A Game-theoretic Framework for Dynamic Spectrum Leasing (DSL) in Cognitive Radios

Sudharman K. Jayaweera, Kamrul Hakim
Department of Electrical and Computer Engineering
University of New Mexico
Albuquerque, NM 87131-0001, USA
Email: {jayaweera, khakim}@ece.unm.edu

Carlos Mosquera
Departamento de Teoría de la Señal y Comunicaciones
Universidad de Vigo
36310 Vigo, Spain
Email: mosquera@gts.tsc.uvigo.es

***Abstract*—In this paper we develop a game-theoretic framework for *dynamic spectrum leasing* in a cognitive radio network. Dynamic Spectrum Leasing, or DSL for short, is a new paradigm for efficient spectrum sharing in cognitive radio networks that was proposed recently in [1]–[3]. DSL allows the primary users to actively encourage, or discourage, secondary access depending on their instantaneous Quality-of-Service (QoS) requirements and affordable power constraints. Secondary users, on the other hand, attempt to maximize throughput, or any other suitably chosen reward function, opportunistically, while not violating the interference limit set by the primary users. In this paper, we have proposed further generalization to the primary user utility function defined in [2] and a new utility function for the secondary users. We establish the conditions on the primary and secondary user utility functions so that the DSL game has desired equilibrium properties. Simulation results show the system behavior and performance at the equilibrium and also help to identify suitable system design parameters.**

I. Introduction

In recent years, there has been a growing consensus that the scarcity of radio spectrum is mainly due to the inefficiency of traditional fixed spectrum allocation policies [4], [5]. *Dynamic spectrum sharing* (DSS) is considered as an effective way to improve inefficient static spectrum utilization by allowing secondary users to access the so-called white spaces in spectrum already licensed to the primary users. Cognitive radios [6], which can be considered as smart radios with built in cognition, are especially suited for realizing such dynamic spectrum access due to their ability to observe, assess, learn from and orient to the observed RF environment.

The hierarchical spectrum access methods that have been considered in recent literature, however, presume that a secondary (cognitive) transmitter may access the spectrum band owned by a primary user only on the no or limited-interference basis to the primary users [7], [8]. Further, the secondary system is almost exclusively responsible for managing the inter-system interference problem due to coexistence. However, recently introduced *dynamic spectrum leasing* (DSL) [1]–[3] is a new paradigm for spectrum sharing in cognitive radios in which primary users are allowed to actively manage the secondary interference they are willing to tolerate at any given time. As opposed to hierarchical dynamic access techniques considered by many in recent literature, in a DSL-based network the primary users are also active decision makers and they are suitably rewarded for allowing secondary users to share their licensed spectrum whenever they can, while meeting their own minimum Quality-of-Service (QoS) requirements. Simultaneously, the secondary users aim to achieve energy efficient transmissions, while not causing excessive interference to the primary users.

In this paper, we further develop the DSL-based spectrum sharing concept for cognitive radios by identifying new payoff functions for both primary and secondary users that are motivated by network utility considerations. We establish the existence of an equilibrium in this primary-secondary spectrum-leasing game, and analyze several example cognitive radio networks based on the proposed DSL framework to investigate their equilibrium behavior.

The remainder of this paper is organized as follows: Section II describes the dynamic spectrum leasing cognitive radio network made of a primary and a secondary communications system. Section III presents the proposed new game-theoretic model for dynamic spectrum leasing in a spectrum sharing cognitive radio network. Section IV discusses the existence and uniqueness of a Nash equilibrium. Section V evaluates the performance of a DSL cognitive radio network based on the proposed game model. We also investigate the dependence of the achieved equilibrium performance on important design parameters. Finally, Section VI concludes the paper by summarizing our results.

II. Dynamic Spectrum Leasing Based Spectrum Sharing Cognitive Radio Network Model

We assume there is one primary wireless communication system that owns the exclusive rights to use the spectrum band of interest. In a bid to improve the spectrum usage efficiency while earning extra revenue, the primary system is willing to allow a secondary system to access this spectrum band. For simplicity of exposition, we assume that there is only one primary transmitter in the primary system, and there are K secondary transmitters in the secondary system which is assumed to operate as an underlay system. The primary user is denoted as user 0, and the secondary users are labeled as users 1 through K. There are one primary receiver and one secondary receiver of interest. The channel gain between the k-th user and the common secondary receiver is h_{sk}, and

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between the k-th user and the primary receiver is h_{pk}, for $k = 0, \cdots, K$. We use p_k to represent transmission power of the k-th user.

In a DSL network, the primary user is assumed to adapt its interference cap (IC), denoted by Q_0, which is the maximum total interference the primary user is willing to tolerate from all secondary transmissions at a given time [1], [2], and thus its reward can be an increasing function of the interference cap. However, in reality, the primary user should maintain a target signal-to-interference-plus-noise ratio (SINR) to ensure its required QoS. Moreover, an unnecessarily large interference cap by the primary user could hinder the secondary system performance due to resulting high primary interference. The goal of secondary system is to fully utilize the spectrum activity allowed by the primary user. However, their transmission powers must be carefully self-regulated in order to ensure low interference to the primary user (within the IC) as well as to other secondary users.

The signal received at the primary receiver can be written as $r_p(t) = A_0 b_0 s_0(t) + \sum_{k=1}^{K} \Theta_k A_k b_k s_k(t) + \sigma_p n(t)$ where $A_k = h_{pk}\sqrt{p_k}$, for $k = 0, 1, \cdots, K$, $n(t)$ is AWGN with unit spectral height, σ_p^2 is the variance of the zero-mean, additive noise at the primary receiver, and Θ_k is a Bernoulli random variable such that $Prob(\Theta_k = 1) = q_k$ and $Prob(\Theta_k = 0) = 1 - q_k$. Note that, Θ_k represents the randomness in secondary-user collisions with the primary user transmissions. Assuming M discrete-time projections $r_m^{(p)} =< r_p(t), \psi_m^{(p)}(t) >$, for $m = 1, \cdots, M$, of the continuous time signal on to a set of M orthonormal directions specified by $\psi_1^{(p)}(t), \cdots, \psi_M^{(p)}(t)$, and letting $\mathbf{r}^{(p)} = \left(r_1^{(p)}, \cdots, r_M^{(p)}\right)^T$, we may obtain the discrete-time representation of the received signal at the primary receiver as $\mathbf{r}^{(p)} = A_0 b_0 \mathbf{s}_0^{(p)} + \sum_{k=1}^{K} \Theta_k A_k b_k \mathbf{s}_k^{(p)} + \sigma_p \mathbf{n}^{(p)}$, where $\mathbf{s}_k^{(p)} = \left(s_{k1}^{(p)}, \cdots, s_{kM}^{(p)}\right)$, for $k = 0, 1, \cdots, K$, is the M-vector representation of $s_k(t)$ in the M-dimensional basis employed by the primary system, where $s_{km}^{(p)} =< s_k(t), \psi_m^{(p)}(t) >$, and $\mathbf{n}^{(p)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_M)$. With the conventional matched-filter (MF) detector at the primary receiver, and assuming that primary modulation is BPSK so that $b_0 \in \{+1, -1\}$, the primary decisions are given by $\hat{b}_0 = sgn\left(y_0^{(p)}\right)$ where

$$y_0^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{r}^{(p)} = A_0 b_0 + \sum_{k=1}^{K} \Theta_k \rho_{0k}^{(p)} A_k b_k + \sigma_p \eta^{(p)},$$

with $\rho_{0k}^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{s}_k^{(p)}$. Note that $\eta^{(p)} \sim \mathcal{N}(0, 1)$. It is straightforward to observe that the total secondary interference I_0 from all secondary transmissions to the primary-user is

$$I_0 = \sum_{k=1}^{K} q_k \left(\rho_{0k}^{(p)}\right)^2 A_k^2 = \sum_{k=1}^{K} q_k \left(\rho_{0k}^{(p)}\right)^2 h_{pk}^2 p_k. \quad (1)$$

Similarly, the received signal at the secondary-system receiver can be written as $r_s(t) = \sum_{k=1}^{K} B_k b_k s_k(t) + B_0 b_0 s_0(t) + \sigma_s n(t)$ where $B_k = h_{sk}\sqrt{p_k}$, for $k = 0, 1, \cdots, K$ and σ_s^2 is the variance of secondary receiver noise. A discrete-time representation of $r_s(t)$ with respect to an N-dimensional orthonormal basis $\psi_1^{(s)}(t), \cdots, \psi_N^{(s)}(t)$ used by the secondary system can be obtained as $\mathbf{r}^{(s)} = \sum_{k=1}^{K} B_k b_k \mathbf{s}_k^{(s)} + B_0 b_0 \mathbf{s}_0^{(s)} + \sigma_s \mathbf{n}^{(s)}$, where $\mathbf{r}^{(s)} = \left(r_1^{(s)}, \cdots, r_N^{(s)}\right)^T$, $r_n^{(s)} =< r_s(t), \psi_n^{(s)}(t) >$, for $n = 1, \cdots, N$, is the projection of the received signal at the secondary receiver on the the n-th orthonormal basis function, $\mathbf{s}_k^{(s)} = \left(s_{k1}^{(s)}, \cdots, s_{kN}^{(s)}\right)$, for $k = 0, 1, \cdots, K$, is the N-vector representation of $s_k(t)$ with respect to the N-dimensional basis employed by the secondary system with $s_{kn}^{(s)} =< s_k(t), \psi_n^{(s)}(t) >$, and $\mathbf{n}^{(s)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$. In the following we assume that all secondary transmissions are BPSK, and the secondary detector is based on the MF. Hence the secondary receiver detects the k-th secondary user's symbols as $\hat{b}_k = sgn\left(y_k^{(s)}\right)$ where, for $k = 1, \cdots, K$,

$$\begin{aligned} y_k^{(s)} &= \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{r}^{(s)} \\ &= B_k b_k + \sum_{j=1, j\neq k}^{K} \rho_{kj}^{(s)} B_j b_j + \rho_{k0}^{(s)} B_0 b_0 + \sigma_s \eta_k^{(s)} \end{aligned}$$

with $\rho_{kj}^{(s)} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{s}_j^{(s)}$, for $j = 1, \cdots, K$, $\rho_{k0}^{(s)} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{s}_0^{(s)}$, and $\eta_k^{(s)} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{n}^{(s)} \sim \mathcal{N}(0, 1)$.

For notational convenience, let us denote by i_k the total interference from all secondary users to the k-th secondary user signal, excluding that from the primary user, so that $i_k = \sum_{j=1, j\neq k}^{K} \left(\rho_{kj}^{(s)}\right)^2 h_{sj}^2 p_j$. Then, the total interference from all secondary and primary transmissions to the k-th secondary user decisions, for $k = 1, \cdots, K$, is $I_k^{(s)} = i_k + \left(\rho_{k0}^{(s)}\right)^2 h_{s0}^2 p_0$.

III. GAME MODEL FOR DYNAMIC SPECTRUM SHARING

In the proposed DSL-based cognitive networks, the primary and secondary users interact with each other by adjusting their interference cap and transmit power levels, respectively, in order to maximize their own utilities. We formulate the above system as in the following noncooperative DSL game $(\mathcal{K}, \mathcal{A}_k, u_k(.))$:

1) Players: $\mathcal{K} = \{0, 1, 2, \cdots, K\}$, where we assume that the 0-th user is the primary user and $k = 1, 2, \cdots, K$ represents the k-th secondary user.
2) Action space: $\mathcal{P} = \mathcal{A}_0 \times \mathcal{A}_1 \times \mathcal{A}_2 \cdots \times \mathcal{A}_K$, where $\mathcal{A}_0 = \mathcal{Q} = [0, \bar{Q}_0]$ represents the primary user's action set and $\mathcal{A}_k = \mathcal{P}_k = [0, \bar{P}_k]$, for $k = 1, 2, \cdots, K$, represents the k-th secondary user's action set. Note that $\bar{Q}_0$ and $\bar{P}_k$ represent, respectively, the maximum IC of the primary user and the maximum transmission power of the k-th secondary user. We denote the action vector of all users by $\mathbf{a} = [Q_0, p_1, \cdots, p_K]^T$, where $Q_0 \in \mathcal{Q}$ and $p_k \in \mathcal{P}_k$. It is customary to denote the action vector excluding the k-th user, for $k = 0, 1, 2, \cdots, K$, by $\mathbf{a}_{-k}$.
3) Utility function: We denote by $u_0(Q_0, \mathbf{a}_{-0})$ the primary user's utility function, and by $u_k(p_k, \mathbf{a}_{-k})$, for $k = 1, 2, \cdots, K$, the k-th secondary user's utility function.

As proposed in [1], [2], at any given time, the primary user's *target* SINR is defined in terms of its assumed worst-case secondary interference $\bar{\gamma}_0 = \frac{h_{p0}^2 p_0}{Q_0+\sigma_p^2}$, where p_0 and Q_0 represent the primary user's transmission power and the interference cap, respectively. Since Q_0 is the maximum possible interference the primary user is willing to tolerate from secondary users, $\bar{\gamma}_0$ represents the least acceptable transmission quality of the primary user. On the other hand, the primary user's actual *instantaneous* SINR is given by

$$\gamma_0 = \frac{h_{p0}^2 p_0}{I_0 + \sigma_p^2} = \bar{\gamma}_0 \left(1 + \frac{Q_0 - I_0}{I_0 + \sigma_p^2}\right). \tag{2}$$

In DSL-based spectrum sharing, the utilities of primary and secondary users are coupled via mutual interference. Thus, we introduce the following utility function for the primary user:

$$u_0\left(Q_0, \mathbf{a}_{-0}\right) \quad = \quad \left(\bar{Q}_0 - \left(Q_0 - I_0(\mathbf{a}_{-0})\right)\right) F(Q_0) \tag{3}$$

where $F(.)$ is a suitable continuous reward function for the primary system. For example, in [2] the authors proposed a linear reward function $F(Q_0) = Q_0$. In this paper, we allow $F(.)$ to be any general reward function and establish conditions on $F(.)$ so that the proposed DSL game has desired equilibrium properties. Note that (3) also assumes that the utility of the primary user is proportional to demand in addition to the reward function $F(.)$. The demand is taken to be decreasing when the extra interference margin $Q_0 - I_0$ increases. This discourages the primary user from swamping all other transmissions (both primary and secondary) by setting too large an interference cap that will lead to higher transmission power. As a special case of (3), we may choose $F(Q_0) = \log(1 + Q_0)$, so that the primary user utility is proportional to the capacity attained by the secondary system with respect to the primary receiver. We believe this model for primary user utility is more sensible in a dynamic spectrum leasing cognitive radio network compared to [2], when the secondary system is more concerned about the rate achieved rather than the transmit power.

Observe from (2) that as long as the secondary user interference I_0 is below the interference cap set by the primary user, the primary user quality of service will be guaranteed. To ensure this the utilities of secondary users must be fast decaying functions of $I_0 - Q_0$ when this difference is positive. From a system perspective the secondary system would be interested in maximizing the sum capacity of all its users. However, from a particular user's perspective it would be interested in gaining the maximum possible data rate it can get. Thus, each user would like to maximize its own achieved capacity. Motivated by this argument,we propose the following utility function for the secondary users:

$$u_k\left(p_k, \mathbf{a}_{-k}\right) \quad = \quad \frac{f(p_k)}{1 + e^{\lambda(I_0 - Q_0)}} \tag{4}$$

where $f(.)$ is a suitable reward function chosen by the secondary system and the weighting term $\frac{1}{1+e^{\lambda(I_0-Q_0)}}$ in (4) is a sigmoidal function with the property that it goes to either +1 or 0, as $I_0 - Q_0$ tends to either negative or positive infinity, respectively, while parameter λ can be used to adjust the steepness of the transition region.

Again we may use $f(p_k) = W\log(1+\gamma_k)$ as a special case of (4), where the received SINR of the k-th secondary user, for $k = 1, \cdots, K$, is given by $\gamma_k = \frac{|h_{sk}|^2 p_k}{i_k + \tilde{\sigma}_s^2} = \frac{p_k}{N_k}$ where $\tilde{\sigma}_s^2 = \left(\rho_{k0}^{(s)}\right)^2 |h_{s0}|^2 p_0 + \sigma_s^2$ is the effective noise seen by the k-th user and $N_k = \frac{i_k + \tilde{\sigma}_s^2}{|h_{sk}|^2}$. Hence,

$$u_k\left(p_k, \mathbf{a}_{-k}\right) = \frac{W\log\left(1 + \frac{p_k}{N_k}\right)}{1 + e^{\lambda(I_{0,-k}(\mathbf{a}_{-k}) - Q_0)} e^{c_k p_k}}, \tag{5}$$

where $W > 0$ is a scaling parameter that can be taken as proportional to the bandwidth, $I_k^{(s)}\left(\mathbf{a}_{-k}\right) = i_k + \left(\rho_{k0}^{(s)}\right)^2 h_{s0}^2 p_0$ is the total interference seen by the k-th user from both primary as well as other secondary users and $I_{0,-k}\left(\mathbf{a}_{-k}\right) = I_0 - q_k \left(\rho_{0k}^{(p)}\right)^2 h_{pk}^2 p_k = \sum_{j \neq k} q_j \left(\rho_{0j}^{(p)}\right)^2 h_{pj}^2 p_j$ is the interference from all secondary transmissions at the primary receiver excluding that from the k-th secondary user. In writing (5) we have defined $c_k = \lambda q_k \left(\rho_{pk}^{(p)}\right)^2 h_{pk}^2 \geq 0$.

IV. Existence of a Nash Equilibrium and the Best Response Adaptation

A. Existence of a Nash Equilibrium

In the following we investigate equilibrium strategies on the game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$, where users are interested in maximizing the utility functions in (3) and (5). Of course the most commonly used equilibrium concept in game theory is the Nash equilibrium:

Definition 1: A strategy vector $\mathbf{p} = (a_0, a_1, \cdots, a_K)$ is a Nash equilibrium of the primary-secondary user power control game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$ if, for every $k \in \mathcal{K}$, $u_k\left(a_k, \mathbf{a}_{-k}\right) \geq u_k\left(a'_k, \mathbf{a}_{-k}\right)$ for all $a'_k \in \mathcal{A}_k$.

The best response correspondence of a user gives the best reaction strategy a rational user would choose in order to maximize its own utility, in response to the actions chosen by other users.

Definition 2: The user k's best response $r_k : \mathcal{A}_{-k} \longrightarrow \mathcal{A}_k$ is the set

$$r_k\left(\mathbf{a}_{-k}\right) \quad = \quad \left\{a_k \in \mathcal{A}_k : u_k\left(a_k, \mathbf{a}_{-k}\right) \geq u_k\left(a'_k, \mathbf{a}_{-k}\right)\right\} \tag{6}$$

where $a'_k \in \mathcal{A}_k$ for all $k = 0, 1, \cdots, K$. Note that, with the assumed form of action sets $\mathcal{A}_k$, it is easy to see that they are both compact and covex for all $k = 0, 1, \cdots, K$ [2]. Further, both $u_0(\mathbf{p})$ and $u_k(\mathbf{p})$ are continuous in the action vector $\mathbf{p}$.

For the existence of a Nash Equilibrium, the remaining condition that we need to ensure is the quasi-concavity of u_k's for all $k = 0, 1, \cdots, K$. Let us define a function $\Phi\left(Q_0\right)$ as,

$$\Phi(Q_0) \quad = \quad \frac{F(Q_0)}{F'(Q_0)} + Q_0\,. \tag{7}$$

It can be seen that u_0 has a local maximum that is indeed a global maximum if $\Phi(Q_0) = \bar{Q}_0 + I_0\left(\mathbf{a}_{-0}\right)$ has only one solution for $Q_0 \in \mathcal{Q}$. Clearly this equation has a solution if

$\Phi(Q_0)$ is continuous and $\lim_{Q_0\to 0}\Phi(Q_0) \le \bar{Q}_0 + I_0(\mathbf{a}_{-0}) < \lim_{Q_0\to\infty}\Phi(Q_0)$. This solution would be a global maximum if in addition $\Phi'(Q_0) > 0$ for $Q_0 > 0$. It can be easily verified that $\Phi'(Q_0) > 0$ will be true if $F(Q_0)$ is such that $\frac{F(Q_0)F''(Q_0)}{(F'(Q_0))^2} < 2$. Note also that $\lim_{Q_0\to\infty}\Phi(Q_0) = \infty$ if $\lim_{Q_0\to\infty}\frac{F(Q_0)}{F'(Q_0)} > -\infty$. Hence, a set of necessary conditions on $F(Q_0)$ for u_0 to be quasi-concave is:

1. $F(Q_0)$ is continuous and strictly monotonic for $Q_0 > 0$
2. $F(0) = 0, F'(0) > 0$ and $\lim_{Q_0\to\infty}\frac{F(Q_0)}{F'(Q_0)} > -\infty$
3. $\frac{F(Q_0)F''(Q_0)}{(F'(Q_0))^2} < 2$ for $Q_0 > 0$
4. $0 \le \bar{Q}_0 + I_0(\mathbf{a}_{-0}) < \infty$.

As can be seen from (5), clearly $u_k(\mathbf{a})$ is continuous in $\mathbf{a}$. Next, consider its first order derivative w.r.t. p_k:

$$\frac{\partial u_k(p_k)}{\partial p_k} = \frac{W e^{c_k p_k} g_k(p_k)}{\left(1+\frac{p_k}{N_k}\right)\left(1+e^{\lambda(I_{p,-k}-Q_0)}e^{c_k p_k}\right)^2}. \quad (8)$$

Note that at an interior local extremum point for $p_k \in [0,\infty)$, we should have $g_k(p_k) = \frac{1}{N_k}e^{-c_k p_k} + \frac{1}{N_k}e^{\lambda(I_{p,-k}-Q_0)} - c_k e^{\lambda(I_{p,-k}-Q_0)}\left(1+\frac{p_k}{N_k}\right)\log\left(1+\frac{p_k}{N_k}\right) = 0$. Since $g_k(0) = \frac{1}{N_k}\left(1+e^{\lambda(I_{p,-k}-Q_0)}\right) > 0$, $g_k(\infty) \longrightarrow -\infty$ and $g_k(p_k)$ is continuous in p_k, clearly function $g_k(.)$ must have at least one zero crossing. However, since $g_k'(p_k) < 0$ for $p_k \ge 0$, there is exactly one zero of $g_k(p_k)$ on $[0,\infty]$, implying that $u_k(p_k)$ only has one local extremum point on $p_k \in [0,\infty)$. If this extremum point is denoted as p^*, since $g_k(p_k) > 0$ for $0 \le p_k < p^*$, we have that $u_k(p_k)$ is monotonic increasing on $0 \le p_k < p^*$. Similarly, since since $g_k(p_k) < 0$ for $p_k > p^*$, we have that $u_k(p_k)$ is monotonic decreasing on $p_k > p^*$. Hence, it follows that the local extremum point $p_k = p^*$ is indeed a global maximum of $u_k(.)$ on $[0,\infty)$, implying that $u_k(p_k)$ is quasi-concave in p_k, for each $k = 1,\cdots,K$.

Thus with $\mathcal{A}_k$'s and u_k's as defined above, the *dynamic spectrum leasing* game defined by (3) and (5) has a Nash Equilibrium.

B. Best Response

The best response of the primary user is obtained by setting $u_0'(Q_0) = 0$. The unique interior solution is given by

$$Q_0^*(I_0) = \left(\bar{Q}_0 + I_0\right) - \frac{F(Q_0^*)}{F'(Q_0^*)}. \quad (9)$$

Since $u_0(Q_0)$ is monotonic increasing for $Q_0 < Q_0^*$, if the maximum interference cap is such that $\bar{Q}_0 < Q_0^*$, the best response is given by

$$r_0(\mathbf{a}_{-0}) = \min\{\bar{Q}_0, Q_0^*(I_0)\}. \quad (10)$$

Thus, in order to determine its best response for a chosen power vector by the secondary users , the only quantity that the primary user needs to know is the total secondary interference I_0 at the primary receivers given in (1). On the other hand, the best response of the k-th secondary user to the transmit powers of the other secondary users as well as the interference cap set by the primary user is given by the (unique) solution $p_k = p_k^*(Q_0, I_{0,-k}, i_k)$ to the equation $g_k(p_k) = 0$.

Again, since u_k is quasi-concave in p_k, if $p_k^*(Q_0, I_{0,-k}) > \bar{P}_k$, the best response of the k-th secondary user, for $k = 1,\cdots,K$, is:

$$r_k(\mathbf{a}_{-k}) = \min\{\bar{P}_k, p_k^*(Q_0, I_{0,-k}, i_k)\}. \quad (11)$$

Observe that in general the best response of the k-th secondary user is a function of the residual interference $Q_0 - I_{0,-k}$ of all other secondary users at the primary receiver and the total interference i_k from all secondary and primary users to the k-th user's received signal at the secondary receiver. In this work, we assume that the primary base station broadcasts both Q_0 and I_0 whenever it adjusts its interference cap to a new value. This is the only interaction that the primary system will need to have with the secondary system. Observe that by knowing, I_0, each secondary user can compute the residual interference $I_{0,-k} = I_0 - q_k\left(\rho_{0k}^{(p)}\right)^2 h_{pk}^2 p_k$ since it knows its own transmit power and it may estimate the channel state information (CSI) if the reverse link signals are available in the same band. Also, the approximation $I_{0,-k} \approx I_0$ proposed in [2] can be shown to perform reasonably well in practice, especially when the number of secondary users K is sufficiently large, as we will see in Section V below.

V. Performance of the Proposed Dynamic Spectrum Leasing System

In this section, we consider a dynamic spectrum leasing cognitive radio system that employs the proposed game-theoretic framework for their interactions and show how the resulting equilibrium point of the proposed game leads to a desirable coexistence. We look at the practical situation in which there are non-identical secondary users in the presence of wireless channel fading. It is expected that in this case the Nash equilibrium transmit powers of individual secondary users will be different for each user. We have assumed all channel gains in the system to follow Rayleigh distribution with all channel coefficients normalized so that $\mathbb{E}\left\{h^2\right\} = 1$. This essentially allows us to consider, without any loss in generality, the transmit powers p_k to be equal to the average received power (averaged over fading).

In Fig. 1, we have shown the game outcome at the Nash equilibrium as a function of the number of secondary users K, both with and without CSI for $\lambda = 5$. It is to be noted from Fig. 1 that, with CSI available to the secondary users, the primary system can accommodate up to 12 secondary users without violating the $I_0 \le Q_0$ requirement. As we may also observe from Fig. 1, the system that does not rely on the knowledge of CSI demonstrate almost the same performance trends at the equilibrium as compared to that with exact CSI. It seems that another effect of not having the exact $I_{0,-k}$ is that the safety margin $Q_0 - I_0$ at the equilibrium is slightly larger. This is essentially due to the fact that each secondary user believes an exaggerated residual interference $I_{0,-k}$ making it to decrease its transmit power

Fig. 2 shows the corresponding primary and secondary user utilities achieved at the Nash equilibrium in the presence of

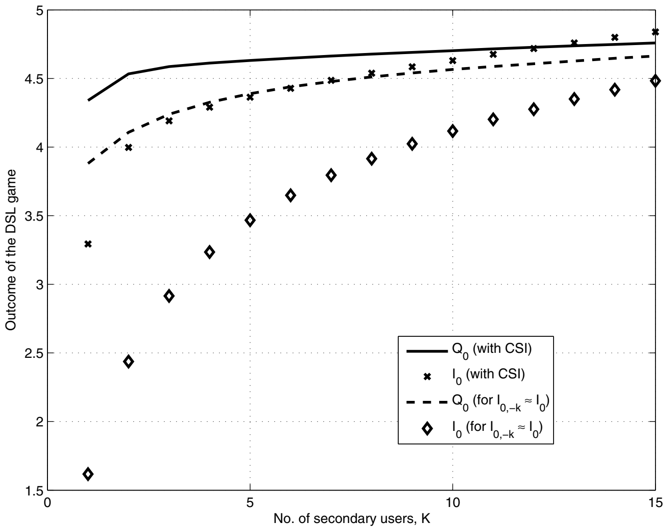

Fig. 1. Outcome of the DSL game at the system Nash equilibrium, with both exact CSI and using approximation $I_{0,-k} \approx I_0$, as a function of secondary system size K in the presence of channel fading, when $F(Q_0) = \log(1 + Q_0)$ and $f(p_k) = W \log\left(1 + \frac{p_k}{N_k}\right)$. Other parameters are $\lambda = 5$, $W = 1$, $\bar{Q} = 10$, $\bar{P}_k = 12$, $\bar{\gamma}_0 = 1$, $q_k = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$ for all k, and $\sigma_s^2 = \sigma_p^2 = 1$.

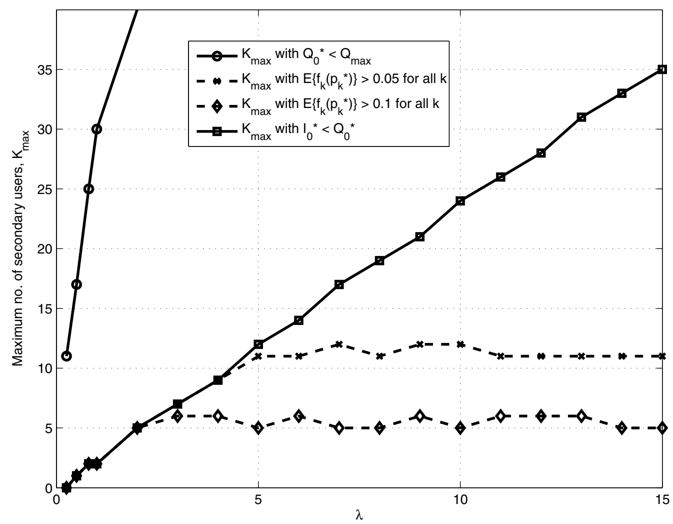

Fig. 3. Maximum secondary system size supportable for a required QoS requirement f_{min} and without violating the primary interference cap as a function of λ in the presence of channel fading with exact CSI, when $F(Q_0) = \log(1 + Q_0)$, $f(p_k) = W \log\left(1 + \frac{p_k}{N_k}\right)$. Other parameters are $W = 1$, $\bar{Q} = 10$, $\bar{P}_k = 12$, $\bar{\gamma}_0 = 1$, $q_k = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$ for all k, and $\sigma_s^2 = \sigma_p^2 = 1$.

channel fading. It can be seen from Fig. 2(a) that primary user utility is reduced when secondary system does not have the exact knowledge of the channel state information. Both the sum-rate $\sum_{k=1}^{K} \log\left(1 + \frac{p_k^*}{N(p_k^*)}\right)$ as well as the per-user rate $\frac{1}{K}\sum_{k=1}^{K} \log\left(1 + \frac{p_k^*}{N(p_k^*)}\right)$ achieved by the secondary system at the Nash equilibrium, with and without exact CSI are shown in Fig. 2(b). It is to be noted that, due to interference averaging in the presence of fading, in this case the secondary system is able to achieve better sum- and per-user rates compared to that with non-fading channels (not shown here to save space). This is because of interference averaging effect due to fading that de-emphasized the interference among users leading to better SNR. It can be seen that the per-user rate is typically a decreasing function of the increasing secondary system size. It is because all secondary users in the system must share the allowed interference level set by the primary system.

Fig. 3 shows the maximum secondary system size K_{max} supportable by the primary system at the Nash equilibrium without violating the $I_0^* \le Q_0^*$ requirement as a function of λ. We have also included in Fig. 3 the maximum tolerable secondary system size by the primary system before maximum interference cap $\bar{Q}_0$ is exceeded at the equilibrium. Although not desirable, it shows the maximum number of secondary users the primary system can support until Q_0^* reaches $\bar{Q}_0$ for a particular value of λ. As we may observe from Fig. 3 that when $\lambda < 1$ there is a likelihood that the primary interference cap Q_0 or the maximum allowable interference cap $\bar{Q}_0$ might be exceeded by an even relatively smaller size secondary system. However, as we can see from Fig. 3 when $\lambda > 1$, the number of secondary users who simultaneously transmit without violating the primary interference condition dramatically increases. In essence, K_{max}'s corresponding to $Q_0^* \le \bar{Q}_0$ and $I_0^* \le Q_0^*$ provide safety margin for desirable coexistence of the primary and secondary system.

We define the required QoS in the secondary system in terms of the average (over fading) reward achieved by a user at the equilibrium. We denote this minimum required QoS of all secondary users as f_{min}. In Fig. 3, we also show the maximum secondary system size K in the presence of fading for different minimum quality of service requirements f_{min} as a function of the parameter λ with exact CSI.

Fig. 4 shows the outage probability $Pr\left(f_k\left(p_k^*\right) < f_{min}\right)$ of a typical secondary user as the secondary system size increases with exact channel state information. It is seen from Fig. 4 that the outage probability increases with K as well as with the minimum QoS requirement. The maximum secondary system size which can be supported according to Fig. 3 thus needs to be interpreted in conjunction with the outage probabilities shown in Fig. 4. For example, although as shown by Fig. 3, about 5 secondary users can (on average) meet the $f_{min} = 0.1$ QoS requirement with $\lambda = 5$, according to Fig. 4 each of the these users may be in outage about 65% of time. This, of course, is the price of operating as a secondary system. However, outage probability with the model proposed in this paper is slightly better than that in [2].

VI. Conclusion

In this paper, we developed a new game-theoretic framework for dynamic spectrum leasing in cognitive radio networks. In our proposed framework we further generalized

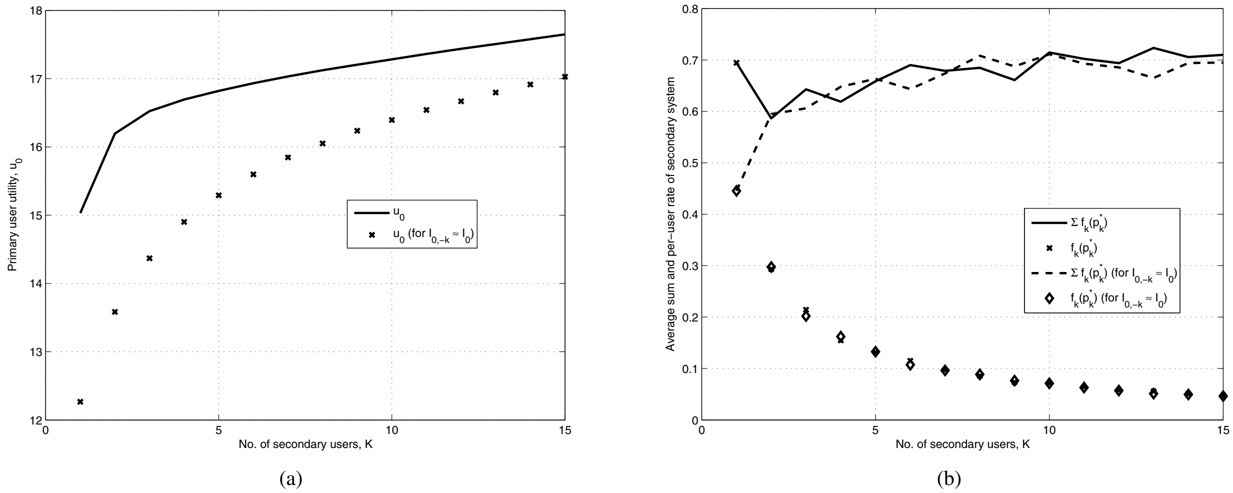

Fig. 2. Primary and secondary utilities at the system Nash equilibrium as a function of secondary system size K for $\lambda = 5$ in the presence of channel fading when $F(Q_0) = \log(1 + Q_0)$ and $f(p) = W \log\left(1 + \frac{p}{N(p)}\right)$. (a) Primary user utility, (b) Average sum-rate and the per-user rate achieved by the secondary system at the Nash equilibrium with $W = 1$, $\bar{Q} = 10$, $\bar{P}_k = 12$, $\bar{\gamma}_0 = 1$, $q_k = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$ for all k, and $\sigma_s^2 = \sigma_p^2 = 1$..

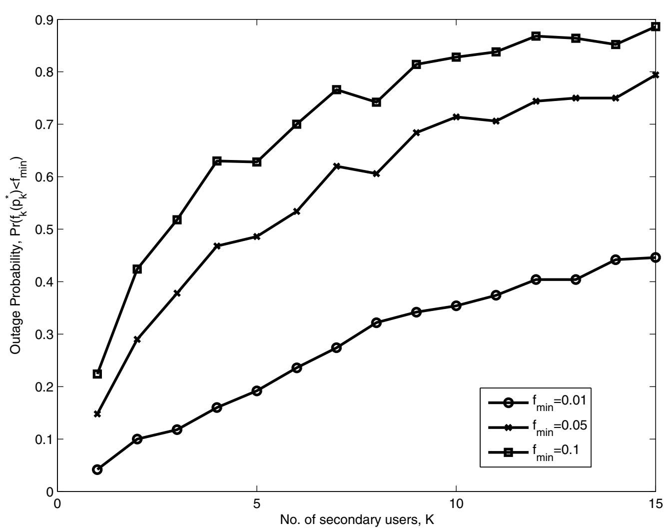

Fig. 4. Outage probability $Pr\left(f_k\left(p_k^*\right) < f_{min}\right)$ of a typical secondary user at the Nash equilibrium of the DSL game in fading channels as a function of secondary system size K for a required QoS requirement f_{min} with $f(p_k) = W \log\left(1 + \frac{p_k}{N_k}\right)$. Other parameters are $W = 1$, $\bar{Q} = 10$, $\bar{P}_k = 12$, $\bar{\gamma}_0 = 1$, $q_k = 1$, $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$ for all k, and $\sigma_s^2 = \sigma_p^2 = 1$.

the primary user utility function defined in [2], [3]. We also defined a new utility function for the secondary system. We formulated the dynamic spectrum leasing cognitive system as a non-cooperative DSL game between the primary and the secondary users and established the conditions on the existence and uniqueness of the Nash equilibrium. Next, we analyzed a cognitive radio DSL network in the presence of fading in detail to investigate the proposed system behavior at equilibrium. For such a system, we observed that the proposed dynamic spectrum leasing leads to a design that determines the maximum number of secondary users based on the required minimum QoS of the primary user.

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