} &= r_j^o \left(\frac{u_j}{\sum_{k=1}^N \rho_k\, r_k^o} + \frac{\beta_j}{\rho_j\, r_j^o - r_{j-min}} - \lambda \right) \\ &= r_j^o\, f_j(D^\rho \mathrm{r}^o), \forall\, j \in \mathcal{N}\end{aligned} \tag{6.2.10}$$

where $D^{\rho} = diag(\rho_1, \rho_2, \cdots, \rho_N)$. One can check easily that $f_j(D^{\rho}\mathrm{r}^o) < f_j(\mathrm{r}^o) = 0, \forall j \in \mathcal{N}$. Henceforth, $\frac{\partial L_j(\mathrm{r}^*)}{\partial \rho_j} < 0, \forall j \in \mathcal{N}$, that is, $L_j(\mathrm{r}^*)$ is decreasing over $\rho_j > 1$ for all users, and by this we conclude the proof of r^o being a Pareto optimal NE point of NRGP game $G1$. Then we prove that p^o is a Pareto optimal NE point of $G2$. It is enough to prove that $p_j^o \in P_j$ is the minimum required transmit power to support $r_j^o \in R_j$ for all $j \in \mathcal{N}$. This is fairly simple to conclude by examining (6.2.5), which can be written as:

$$\gamma_j^o = \frac{p_j^o}{I_j} = \exp(r_j^o), \forall j \in \mathcal{N} \tag{6.2.11}$$

And from (6.1.6), we can say that p^o is indeed the minimum power vector that can support r^o. By this we conclude the proof of lemma 6.2.2. □

It was proved in [2] that synchronous and asynchronous algorithms with standard best response functions converge to the same point. Therefore, we consider an asynchronous power and rate control algorithms which converge to the unique Nash equilibrium point $(\mathrm{r}^o, \mathrm{p}^o)$ of games $G1$ and $G2$. In these algorithms, users update their transmission rates and powers in the same manner as in [8]. Assume user j updates its transmission rate at time instances in the set $T_j = \{t_{j1}, t_{j2}, ...\}$, with $t_{jk} < t_{jk+1}$ and $t_{j0} = 0$ for all $j \in \mathcal{N}$. Let $T = \{t_1, t_2, ...\}$ where $T = T_1 \bigcup T_2 \bigcup ... \bigcup T_N$ with $t_k < t_{k+1}$ and define $\underline{r}$ to be the transmission rates vector picked randomly from the total strategy space $R = R_1 \bigcup R_2 \bigcup ... \bigcup R_N$.

Algorithm 6.2.1. *Consider the game $G1$ given in (6.1.1) and generate a sequence of transmission rates vectors as follows:*

1. *Set the transmission rate vector at time $t = 0$: $r(0) = \underline{r}$, let $k = 1$*

2. *For all $j \in \mathcal{N}$, such that $t_k \in T_j$:*

   (a) *Given $r(t_{k-1})$, calculate $r_j^o(t_k) = \underset{r_j \in R_j}{argmax} L_j(r_j, r_{-j}(t_{k-1}), \lambda)$*

(b) *Let the transmission rate* $r_j(t_k) = \rho_j(t_k) = min(r_j^o(t_k), r_{j-max})$

3. *If* $r(t_k) = r(t_{k-1})$ *stop and declare the Nash equilibrium transmission rates vector as* $r(t_k)$, *else let* $k := k+1$ *and go to 2.*

4. *For all* $j \in \mathcal{N}$, *calculate* $\mathcal{M}_j$ *and provide it to algorithm 6.2.2.*

Algorithm 6.2.2 below finds the optimal transmit power levels p^o to support r^o. Suppose user j updates its power level at time instances in the set $T_j^p = \{t_{j1}, t_{j2}, ...\}$, with $t_{jk} < t_{jk+1}$ and $t_{j0} = 0$ for all $j \in \mathcal{N}$. Let $T^p = \{t_1, t_2, ...\}$ where $T^p = T_1^p \bigcup T_2^p \bigcup ... \bigcup T_N^p$ with $t_k < t_{k+1}$ and define $\underline{p}$ to be the power vector picked randomly from the total strategy space $P = P_1 \bigcup P_2 \bigcup ... \bigcup P_N$.

Algorithm 6.2.2. *The game G2 as given in (6.1.2) generates a sequence of power vectors as follows:*

1. *Set the power vector at time* $t = 0$: $p(0) = \underline{p}$, *let* $k = 1$

2. *For all* $j \in \mathcal{N}$, *such that* $t_k \in T_j^p$:

   (a) *Given* $p(t_{k-1})$, *then calculate* $p_j^o(t_k) = \underset{p_j \in P_j}{argmax}\, g_j(p_j, p_{-j}(t_{k-1}), \mathcal{M}_j)$

   (b) *Let the transmit power* $p_j(t_k) = \nu_j(t_k) = min(p_j^o(t_k), p_{j-max})$

3. *If* $p(t_k) = p(t_{k-1})$ *stop and declare the Nash equilibrium power vector as* $p(t_k)$, *else let* $k := k+1$ *and go to 2.*

The solution point for both algorithms is $(\mathrm{r}^o, \mathrm{p}^o)$ in the strategy space $R \times P$.

6.3 Simulation Results

In this Section we study the performance of the two joint game-theoretic distributed power and rate control algorithms for wireless data systems with $N = 50$ Tx/Rx

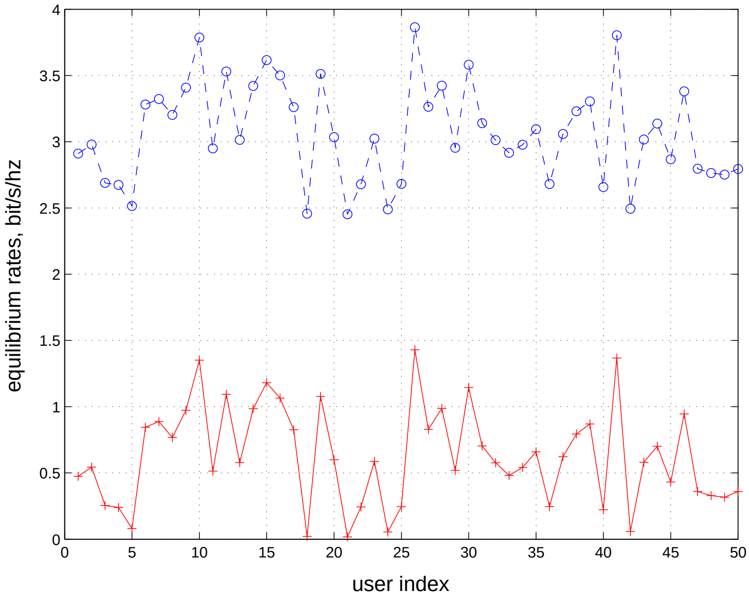

Figure 6.1: Normalized equilibrium rates of the game $G1$ ($\circ$) and the normalized minimum required rates of the users (+) versus the user index with pricing factor $\lambda = 10^3$ and utility factors $u_i = 10^5$.

pairs. The path gains $G_{i,k}$ were generated according to a uniform distribution on the interval $[0, 0.001]$ for all $i \neq k \in \mathcal{N}$ and $G_{i,i} = 1 \; \forall i \in \mathcal{N}$. The additive-white-gaussian noise (AWGN) variance was set $\sigma^2 = 5 \times 10^{-10}$. Game $G1$ was run for different values of the minimum transmission rates for different users. In Fig. 6.1, we present the normalized resulting transmission rates of game $G1$ ($\circ$) and the minimum normalized required transmission rates of all users (+) versus the user index. Results show that all users were able to attain reasonable transmission rates with low transmit power levels resulting from game $G2$ as shown in Fig. 6.2.

6.4 Summary

In this Chapter two joint game-theoretic distributed rate and power control algorithms for wireless data systems were proposed. We presented target functions which

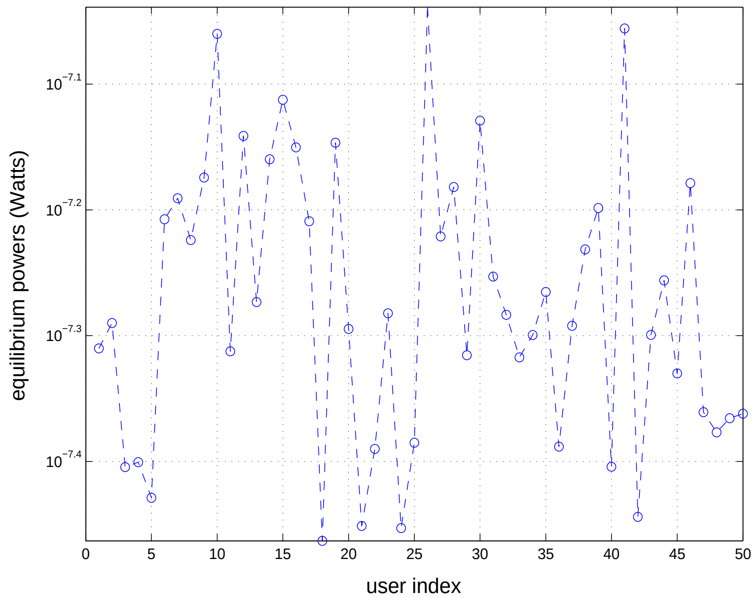

Figure 6.2: Equilibrium powers of the game $G2$ (○) required to support the equilibrium rates versus the user index with pricing factor $\lambda = 10^3$ and utility factors $u_i = 10^5$.

are composed of the difference between a utility function and a pricing function to set the rules of the games among the users. We established the existence, uniqueness and Pareto optimality (efficiency) of the Nash equilibrium point of both games. All 50 users in the studied example were able to attain transmission rates that are higher than their minimum required transmission rates at low transmit power levels.

Chapter 7

Conclusions and Future work

In this Chapter we conclude our results presented in this dissertation and present possible future extension of these results.

7.1 Conclusions

A tremendous demand for wireless services as in the new generations of CDMA communications systems (3G, B3G and 4G) and *adhoc* networks has been increasing. As these new generations are expected to support multirate services (multimedia applications, email, Internet, etc.) in addition to telephone service (fixed rate service) which was the only service offered by 1G and 2G. Unlike 1G and 2G, different users in 3G, B3G and 4G have different QOSs, moreover each user may have different QoSs (e.g. SIR, FER and data rate) that he/she is willing to fulfill by accessing the common radio interface. This establishes a strong need for new algorithms that enable the efficient spectral use of the common radio interface.

The different QoSs needs of each user restrict attention to more realistic algorithms that optimize multiple objectives: power consumption, SIR, FER and data rate. Multi-objective algorithms are important and very informative about the performance of algorithms and protocols.

As one more step toward more realistic multiple objectives optimization algorithms, we proposed joint power and rate control algorithms for wireless data in game theoretic framework. The joint optimization problem was handled by two layered games: First game takes care of efficiently allocating the transmission rates of all users. The second is responsible for allocating the minimum power vector that support the resulting transmission rate vector from the first game. The two games however are connected by a scalar that is evaluated by the first game and delivered to the second game to work as a pricing factor. Then we established the existence, uniqueness and Pareto optimality of the resulting Nash equilibrium point of both games. The only game theoretic joint rate and power control algorithm [38] prior to our work has the rates of all users forced to be equal to guarantee the uniqueness of Nash equilibrium point, and also Nash equilibrium point was Pareto inefficient.

In terms of power control algorithms, we proposed two new games: First game was inspired by [26] to address the transmit power optimization for CDMA uplink (multi-access). The proposed utility was a modification of the cost function proposed by [26] to solve two limitations associated with the cost function: First limitation is that if all users in a market-based pricing has the same utility factors, then users closer to the BS need to transmit at higher power levels than a distant users. Second limitation is that transmit powers at equilibrium are unnecessary large. Second game was inspired by [12] to optimize the fading induced outage probability and maximize the certainty equivalent margin in interference limited CDMA uplink under two channel models, namely Rayleigh and Nakagami channels.

Also, we extended already existed game theoretic algorithms proposed by [8] to more realistic channels. Where the authors only studied NPG and NPGP games for wireless data in nonfading channels.

Finally, We applied statistical learning theory, in particular distribution- free learning theory to study the performance of the NPG and NPGP games in realistic wireless fading channels whenever the channel model is not available a priori.

7.2 Future Work

One of the expansions of this research is to consider BER in addition to rate and power as a multi objectives optimization problem. The motivation for this expansion is that usually there is a trade of between the transmission rate and the BER, so involving explicitly BER in the optimization problem will give more realistic picture of the performance of the algorithm. Involving BER in multi objectives optimization problem may start from equation 6.1.5.

Another expansion can be modifying our joint rate power control algorithms in realistic fading wireless channels. In our algorithms we captured the fading effect, path attenuation, shadowing effect and cross correlation between spreading codes by the constants $G_{i,k}$ (see Chapter 6). Therefor, it will be interesting to see how the statistical variations of the powers may affect our results.

Finally, the results of Chapter 4 can be extended by testing the relationship between maximizing the certainty- equivalent-margin (CEM) and minimizing the outage probability in an interference limited Nakagami ($m > 2$) and Rician fading wireless channels in terms of power allocation. This helps finding out if the results in Chapter 4 are only special for Rayleigh (Nakagami with $m = 1$) and Nakagami ($m = 2$) fading channels or still valid in general. If results of Chapter 4 turns out to be general for Nakagami, Rayleigh and Rician fading channels, then the best policy for all users in an interference limited wireless is to set their transmit power levels at the minimum level to guarantee a good QoS at the BS.

Appendices

Appendix A

Game Theory

In this appendix we describe and define some of the vocabulary of game theory that is needed in this dissertation.

A.1 Quasiconcavity and Quasiconvexity

Economists has been interested in quasinconcave functions because it represents the best model of an ordering over a set of alternatives (strategy space) that a decision-maker has.

Consider a multivariable function $l(x)$ defined on a convex set $\mathcal{C}$. Let U^b be an upper level set for any given real number b defined by

$$U^b = \{x \in \mathcal{C} : l(x) \geq b\}$$

Definition A.1.1. The function $l(x)$ is quasiconcave if U^b is convex set for any b.

On the other hand, let L_b be a lower level set, i.e.,

$$L_b = \{x \in \mathcal{C} : l(x) \leq b\}$$

Definition A.1.2. Function $l(x)$ is quasiconvex if L_b is convex set for any b.

If the function $l(x)$ is concave (convex) then it is quasiconcave (quasiconvex). For proof and more information see [39]

A.2 Standard Vector Function

we introduce the definition of an arbitrary standard vector function $\phi(\mathrm{p})$ as follows:

Definition A.2.1. *[2] A vector function $\phi(p) = (\phi_1(p), \phi_2(p), \cdots, \phi_l(p))$ is called a standard vector function if it satisfies the following:*

1. *Positivity: $\phi(p) > 0$, i.e., each element $\phi_i(p), 1 \leq i \leq l$ is positive.*
2. *Monotonicity: if $p > \hat{p}$ then $\phi(p) \geq \phi(\hat{p})$ (component wise), $\phi_i(p) \geq \phi_i(\hat{p})$.*
3. *Scalability: $\forall \delta > 1,\ \ \delta\phi(p) \geq \phi(\delta\, p)$ (component wise), $\delta\phi_i(p) \geq \phi_i(\delta\, p)$.*

Best response based games in which the best response of all players (users) is a standard vector function are preferable because the algorithms that implement these games are guaranteed to converge to a fixed *unique* point (Nash equilibrium point).

A.3 S-Modular Games

In S-modular games the strict condition of quasiconcavity and quasiconvexity are not needed to guarantee the existence of Nash equilibria. A game is said to be S-modular if it is supermodular or submodular game. Before introducing the formal definition of S-modular games, we need to introduce the notion of nondecreasing difference (NDD) property of an arbitrary function $\varphi_j(p_j, \mathrm{p}_{-j})$ [27], [8]:

Definition A.3.1. A function $\varphi_j(p_j, \mathrm{p}_{-j})$ *has the NDD property in* (p_j, p_{-j}) *if* $\forall\ \mathrm{p}^*_{-j}, \mathrm{p}_{-j}$ *such that* $\mathrm{p}^*_{-j} > \mathrm{p}_{-j}$ *(component wise) then* $\varphi_j(p_j, \mathrm{p}^*_{-j}) - \varphi_j(p_j, \mathrm{p}_{-j})$ *is non-decreasing in* p_j. *Equivalently if* $\varphi_j(p_j, \mathrm{p}_{-j})$ *is twice differentiable, then* $\varphi_j(p_j, \mathrm{p}_{-j})$ *has NDD property if* $\frac{\partial^2 \varphi_j(p_j, \mathrm{p}_{-j})}{\partial p_j\, \partial p_i} > 0,\ \forall i \neq j$. And $\varphi_j(p_j, \mathrm{p}_{-j})$ *has decreasing difference (DD) in* (p_j, p_{-j}) *if* $\varphi_j(p_j, \mathrm{p}^*_{-j}) - \varphi_j(p_j, \mathrm{p}_{-j})$ *is decreasing in* p_j.

Let us then introduce the formal definition of S-modular games [27], [8], [18]:

Definition A.3.2. $\forall\ p_j \in P_j \subset \mathbb{R}^+$, *if* $\varphi_j(p_j, \mathrm{p}_{-j})$ *is a utility function of user* j, *then* $\varphi_j(p_j, \mathrm{p}_{-j})$ *is supermodular if* $\varphi_j(p_j, \mathrm{p}_{-j})$ *has NDD property in* (p_j, p_{-j}). *If* $\varphi_j(p_j, \mathrm{p}_{-j})$ *is a cost function of user* j, *then* $\varphi_j(p_j, \mathrm{p}_{-j})$ *is submodular if* $\varphi_j(p_j, \mathrm{p}_{-j})$ *has decreasing difference (DD) in* (p_j, p_{-j}). *If a game with target function* $\varphi_j(p_j, \mathrm{p}_{-j})$ *is supermodular or submodular, it is called an S-modular game.*

If $\varphi_j(p_j, \mathrm{p}_{-j})$ is a utility function that has the NDD property in (p_j, p_{-j}), then user j needs to increase his/her transmit power level p_j to maximize $\varphi_j(p_j, \mathrm{p}_{-j})$ given that the other users have chosen to increase their transmit power levels. In other words, each user's tendency is to increase his transmit power level in response to the other users' decisions of increasing their transmit power levels.

Sometimes a game may have a parameter that users have no control over (e.g. pricing factor in market-based pricing scheme). Such parameter is called an exogenous parameter in the S-modular games context, and the corresponding S-modular game is called a *parameterized game with complementarities*. The supermodularity definition for a *parameterized game with complementarities* may be found in [28],[8]:

Definition A.3.3. *A game* G_1^{ω} *with an exogenous parameter* ω *is called a supermodular game or a parameterized game with complementarities if the utility function* $\varphi_j(p_j, \mathrm{p}_{-j}, \omega)$ *has a NDD in* (p_j, p_{-j}) *and in* (p_j, ω).

Parameterized games with complementarities offer a very rich framework to study non-cooperative games with pricing in Economics and power and rate control algorithms for wireless and wireline data as we mentioned in the Chapter 1 of this dissertation.

Appendix B

Channel Models

This appendix is devoted to introducing some of the most popular channel models that researchers use to describe small-scale fading channels a user faces in wireless networks such as mobile cellular CDMA systems.

B.1 Rayleigh Channel

In the following situations [40, 41]:

- Macrocell deployment with BS is placed far away from the scatterers and the mobile station (MS) is surrounded by infinitely many scatterers
- In an indoor environment, with rich scattering environment (walls, furniture, etc.)

The channel is modeled as a Rayleigh channel. Mathematically speaking, the fading coefficient α has Rayleigh PDF

$$f^{\alpha}(\omega) = \frac{\omega}{\sigma_r^2}\, e^{-(1/2\sigma_r^2)\omega^2} \tag{B.1.1}$$

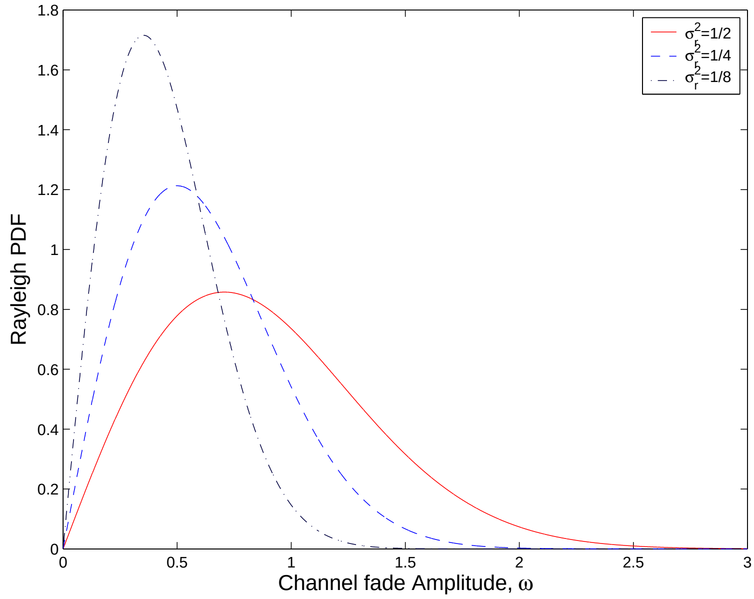

Figure B.1: Rayleigh probability density function with different fading power.

Where $\sigma_r^2 = E[\alpha^2]/2$ is the measure of the spread of the distribution (See Fig. B.1). In other words, In Rayleigh channel the main contribution of the received signal is from scattering.

Rayleigh PDF has only one parameter, that is σ_r. Henceforth, the ability of Rayleigh PDF to describe the channel mentioned above is limited by this parameter (one degree of freedom), another model that better model this channel with two parameters (two degrees of freedom) is the Nakagami model.

B.2 Nakagami Channel

Nakagami PDF has two parameters, namely $\Omega = E[\alpha^2]$ that measures the spread of the density function and m which is called the fading figure represents the reciprocal of the amount of fading around the desired signal, i.e.,

$$m = \frac{\Omega^2}{E[(\alpha^2 - \Omega)^2]}, \quad 1/2 \leq m \leq \infty$$

Therefore m captures the severity of the channel where $m = \infty$ represents a non-fading channel. For $m = 1$, Nakagami density function reduces to Rayleigh density function. A channel with $m < 1$ is more severe channel than a Rayleigh channel, on the other hand a channel $m > 1$ represents less severe channel than Rayleigh. For $m = 1/2$ the Nakagami channel is called Half-Gaussian fading channel, which is the most severe Nakagami fading channel (see Fig. B.2). Nakagami PDF has this formula

$$f^{\alpha}(\omega) = \frac{2m^m}{\Gamma(m)\Omega^m}\, \omega^{2m-1}\, e^{(-\frac{m}{\Omega})\omega^2} \tag{B.2.1}$$

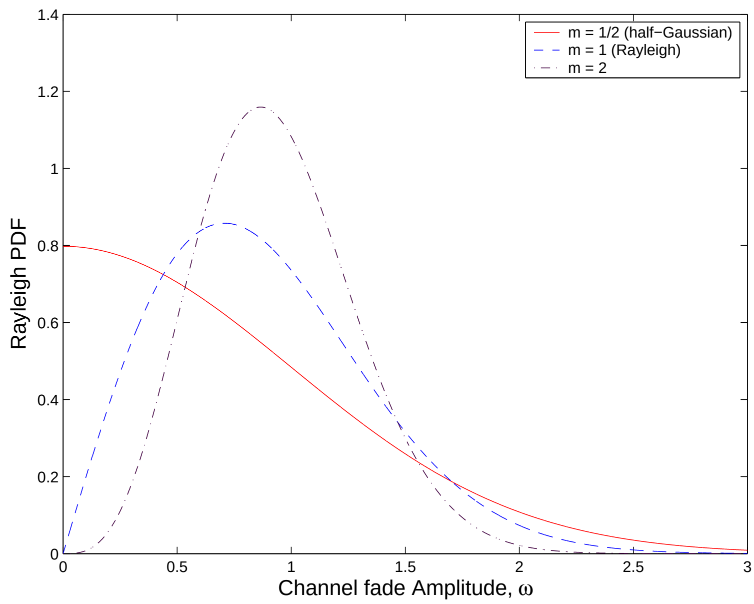

Figure B.2: Nakagami probability density function with different fading figures.

B.3 Rician Channel

Rician channels are those channels where the main contribution of the received signal comes from a fixed scatterers or from a direct path between the MS (mobile station) and BS, line-of-sight (LOS). Such channels can be found in metropolitan areas where microcell deployment exists. In microcell deployment, the height of the transmit antenna at the BS is comparable to the height of street lamps, and the coverage radius is few hundred meters, and hence such deployment exhibit a LOS path between a BS and MS. The Rician PDF is given by:

$$f^{\alpha}(\omega) = \frac{\omega}{\sigma_r^2}\, e^{-(\frac{\omega^2+s^2}{2\sigma_r^2})} I_0(\frac{\omega\, s}{\sigma_r^2}) \qquad \text{(B.3.1)}$$

where s^2 represents the power in the nonfading signal components, and is sometimes called a specular component of the received signal or the noncentrality parameter of the PDF [15, 42]. $I_0(z)$ is the zero-order, modified first-kind Bessel function. As one can notice, Rician PDF has also two parameters, namely s and σ_r. Note that with $s = 0$ Rician density function reduces to Rayleigh PDF, i.e., there is no LOS (See Fig. B.3). In the next Section we discus briefly the time variation of the fading channel.

B.4 Slow and Fast Fading Channels

A fading channel is slow if the signalling interval of the signal T_s is less than the coherence time of the channel Δt_c, where $\Delta t_c \approx 1/B_d$ and B_d is the doppler spread of the channel. On the other hand, the channel is fast if $T_s > \Delta t_c$. Therefore, in fast fading channels a significant spectral broadening of the transmitted signal is observed (B_d is large) [15].

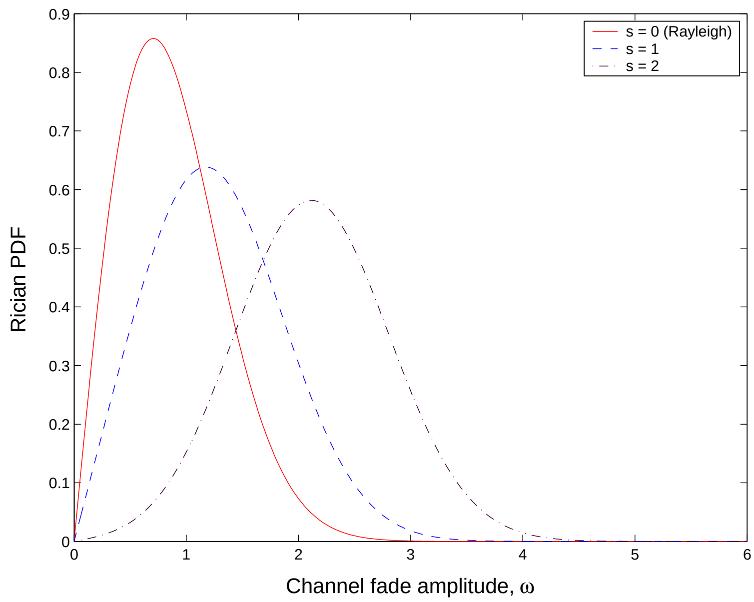

Figure B.3: Rician probability density function with different values of noncentrality parameter.

B.5 Frequency Selective and Frequency Nonselective Channel

If a signal with bandwidth B_w is transmitted through a fading channel with coherence bandwidth Δf_c such that $B_w < \Delta f_c$, then the channel is said to be frequency nonselective (flat fading) channel. That is, all spectral components of the transmitted signal will undergo the same fading effects (no distortion). While if $B_w > \Delta f_c$, then the channel is frequency selective channel, and hence different spectral components of the transmitted signal will undergo different fading effects (distortion). Channel coherence bandwidth is related to multipath delay spread T_m as given by $\Delta f_c \approx 1/T_m$. So, a good channel for reliable transmission will have small multipath delay spread T_m (flat fading channel) and small Doppler spread B_d (slow fading channel).

Appendix C

Machine Learning Theory

The elements of machine learning theory are [23]:

- A measurable space ($\mathcal{X}$,$\mathcal{S}$), where $\mathcal{X}$ is a set (observation set) and $\mathcal{S}$ is a σ-algebra of subsets of $\mathcal{X}$.
- A family of probability measures $\mathcal{P}$ on ($\mathcal{X}$,$\mathcal{S}$).
- A family of measurable functions $\mathcal{F}$ with respect to $\mathcal{S}$ such that $f : \mathcal{X} \to [0, 1]$, $\forall\, f \in \mathcal{F}$, or
- A concept class $\aleph \subseteq \mathcal{S}$ such that $\varrho : \mathcal{X} \to \{0, 1\}$, $\forall \varrho \in \aleph$.

C.1 Concept Learning

In concept learning the problem is to learn a target concept $\varrho^* \in \aleph$ (e.g. binary classification problem) based on a learning multisample $\Psi = (x_1, x_2, \cdots, x_n)$ (observations) drawn from the observation set $\mathcal{X}$ according to a probability measure $P \in \mathcal{P}$ which is usually unknown. The learnability of the target concept ϱ^* depends on a complexity measure of the concept class $\aleph$ called the VC (Vapnik-Chervonenkis)-dimension ($VC - dim(\aleph)$), and it is defined as:

Definition C.1.1. Let ($\mathcal{X}$,$\mathcal{S}$) be a measurable space, and $\aleph \subseteq \mathcal{S}$. Suppose $\Psi = (x_1, x_2, \cdots, x_n) \subset \mathcal{X}$, then the set Ψ is said to be shattered by the concept class $\aleph$ if $\forall \psi \subseteq \Psi$, $\exists$ a set $A \in \aleph$ such that $A \bigcap \Psi = \psi$. Then $VC - dim(\aleph)$ is the largest cardinality (n) of Ψ such that $\exists \Psi$ that is shattered by $\aleph$.

C.2 Function Learning

The problem in function learning is almost the same as in concept learning with the difference however is that the target is to learn a function $f^* : \mathcal{X} \rightarrow [0, 1]$ where $f^* \in \mathcal{F}$. Also, the learnability of the target function f^* depends on P (Pollard)-dimension of the function class $\mathcal{F}$ ($P - dim(\mathcal{F})$). P-dimension of the function class $\mathcal{F}$ ($P - dim(\mathcal{F})$) is defined as :

Definition C.2.1. Assume ($\mathcal{X}$,$\mathcal{S}$) is a measurable space and $\mathcal{F}$ is a set of measurable functions with respect to $\mathcal{S}$. Then $\Psi = (x_1, x_2, \cdots, x_n) \subseteq \mathcal{X}$ is P-shattered by the function class $\mathcal{F}$ if $\exists$ a real vector $\theta = [0, 1]^n$ such that for every $\forall\, e = \{0, 1\}^n$ (binary vector) $\exists$ a corresponding function $f_e \in \mathcal{F}$ such that

$$f_e(x_i) \begin{cases} \geq \theta_i, & \text{if } e_i = 1 \\ < \theta_i, & \text{if } e_i = 0 \end{cases}$$

then $P - dim(\mathcal{F})$ equals the maximum cardinality of the set Ψ that is P-shattered by the function class $\mathcal{F}$.

References

[1] G. J. Foschini and Z. Miljanic, " A simple distributed autonomous power control algorithm and its convergence", *IEEE Transactions on Vehicular technology,* Vol. 42, No. 4, pp. 641-646, Nov., 1993.

[2] R. D. Yates, "A framework for uplink power control in cellular radio systems", *IEEE Journal on Selected Areas in Communication,* Vol. 13, No. 7, pp. 1341-1347, Sep. 1995.

[3] D. Mitra, "An asynchronous distributed algorithm for power control in cellular radio systems", *Proc. 4th WINLAB Workshop on 3rd Generation Wireless Information Networks,* pp. 177-186, NJ, Oct. 1993.

[4] J. Zander. "Distributed cochannel interference control in cellular radio systems,"*IEEE Tran. Veh. Tchnol.*, Vol. 41, No. 2, pp. 305- 311, Aug. 1992

[5] V. Shah, N. B. Mandayam, and D. J. Goodman. "Power control for wireless data based on utility and pricing." In *Proceedings of PIMRC,* pp. 1427-1432, Sep., 1998.

[6] D. Goodman and M. Mandayam, "Power control for wireless data", *IEEE Personal Communications,* Vol. 7, No. 2, pp. 4854, Apr. 2000.

[7] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Pricing and power control in multicell wireless data network," *IEEE JSAC*, Vol. 19, No. 10, pp. 1883- 1892, Oct. 2001.

[8] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks." *IEEE Tras. Comm.*, Vol. 50, No. 2, pp. 291- 303, Feb. 2002.

[9] Allen B. Mackenzie and Stephen B. Wicker, "Game theory and the design of self-configuring, adaptive wireless networks," *IEEE Communications Magazine,* Vol. 39, No. 11, pp. 126 -131, Nov. 2001.

[10] Allen B. Mackenzie and Stephen B. Wicker, "Game theory in communications: motivation, explanation, and application to power control ," *IEEE Global Telecommunications Conference,* Vol. 2, pp. 821-826, Nov., 2001.

[11] J. Zander, "Performance of optimum transmitter power control in cellular radio systems", *IEEE transactions on Vehicular Technology,* Vol. 41, No. 1, pp. 57-62, Feb., 1992.

[12] Sunil Kandukuri and Stephen Boyd, "Optimal power control in interfernce limited fading wireless channels with outage probability specifications." *IEEE Trans. Wireless Comm.*, Vol. 1, No. 1, pp. 46-55, Jan., 2002.

[13] S. A. Grandhi, R. Vijayan and D. Goodman, "Centralized Power Control in Cellular Radio Systems", *IEEE Transactions on Vehicular Technology,* Vol. 42, No. 4, pp. 466-468, Nov., 1993.

[14] Q. Wu, "Performance of Optimum Transmitter Power Control in CDMA Cellular Mobile Systems ", *IEEE Transactions on Vehicular Technology,* Vol. 48, No. 2, pp. 571-575, Mar., 1999.

[15] J. G. Proakis, *Digital Communications*, The McGraw Hill Press, New York, NY , 2000.

[16] Roger L. Peterson, Rodger E. Ziemer and David E. Borth, *Introduction to Spread Spectrum Communications*, Prentice Hall, Upper Saddle River, NJ, 1995.

[17] J. V. Neumann and O. Morgenstern, *Theory of Games and Economic Behavior,* Princeton University Press, Princeton, 1944.

[18] Eitan Altman and Zwi Altman, "S-Modular games and power control in wireless networks", *IEEE Transactions On Automatic Control,* Vol. 48, No. 5, pp. 839-842, May 2003.

[19] Don Ross, *What People Want: The concept of utility from Bentham to game theory,* University of Cape Town Press, South Africa, 1999.

[20] I. S. Gradshteyn and I. M. Ryzhik - *Table of Integrals, Series, And Products.* Academic Press, Orlando, FL, 1980.

[21] Valentin V. Petrov, *Limit Theorems of Probability Theory: Sequences of Independent random variables,* Clarendon Press, Oxford, 1995

[22] P. Ligdas and N. Farvardin, "Finite-State Power Control for Fading Channels," *Proc. Conference on Information Sciences and Systems*, Johns Hopkins University, Baltimore, MD, pp. 550-555, Mar., 1997

[23] M. Vidyasagar, *A Theory of Learning and Generalization with Applications to Neural Networks and Control Systems.* Springer-Verlag, Berlin, Germany, 1996.

[24] Simon Haykin, *Neural Networks A Comprehensive Foundation*, Prentice Hall, Upper Saddle River, NJ, 1994.

[25] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal,* vol. 27, pp. 379-423 and 623-656, July and Oct. 1948.

[26] Tansu Alpcan, Tamer Başar, R. Sirkant and Eitan Altman, "CDMA Uplink Power Control as a Noncooperative Game", *Wireless Neworks,* Vol. 8, No. 6, pp. 659-669, Nov. 2002.

[27] D. M. Topkis, "Equilibrium points in non-zero sum n-person submodular games." *SIAM J. Control and Optimization,* Vol. 17, No. 6, pp. 773-787, 1979.

[28] D. M. Topkis, *Supermodularity and Complementarity.* Princeton University Press, Princeton, New Jersey, 1998

[29] D. Fudenberg and J. Tirole. *Game Theory*, The MIT Press, Cambridge, MA, 1991.

[30] C. W. Sung and W. S. Wong. "Power control and rate management for wireless multimedia CDMA systems," *IEEE Tran. Comm.*, Vol. 49, No. 7, pp. 1215-1226, Jul. 2001

[31] R. Jäntti and S.-L. Kim, "Selective power control with active link protection for combined rate and power management," in *Proc. IEEE VTC 2000-Spring,* Vol. 3, pp. 1960-1964, May, 2000.

[32] F. P. Kelly, A. Maulloo, and D. Tan. "Rate control in communication networks: shadow prices, proportional fairness and stability",*Journal of the Operational Research Society*, Vol.49, No. 3, pp. 237-252, 1998.

[33] D. ONeill, D. Julian, S. Boyd, "Seeking Foschini's Genie: Optimal Rates and Powers in Wireless networks", Submitted to *IEEE Transactions on Vehicular Technology, April 2003.*

[34] A. Goldsmith, *Wireless Communications,* Stanford, CA: Stanford University EE 359 Course Reader, 2001, http://www.stanford.edu/class/ee359/

[35] C. Douligeris and R. Mazumdar,"A game theoretic approach to flow control in an integrated environment with two classes of users," in *Proceedings of the Computer Network Symposium,* pp. 214-221, Apr. 1988.

[36] C. Douligeris and R. Mazumdar,"A game theoretic approach to flow control in an integrated environment," *J. Franklin Inst.,* Vol. 329, No. 3, pp. 383-402, Mar. 1992.

[37] F. Kelly, P. Key, and S. Zachary, "Distributed Admission Control," *IEEE J. Selected Areas in Comm.*, Vol. 18, No. 12, pp. 2617-2628, Dec. 2000.

[38] N. Feng, N. Mandayam and D. Goodman "Joint Power and Rate Optimization for Wireless Data Services Based on Utility Functions", *In Proceedings of CISS,* Vol. 1, pp. 109-113, Mar. 1999.

[39] M. osborne "Math Tutorial", http://www.chass.utoronto.ca/∼osborne/ MathTutorial/QCCF.HTM.

[40] W. C. Jakes, *Microwave Mobile Communications.* John Wiley and Sons, New York, NY, 1974

[41] D. Gesbert, M. Shafi, D. Shiu, P. J. Smith, and A. Naguib, "From Theory to Practice: An Overview of MIMO Space-Time Coded Wireless Systems (Tutorial Paper)", *Journal on Selected Areas in Communications,* Vol. 21, No. 3, pp. 281-302, Apr., 2003.

[42] M.K. Simon and M. S. Alouini, *Digital Communication Over Fading Channels: A Unified Approach to Performance Analysis,* John Wiley and Sons, New York, NY, 2000.

[43] J. Nash, "Non-cooperative games," *Annals of Mathematics,* Vol. 54, No. 2, pp. 286-295, Sep., 1951.